

Modeling and Simulation of Bridge – Track – Train Systems at High Service Velocities with LS-DYNA[®]

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Abstract

The paper develops a new methodology of FE modeling and simulation of the bridge – track – train systems at high service velocities with the use of selected CAE systems. The methodology is presented on the KNI 140070 viaduct with composite (steel – concrete) superstructure and 14.40 m span length, located on the Central Main Line, Poland. A ballasted track and two types of high speed trains have been modeled physically and numerically. The study includes German ICE-3 (InterCityExpress) train with classic bogies and Korean KTX (Korea Train eXpress close to French TGV) train with classic and Jacobs bogies. A methodology of the FE modeling and simulation of the bridge – track – moving train system is based on the following concept. The physical and numerical modeling of the viaduct – track – train system was performed with Altair HyperMesh[®] and LS-PrePost[®] software. The FE model of the bridge superstructure consisted of 4-node shell elements (main beams) and 8-node 48 DOF solid elements (RC platform). In order to simulate the moving train – track interaction, RAIL_TRACK and RAIL_TRAIN modules available in LS-DYNA system were used. Hughes-Liu beam elements were used for rail modeling whereas rail fastenings were simulated using one-dimensional discrete spring and damper elements. Carbodies, bogie frames and wheelsets were considered as rigid bodies and they were modeled using shell and beam elements. Cylindrical and revolute constrained joints and discrete springs and dampers were applied to connect components of the FE model of rail-vehicles. In the longitudinal direction, the FE mesh of the system is based on a 600 mm length module. DYNAMIC_RELAXATION is omitted via applying the static wheel loads increasing in the cosine shape in the 2-sec initial time interval. The quasi steady-state wave in the track is generated after the initial time interval. Dynamic response of the bridge – track – train system is registered in the form of displacement and acceleration time-histories at the design cross-sections as well as displacement and stress contours in reference to main steel beams.

Introduction

In spite of a large number of papers on design, dynamics, service and maintenance of railway bridges, there still occur serious problems with durability protection of bridge superstructures, tracks and approach zones. First of all, it results from complexity of bridge – track – moving train (BTT) systems which nonlinear models are described by a huge number of parameters. Many of these parameters, describing fasteners, ballast, subsoil layers, rail-vehicles' suspensions, track irregularities, settlements etc., are still known partly and are difficult for identification. Producers and research institutions involved in modern high-speed trains do not bring to light structural details, values of parameters or their research results. These circumstances make realistic prediction of dynamic response of bridges to high-speed moving trains difficult.

In the 2nd half of the 20th century scientists mostly developed analytic-numerical methods in dynamics of railway bridges, summarized in Refs. [1, 2]. They modeled transient and quasi steady-state vibrations of the bridge – train or bridge – track – train systems, without or with snaking of wheel sets taken into account. Problem-oriented computer programs were created and used for simulations.

At present, one may observe various numerical approaches to dynamics of railway bridges but commercial CAE systems based on FEM are still not used in this field, e.g. [2–7]. Authors of Ref. [3] analyzed dynamic response of a simply-supported Euler beam to the passage of the mass elements stream applying the Galerkin method. A 2D linear model of the bridge – track – moving train system has been developed in Ref. [4] using Euler’s beam finite elements and double-mass oscillators. A 750 m long cable-stayed railway bridge is modeled in Ref. [5]. The authors assumed a 6DOF Matsuura model as the basic model of rail-vehicles. A 3D modeling of the bridge – track – moving train system is presented by Zhang *at al.* in Ref. [6]. Like as in previous references, the writers applied the beam mesh for the bridge superstructure and a 3D multi-body model with vertical and horizontal suspensions for rail-vehicles. Song and Hoi [7] developed numerical modeling and simulation of the double-track – TGV train system. They considered continuous beam bridges discretized with 6DOF beam finite elements.

Summing up, vibrations of BTT systems are transient, spatial and nonlinear. Modern bridge spans are designed separately for each track and have quasi-symmetrical cross-sections. Structural solutions in reference to ballasted rectilinear tracks and high-speed trains lead to negligible lateral vibrations of rail-vehicles. In these circumstances, assuming vibrations of the bridge – track – high-speed train system to be 3D but symmetric with respect to the vertical longitudinal plane of symmetry is reasonable. In order to obtain realistic predictions of dynamic behavior of the bridge – track – high-speed train system, computer aided dynamic calculations should be applied. It should be also noted that application of advanced CAE systems in dynamics of bridges is at early stage of its development.

The study develops a new methodology of FE modeling and simulation of transient vibrations of the composite (steel – concrete) bridge – ballasted track – high-speed train system making the use of advanced CAE systems. The KNI 140070 viaduct of span length 14.40 m, located on the Polish Central Main Line (PCML), has been selected for presentation of this methodology. The ballasted track serviced currently on PCML has been redesigned by the authors in order to accommodate it to high service speeds of trains. Two types of high-speed trains have been taken into consideration — German ICE-3 (InterCityExpress) with classic bogies and Korean KTX (Korea Train eXpress modeled on French TGV) with classic and Jacobs bogies.

Description of the BTT System

The viaduct No. KNI 140070, located at 200.794 km on the PCML No. 4 – E 65, was selected for numerical modeling and simulation [8]. There is considered the bridge span under track No. 1 (Figure 1). After recent modernization the viaduct has $k = +2$ rail-line classification coefficient. A track spacing equals 4.57 m, the spans are of 14.40 m theoretical length and 15.34 m total length. Four main beams are 0.77 m high and made of St3M steel. Bottom flanges have been enforced with additional cover plates. The thickness of a new RC platform ranges from 0.29 m in the track axis to 0.25 m at the side wall. The platform is made of C35 concrete reinforced with AII/18G2-b steel rebars. The side wall is made of C30 concrete and has vertical dilatations at 1/4, 1/2 and 3/4 of the span length. The track structure before modernization consists of UIC 60 service rails, sleepers with SB3 fasteners, and ballast of the first class.

In order to conform the track to high service velocities up to 300 km/h, theoretical modernization of the track has been designed, as shown in Figure 2. The 60E1 main rails are fixed to B 320 U60

and B 320 U60–U sleepers with Vossloh 300-1 fasteners. The 60E1 side rails, enclosing the span and the approach slabs, are fixed with SB3 fasteners. The ballast layer is 0.35 m thick under sleepers. The RC (C30 concrete, 18G2 rebars) approach slabs have dimensions $10.2 \times 4.8 \times 0.2$ m and each of them is supported on an abutment with four elastomeric bearings. The embankment in the approach zones contains cement-stabilized subsoil, while outside the approach zones an upper 0.20 m thick sand-gravel mix layer has been applied.



Figure 1: The modernized viaduct No. KNI 140070 composed of two separate spans, located at 200.794 km on the PCML — a side view (*left*) and view on the main steel beams with the bracing system (*right*)

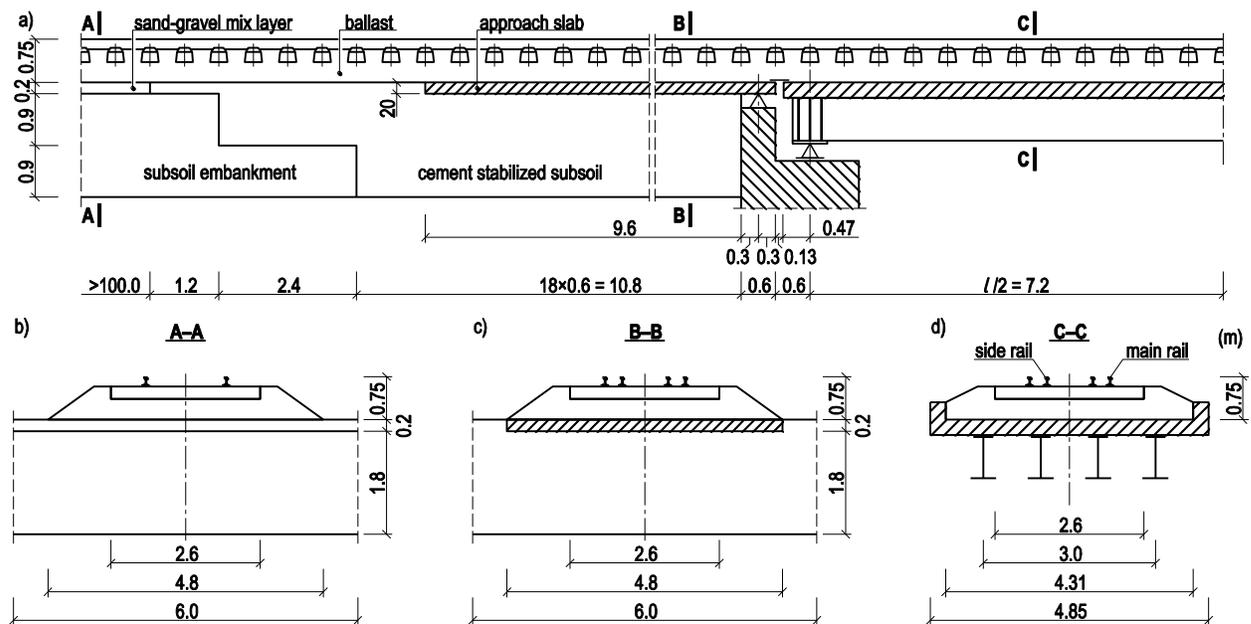
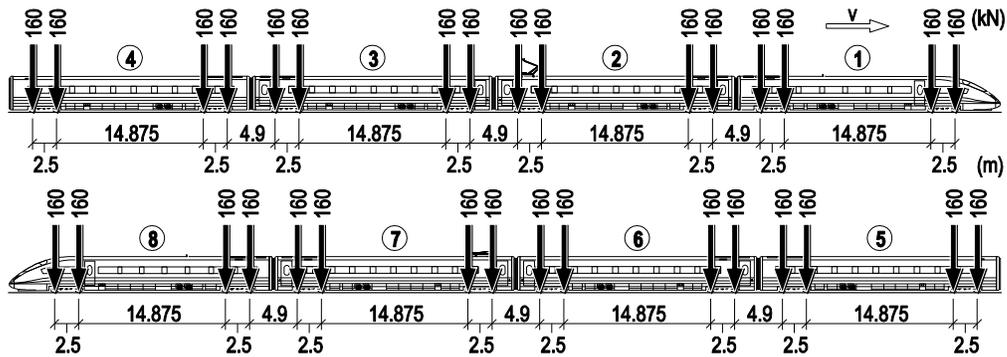


Figure 2: The modernized ballasted track in the KNI 140070 viaduct zone: a longitudinal section (a); a cross-section on the regular track (b), in the approach zone (c), and over the bridge span (d)

The ICE-3 high-speed trains, as presented in Figure 3a, were built by Siemens Company in 2000 and 2001 in the total number of 50 trains (Series 1). The ICE-3 is the third generation of these trains. The main difference in comparison to the previous generations is a multiple unit power system. The train has no end power heads as the ICE-1 train, but it has motor bogies located every second car. It results in improved operating parameters. The total weight is distributed evenly across the entire trainset, therefore, the axle load is reduced to 16 metric tons [11].

A KTX (Korea Train eXpress) high-speed train, operated by Korean Railways, is a modification of the TGV *Réseau* train [17], but it is longer than its French archetype. The trainset consists of 20 cars, in which the first and the last one are the power units and an additional motorized bogie is located in each intermediate car close to the power unit. Hence, the total number of motorized bogies is six for 20-car trainset. A schematic diagram of the KTX trainset is depicted in Figure 3b. The carriages are equipped with the Jacobs bogies which are common for two adjacent cars. The KTX train is supported on 6 motor bogies – type Y230A, and 17 articulated bogies (Jacobs bogies) – type Y237A. Static axle loads for the KTX trainset are depicted in Figure 3b [18]. The top speed in service equals 300 km/h.

a)



b)

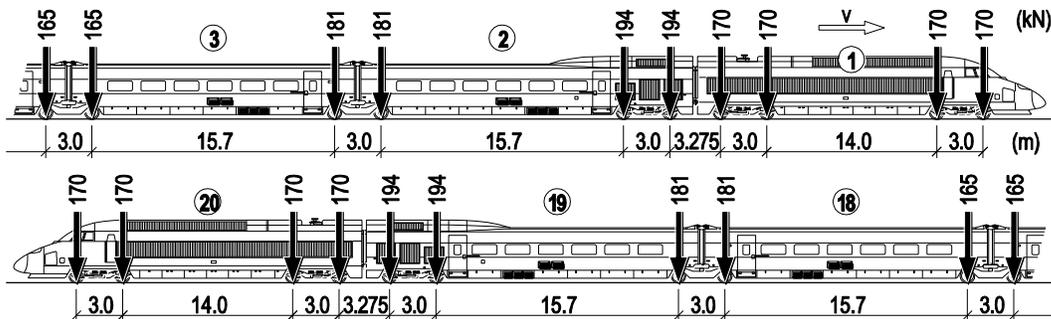


Figure 3: Schematic diagrams and static axle loads of ICE-3 (a) [11] and KTX (b) [18] trains

Physical and FE Modelling of the BTT System

The following concept in physic modeling of the viaduct has been developed. The reinforcement of the RC platform and the side RC wall is distributed quasi-uniformly in the specified platform sectors. The slab and the wall of the bridge platform is homogenized according to the mixtures rule [12]. After homogenization, the slab and the wall are reflected by linear viscoelastic orthotropic materials described by 3 Young's modules, 3 Poisson's ratios, and 3 shear modules in each subarea. A cross-section of the platform slab is approximated by a rectangle of 270 mm height. The platform is symmetrized via replacing a single dilated wall with two smaller dilated walls on both sides of the platform slab. The original and approximate structures have the same values of mass, cross-section area and geometrical moments of inertia. The material of steel main beams is linear viscoelastic and isotropic. The vertical and horizontal bracing in the main beams set are

neglected, hence all bearings on the abutments are assumed to be unmovable in the lateral direction.

Schemes of the KNI 140070 viaduct in the xz plane are depicted in Figure 4 where all elements taken in the FE modeling are marked, i.e. the homogenized platform (the slab and the walls), the main beams set, the vertical ribs welded to webs of the main beams, the horizontal bearing plates welded to the bottom flanges of the main beams over the bearings.

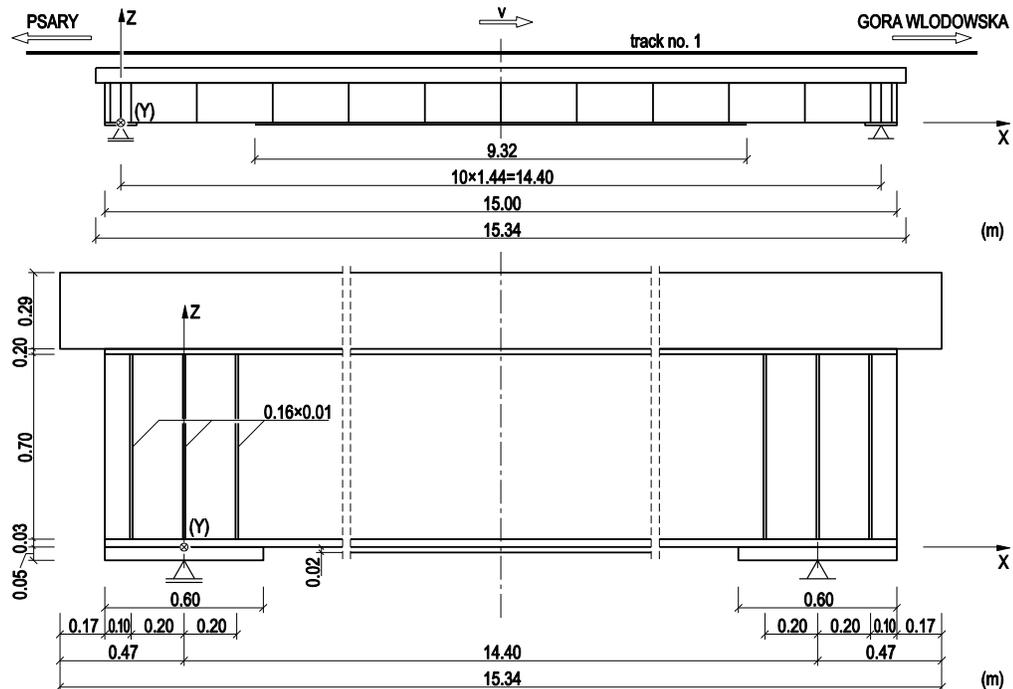


Figure 4: The longitudinal sections of the KNI 140070 viaduct in the XZ plane

Figure 5 illustrates the original (grey lines) and symmetrized (black lines) cross-sections, the longitudinal section of the viaduct and the symmetrized reinforcement close to the original one. The mixture rule for homogenization of the RC platform has been applied and four homogenized orthotropic subareas in the cross-section have been distinguished with the orthotropy directions coinciding the x , y , z directions.

The FE model of the bridge superstructure has been created in Altair HyperMesh software (Figure 6). The numerical model of the bridge superstructure consists of 3896 4-node shell elements (steel main beams) and 5568 8-node 48 DOF solid elements (the homogenized RC platform divided into orthotropic parts). Cover plates of the bottom beam flanges are modeled as an additional layer of finite shell elements. Roller bearings on the left support and pivot bearings on the right support have been reflected by respective constraints.

The following assumptions have been made in physic modeling of the track. The rail-line axis is rectilinear, and — in the unloading state — the service rails are rectilinear. No rail surface irregularities appear. Vibrations of the track are small and symmetric with respect to the vertical xz plane. The service and side rails are prismatic beams deformable in flexure and shear, made of linear viscoelastic material. Rail fastenings are simulated using massless one-dimensional nonlinear

discrete spring and damper elements oriented vertically. Sleepers are modeled as rigid beams. The crushed stone ballast is modeled as a layer of discrete nonlinear one-way viscoelastic elements (9 elements under each sleeper). The ballast mass is lumped in the FE nodes of the platform slab and the upper layers outside the bridge, respectively. The approach slabs are prismatic, modeled as linear viscoelastic isotropic continuum, and supported on the subsoil and on the abutments with non-deformable pivot bearings. Layers of the embankment are considered as a linear viscoelastic material continuum.

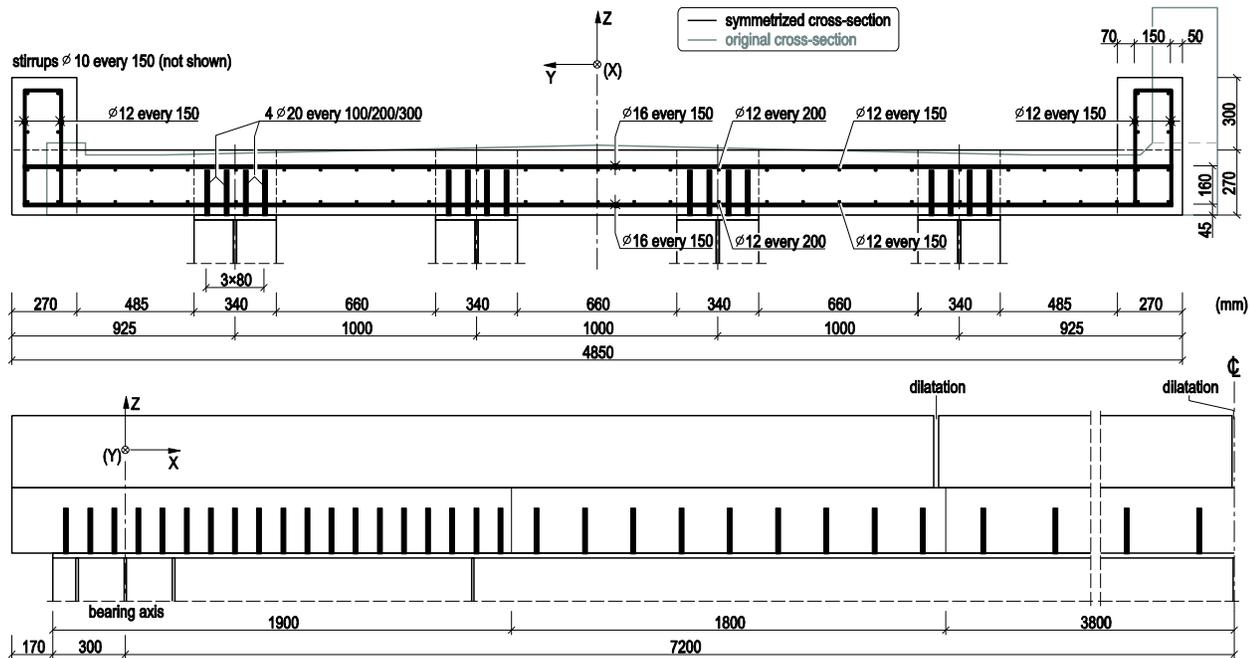


Figure 5: The original and symmetrized cross-sections (a), the longitudinal section (b) of the KNI 140070 viaduct and the symmetrized reinforcement

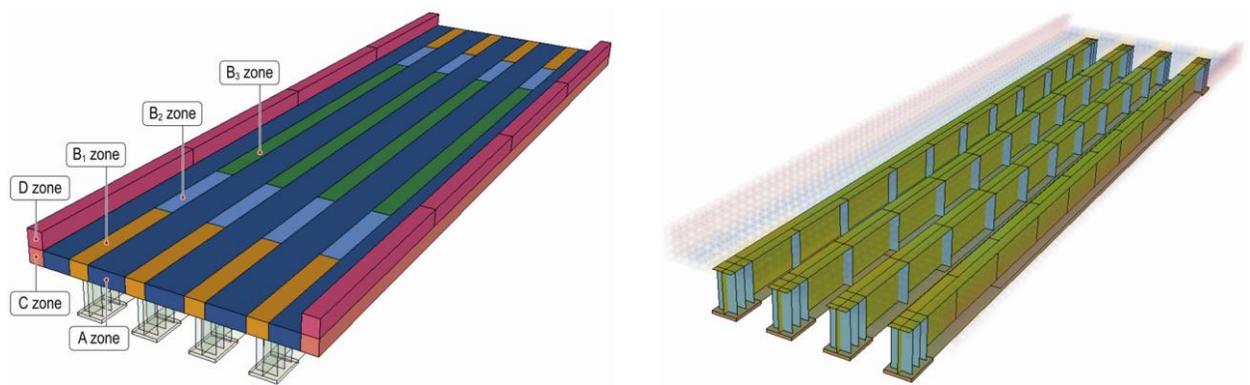


Figure 6: The FE model of the KNI 140070 viaduct superstructure

RAIL_TRACK and RAIL_TRAIN modules available in LS-DYNA [13] were applied for approximate modeling of the train – track interaction (without simulation of wheels’ rotation). The wheel – rail contact stiffness amounts to 2 MN/mm as suggested in Ref. [13]. Hughes-Liu beam elements (2-node elements with 12 DOFs [13]) were used for FE modeling of rails bent in the vertical planes. In order to declare a set of integration points for the rail cross-section, the

INTEGRATION card has been applied. For main and side rails a substitute I asymmetric cross-section was assumed, denoted in Ref. [13] as Type 10: I-Shape 1. The actual values of the centre-of-gravity location, the area and the geometrical moment of inertia with respect to the horizontal principal axis of the cross-section have been saved.

The FE numerical model of the modernized ballasted track has been created in HyperMesh and LS-PrePost software. The main dimensions of the track, the abutments and the embankment are depicted in Figure 2, whereas the FE model of the ballasted track in the approach zone is presented in Figure 7. Rail fasteners were simulated using massless one-dimensional discrete spring and damper elements oriented vertically [13]. The embankment has been reflected approximately by a rectangular prism with unmovable side and bottom boundary surfaces and meshed using 8-node 24 DOF solid elements [13]. Sleepers are modeled as rigid beams vibrating only vertically, using finite beam elements and respective constraints. The ballast layer has been divided into cubicoid columns in coincidence with FE mesh of the parts under the ballast. Each ballast column was reflected by a vertical set of nonlinear spring and damper elements. The lumped mass distribution for the ballast has been applied in the bottom set of the nodal points contacting the platform slab, the approach slabs and the top subsoil layers. The track section modeled numerically in the bridge – track – moving train system is 810 m long. In total, the FE track model contains 141,770 beam, shell, brick and discrete elements and 21,658 point mass elements.

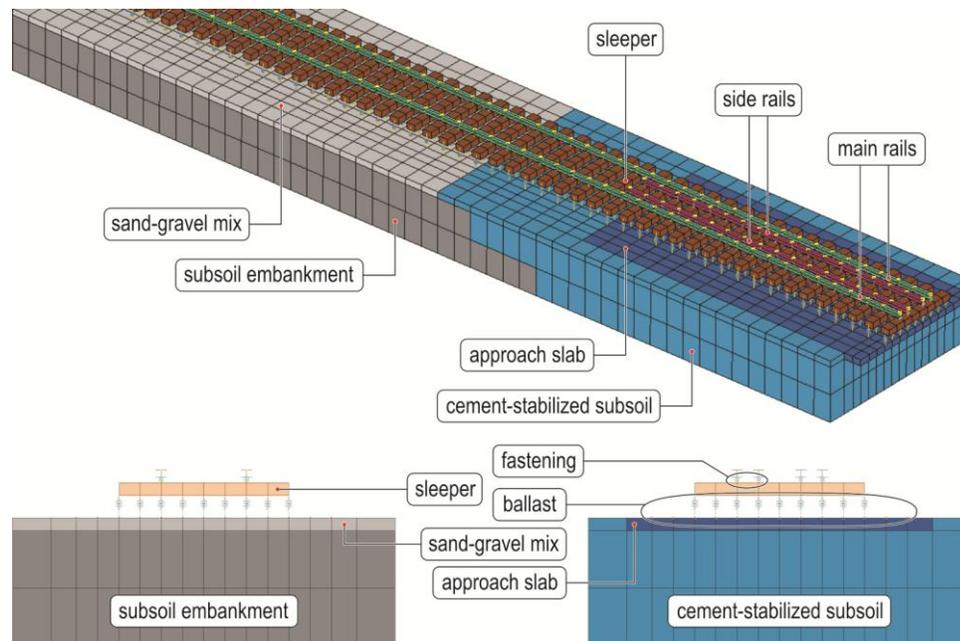


Figure 7: The FE model of the modernized ballasted track in the approach zone to the bridge

A numerical model of the train consists of the following components: carboodies, bogie frames, wheelsets, vertical massless discrete linear viscoelastic elements reflecting the primary and the secondary suspension systems. For example, a side-view scheme of the 3D model of the power car of the ICE-3 train is shown in Figure 8. All mass components were modeled using shell and beam elements and were treated as rigid bodies. Vibrations of the train units are symmetric with respect to the main longitudinal vertical plane of symmetry of the system. Respective constraints

have been put into the train model via incorporating translational `CONSTRAINED_JOINT_CYLINDRICAL` and rotational `CONSTRAINED_JOINT_REVOLUTE` elements [13].

Material parameters describing mass, elasticity, strength and damping properties of the system components have been predicted theoretically based on references including [1, 2, 9–11, 15–19].

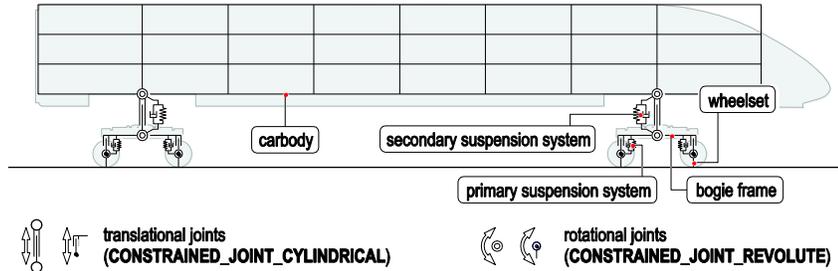


Figure 8: A side-view scheme of the 3D model of the power car of the ICE-3 train

A constant service velocity of the train FE model was declared in two steps. In the first one, for time equal to zero, the `INITIAL_VELOCITY` option was used, whereas in the second step for $t > 0$ the `PRESCRIBED_MOTION_RIGID` was applied for all car bodies and bogies FE models.

The FE models of car bodies, bogie frames and wheel sets were created in such a way that full conformity was achieved between the actual vehicles and their numerical models with respect to masses and principal mass moments of inertia. In total, the FE model of the 8-unit ICE-3 train contains 1568 beam, shell and discrete finite elements and 80 point masses.

In simulations, a non-effective `DYNAMIC_RELAXATION` option [13] has been replaced with loading the system by a set of vertical forces put in the moving vehicle – rail contact points, according to the formula:

$$P(t) = \frac{P_0}{2} \left(1 - \cos \frac{\pi t}{t_0} \right) \quad (1)$$

where P_0 is the static load of a single wheel onto the rail head, $t_0 = 2$ sec is a time of the static load increasing (see Figure 9). For $t > t_0$ vertical forces are constant, i.e. $P(t) = P_0$.

The total length of the track section modeled numerically has been assumed to be equal 810 m, and contains the following subsections: the initial train position (with zero static wheel load), the zone of increasing the static wheel pressures according to Eqn. (1), the train – track vibration stabilization zone (lasting 1 sec), 60 m long main zone (including the approach zones and the viaduct), the zone corresponding to the bridge free damped vibrations, the final train position zone.

Output quantities were registered using `HISTORY_NODE_SET` and `HISTORY_SHELL_SET` options. The sampling frequency amounts 1000 Hz in the main zone. Selected output quantities are depicted in Figure 10. Vertical displacements were registered in the nodal points D1 and D2, for the bottom flange of the main beam No. 2 and the main rail, respectively. Longitudinal normal stress in the bottom and top flanges of the beam No. 2 were registered in shell elements S1 and S2. It was decided to present the results for the internal beam since the obtained vibrations were slightly greater compared to the external main beams.

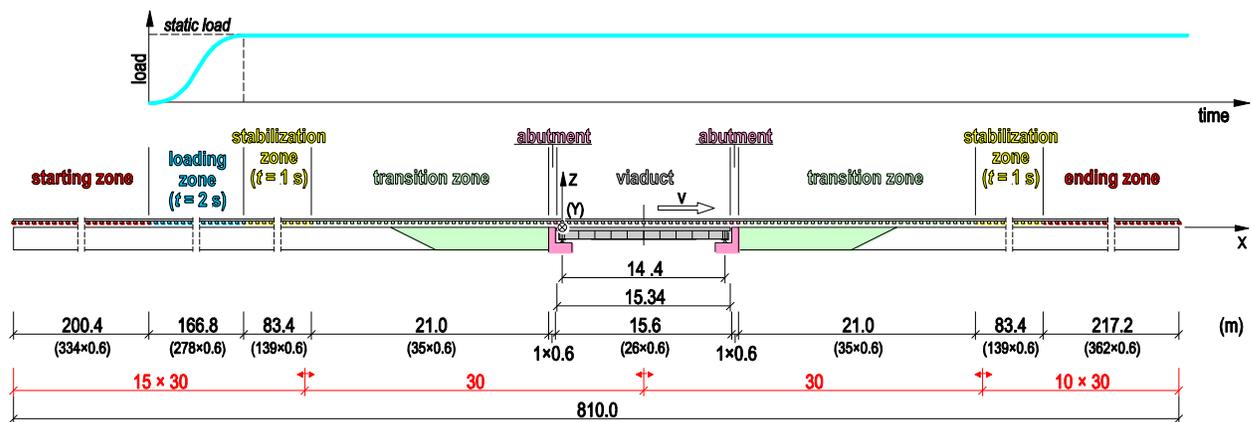


Figure 9: Subsections of the track section included in the FE modeling

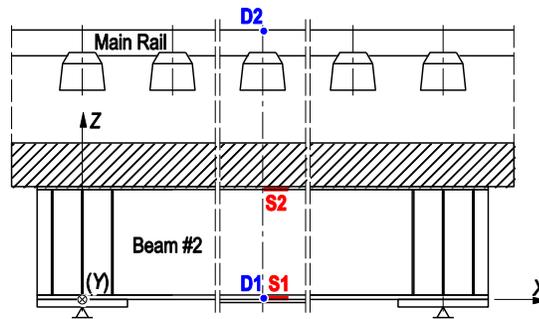


Figure 10: Selected quantities registered during the FE simulations

Simulations of Transient Vibrations of the BTT System

The computations have been made using the 120-processor supercomputer. At service velocity 300 km/h the real time equals 7.1 (ICE-3) and 6.7 sec (KTX), while the CPU time amounts to about 46 and 44 hours, respectively. The time step was equal to $7.13 \cdot 10^{-6}$ sec. Simulations of dynamic behavior of the system have been performed for service velocities 50, 100 km/h, and 150 – 300 km/h stepped by 10 km/h or 5 km/h.

The influence of the service velocity on dynamic response of the bridge span is illustrated in Figure 11 by extreme values of the design quantities, i.e. the dynamic deflection (D1) and the normal stress (S1) in the bottom flange of the internal main beam at the midspan. The system with the ICE-3 train exhibits two resonant velocities, i.e. ~195 and 260 km/h, whereas the system with the KTX train does not show valuable resonant speeds. Two velocities for the extreme deflections and normal stresses are marked in the graph in Figure 11.

Representative simulation results corresponding to the maximum service speed of 300 km/h and the resonant speed of 260 km/h for the system with the ICE-3 train are presented in Figures 12, 13. Selected results in the form of time-histories of vertical displacements are drawn in Figure 12. Selected contours of displacements and effective stresses are shown in Figure 13.

The registered longitudinal normal stresses are dynamic stresses in the native structural steel in the bottom fibers of the main beam at the midspan. These stresses are induced by the characteristic (real) active load in the form of a moving train. The load capacity condition for the native structural material (apart from welds and rivets) can be formulated classically taking into consideration the Schmidt high-cycle fatigue diagram which results in the following durability condition [2]:

$$\sigma_z = \sigma_g + \sigma_m + \beta \sigma_a \leq \sigma_d \tag{2}$$

where σ_z is equivalent normal stress, σ_g is static normal stress caused by characteristic weight of the bridge – track subsystem, σ_m is average normal dynamic stress caused by characteristic active load, β is fatigue coefficient, σ_a is amplitude of the dynamic normal stresses induced by characteristic active load, σ_d is admissible stress.

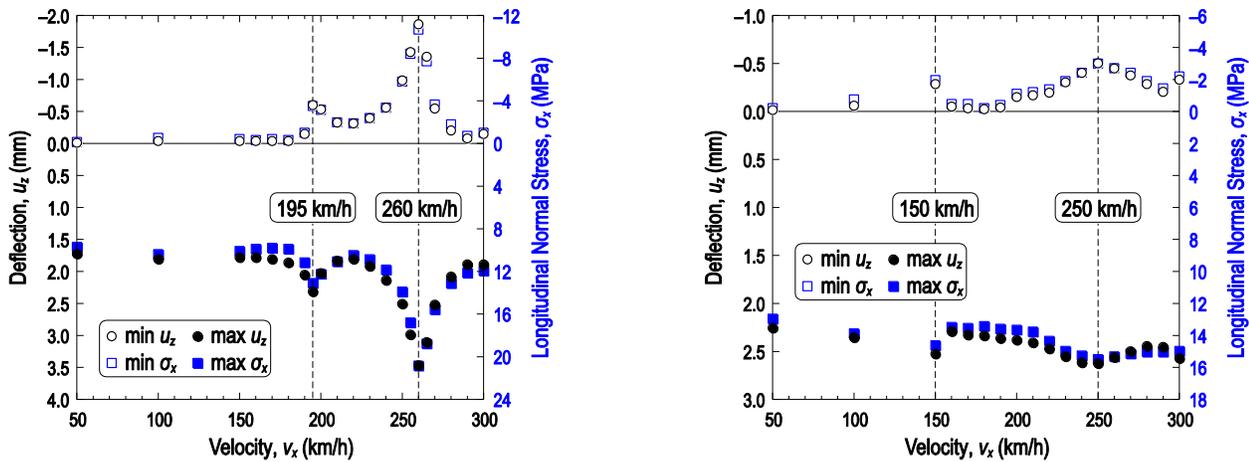


Figure 11: Extreme values of vertical deflections (D1) and longitudinal normal stresses (S1) in the bottom flange of the main beam No. 2 at the midspan, for selected values of the service velocity. The results correspond to the system with ICE-3 (*left*) and KTX (*right*) trains

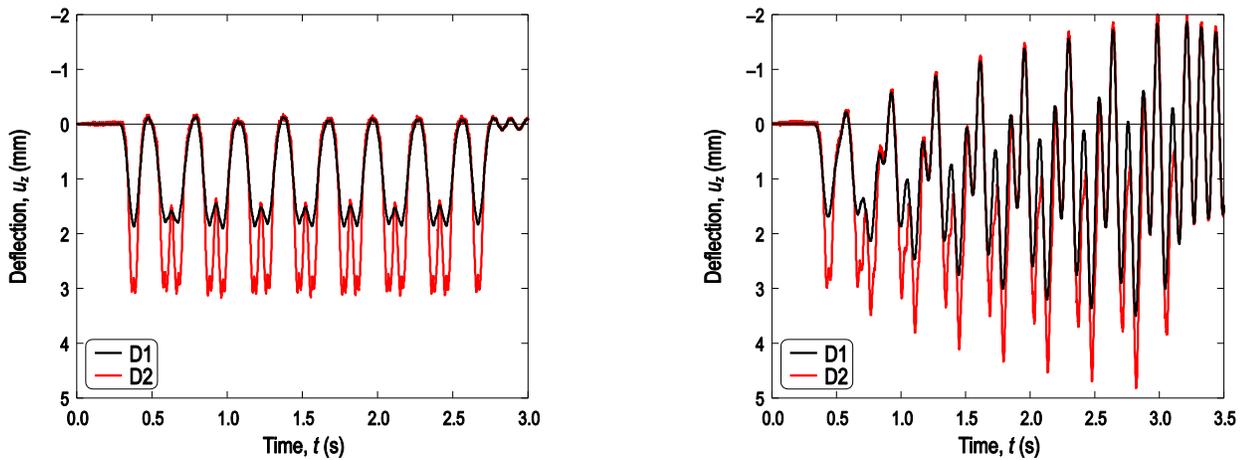


Figure 12: Time-histories of selected vertical displacements at the midspan. The results correspond to the system with the ICE-3 train moving at 300 km/h (*left*) and 260 km/h (*right*)

For St3S native steel material one obtains $\beta = 2.35$, $\sigma_d = 150$ MPa [14]. For KNI 140070 viaduct the static calculations have given $\sigma_g = 30$ MPa. For the system with the ICE-3 train moving at the resonant speed $v = 260$ km/h Equation (2) gives

$$\sigma_z = 30 + 6 + 2.35 \cdot 15 = 71.3 \text{ MPa} < 150 \text{ MPa}. \quad (3)$$

This result proves that the KNI 140070 railway bridge is working with great margin of safety.

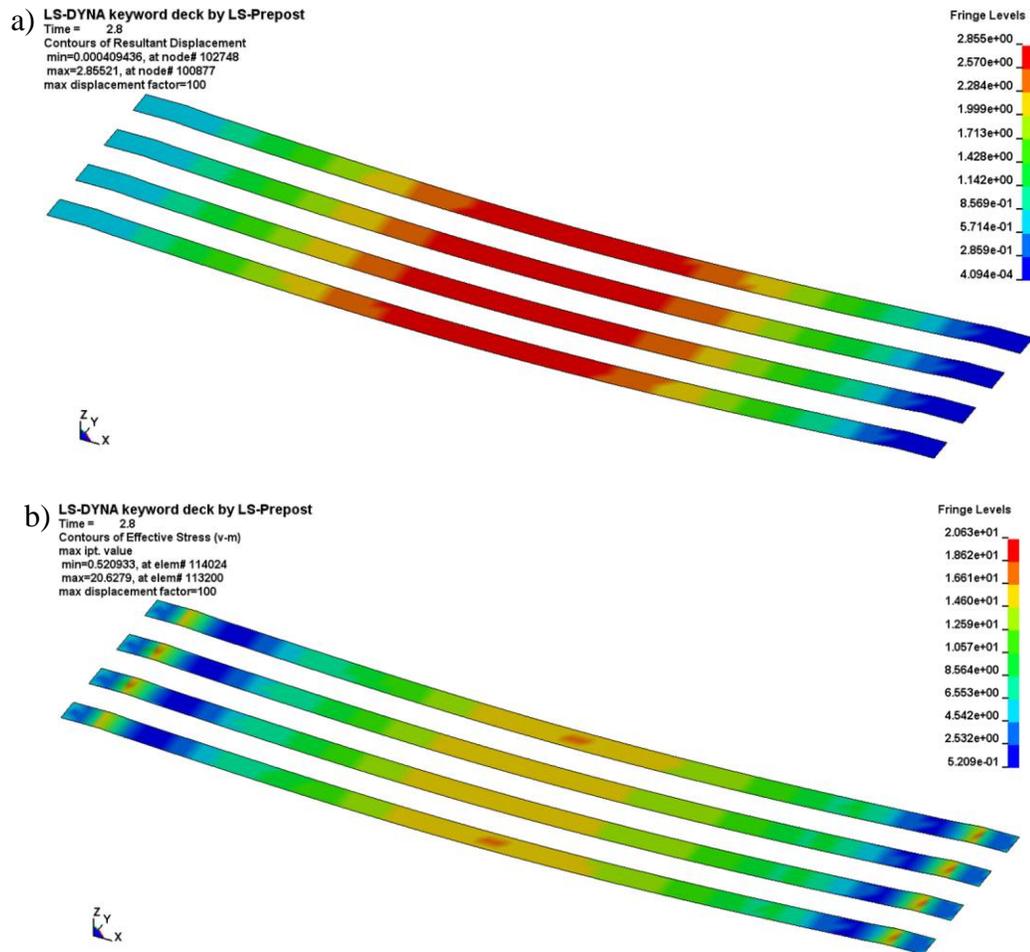


Figure 13: Contours of the total displacements (mm) (a) and the effective stresses (MPa) (b) for the bottom flanges of the main beams at $t = 2.8$ sec (scale coefficient $100\times$ for displacements). The results correspond to the system with the ICE-3 train ($v=260$ km/h)

Summary and Conclusions

The study develops FE modeling and simulation of the composite (steel – concrete) bridge – ballasted track – high-speed train system. In the physical and numerical modeling of the system the following main limitations have been introduced:

- rectilinearity of the rail-line,
- no track irregularities,
- symmetrization of the bridge superstructure and homogenization of the bridge RC platform,
- forced vibrations symmetric to the main longitudinal vertical plane of symmetry of the system.

Selected results of the numerical analysis presented in this study are related to the KNI 140070 viaduct on the Polish Central Main Line (PCML), hence the conclusions cannot be generalized. The analyzed viaduct has appeared to be overdesigned. Numerical modeling and simulation of dynamic processes in the bridge – track – moving train system, developed in the study, has been validated experimentally in Ref. [20]. Thus, the results seem to be very credible and useful in engineering practice.

Practical usability of advanced CAE systems, i.e. Altair HyperMesh, LS-PrePost, LS-DYNA, in FE modeling and simulation of bridge – track – moving train systems has been proved. Other bridges and trains may be modeled and simulated via respective modification of the FE model created for the system undertaken in this study.

Based on the simulation results, the following main conclusions can be formulated:

- 1) The dynamic response of the viaduct has the beam character with dominant influence of the first modal system.
- 2) The longitudinal normal stresses resulting from the dead load amount to 30 MPa, while the maximum dynamic stresses resulting from the train loading equal 21 MPa. The shear stress induced by the train are very small and less than 4 MPa. It testifies to high overdesign of the viaduct.
- 3) Local concentrations of the effective stresses in the bottom flanges of the main beams over the bearings have been detected.
- 4) Beyond the resonant zones, the dynamic response of the bridge superstructure to the moving train is of quasi-static character, but stresses are fast-varying in time with relatively big amplitudes. Thus, in the design calculations high-cycle fatigue should be taken into consideration.
- 5) The approach zones designed by the authors are very stable and minimize the threshold effect.

Acknowledgments

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References

- [1] Klasztorny, M., *Vibrations of single-track railway bridges under high-speed trains* [in Polish], Wroclaw Univ. Techn. Press, Wroclaw, 1987.
- [2] Klasztorny, M., *Dynamics of beam bridges loaded by high-speed trains* [in Polish], WNT Press, Warsaw, 2005.
- [3] Yang, Y.-B., et al., *Vibrations of simple beams due to trains moving at high speeds*, Engineering Structures, 1997, 19, 11, pp. 936–944.
- [4] Cheng, Y.S., et al., *Vibration of railway bridges under a moving train by using bridge-track-vehicle element*, Engineering Structures, 2001, 23, pp. 1597–1606.
- [5] Au, F.T.K., et al., *Impact study of cable-stayed bridge under railway traffic using various models*, J. Sound & Vibration, 2001, 240, 3, pp. 447–465.
- [6] Zhang, Q.-L., et al., *Numerical simulation of train – bridge interaction dynamics*, Computers & Structures, 2001, 79, pp. 1059–1075.
- [7] Song, M.K., Choi, C.K., *Analysis of high-speed vehicle-bridge interactions by a simplified 3-D model*, Struct. Eng. & Mech., 2002, 13, 5, pp. 505–532.
- [8] Modernization of the rail-line 4 –E 65 from Grodzisk Maz. to Zawiercie. Northern branch Knapówka – Psary – Góra Włod. Way. Building Project [in Polish].

- [9] www.cheminots.net, Retrieved on February 3, 2010.
- [10] http://www.hochgeschwindigkeitszuege.com/germany/index_ice_3.htm, Retrieved on February 3, 2010.
- [11] *High Speed Trainset Velaro E for Spanish National Railways RENFE*, Siemens Transportation Systems, <http://www.siemens.com/mobility>.
- [12] Jones, R.M., *Mechanics of composite materials*, London: Taylor & Francis, 1999.
- [13] Hallquist, J.O., *LS-DYNA[®] V971 R4 Beta. Keyword User's Manual*, LSTC Co., CA, USA 2009.
- [14] Karlikowski, J., Madaj, A., Wołowicki, W.: *Composite steel – concrete bridge structures* [In Polish], WKŁ Press, Warsaw, 2007.
- [15] Matsuura A.: *Dynamic behavior of bridge girder for high speed railway bridge*, RTRI Quarterly Reports, 1970, 20, 2, pp. 70-76.
- [16] Niemierko, A., et al., *Reinforcing the track and the subgrade in the approach zones to engineering objects and in other cases of rapid stiffness change of the track subsoil (Stage 1)* [in Polish], Project No. M1-123, Research Institute for Roads and Bridges, Warsaw, 2009.
- [17] *SNCF International in South Korea*, Leaflet No. 1, 2008, www.sncf-international.net.
- [18] *The High Speed Rail System in Korea*, Alstom Leaflet, 2004, www.transport.alstom.com.
- [19] Kwark J. W., Choi E. S., Kim Y. J., Kim B. S., Kim S. I., *Dynamic behavior of two-span continuous concrete bridges under moving high-speed train*, Computers and Structures, Vol. 82, Issue 4-5, 2004, pp. 463–474.
- [20] Szurgott P., Kłasztorny M., Grosel J., Wojcicki Z., *Experimental validation of numerical modelling of the bridge – track – moving train system*, 15th Int. Conf. on Computational Methods & Experimental Measurements, Southampton, England, 2011; in: *Computational Methods & Experimental Measurements XV*, Eds.: G.M. Carlomagno, C.A. Brebbia, WIT Press, Southampton, Boston, 2011, pp. 97–109.

