Random Vibration Fatigue Analysis with LS-DYNA®

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Abstract

Fatigue damage assessment for components under random cyclic loading is an important concern in engineering. A new feature of random vibration fatigue analysis has been implemented to LS-DYNA, to perform the structural fatigue analysis in a random vibration environment. This feature computes cumulative damage ratio and expected fatigue life for structures, based on the Palmgren-Miner's rule of cumulative damage ratio and material's S-N fatigue curve. A series of fatigue analysis methods have been implemented. They include the Steinberg's three band method, Dirlik method, Narrow band method, Wirsching method, Chaudhury and Dover method, Tunna method and Hancock method. Brief introduction of the analysis methods is provided.

To facilitate post-processing of the fatigue analysis, a new binary plot file d3ftg has been implemented in LS-DYNA. This binary plot file provides fatigue analysis information including cumulative damage ratio, expected life, zero-crossing frequency, peak-crossing frequency and irregularity factor for the structure, based on the stress index adopted in the analysis and the load period. This file is accessible to LS-PREPOST.

Several examples are given to demonstrate the effectiveness of the random vibration fatigue analysis feature with LS-DYNA. Some preliminary discussions on the different fatigue analysis methods are included.

Introduction

Machines and mechanical parts are often subjected to cyclic loads which are lower than the material's strength (e.g. tensile stress limit, or yield stress limit), but fail early due to fatigue. Fatigue is defined as the progressive and localized structural damage that occurs when a material is subjected to cyclic loading [1]. Industrial data show that, about 80% to 95% of all structural failures occur through a fatigue mechanism. Thus it is important to study the fatigue life in the early design phase of new products. This paper introduces a new feature of LS-DYNA: random vibration fatigue analysis. This feature is an important tool in structural durability analysis and has wide application in various industries.

Fatigue analysis can be conducted in time domain and frequency domain. In time domain fatigue analysis, people usually use rain-flow counting algorithm to get the number of cycles at each stress/strain level, based on the stress/strain time history. In many situations, a description in frequency domain is more practical. This is because 1) the load for the structure may be random in nature, for example, the wind load on wind turbine, or wave load on an offshore structure – in this case, the best approach for fatigue analysis is to use the statistical method; 2) it may be too intensive to calculate the fatigue life in time domain for large scale structures with long time history of load. This is the motivation for implementing the frequency domain fatigue analysis tool in LS-DYNA.

Implementation of the feature in LS-DYNA

A new keyword *FREQUENCY_DOMAIN_RANDOM_VIBRATION has been introduced in LS-DYNA to perform random vibration analysis since version 971 R5 [2]. Through the keyword, user provides information about the location, direction, range of frequencies for the random excitation. Damping information is also provided through the keyword. The location of the excitation and response area can be given as node, set of nodes, set of segments, or part. The direction of load can be in any of the x, y, z directions or given as a vector by using *DEFINE_VECTOR. Load curve IDs for the Power Spectral Density (PSD) loads in random computation are also specified under the keyword.

The feature of random vibration fatigue is implemented as an option of the keyword *FREQUENCY_DOMAIN_RANDOM_VIBRATION, as it is a natural extension of the random analysis procedure. The method for performing fatigue analysis is defined by the parameter MFTG in card 1. An additional card (Card 6) is needed when the {*FATIGUE*} option appears in the keyword. This card defines the parts or elements where the fatigue analysis is needed, the material's S-N fatigue curve ID, and some other options. The exposure time is defined by the parameter TEXPOS in Card 4.

As the stress state for a physical problem is 3D in the form of stress tensor, a stress invariant is needed to perform fatigue analysis. The typical choices include maximum Principal stress, maximum shear stress and Von-Mises stress, which can be obtained from the stress components.

Material's fatigue characteristics is featured by an S-N (E-N) fatigue curve, which depicts the fatigue life (no. of cycles) for a given cyclic stress (strain) level. The plots are usually given in logarithmic scale. For high cycle, low stress level fatigue, it is more appropriate to use stress index. The S-N fatigue curve is obtained by a large amount of fatigue testing experiments under different stress levels. The S-N fatigue curve can also be given in the form of analytical equations

$$N \cdot S^m = a \tag{1}$$

or

$$\log(S) = a - m \cdot \log(N) \tag{2}$$

where N is the number of cycles for fatigue failure and S is the stress amplitude, and a and m are material parameters determined by experiments. Particularly m is the slope of the S-N curve. Please note that stress variation range (from tension to compression) is used in the paper so S is 2 times of the amplitude if the stress is given as a cyclic function.

Since modal analysis is the first step for running this feature, the keywords *CONTROL_IMPLICIT_GENERAL and *CONTROL_IMPLICIT_EIGENVALUE must be included in the input. Some other keywords related to implicit solution may also be needed, depending on the type of analysis.

The results are given in binary plot file d3ftg which is accessible to LS-PREPOST. Five plot states are included in d3ftg:

State 1: Cumulative damage ratio

State 2: Expected fatigue life
State 3: Zero-crossing frequency
State 4: Peak-crossing frequency
State 5: Irregularity factor

Frequency domain fatigue analysis methods

A series of frequency domain fatigue analysis methods have been implemented in LS-DYNA. They are all based on Palmgren-Miner's rule of cumulative damage ratio

$$E[D] = \sum_i \frac{n_i}{N_i} \tag{3}$$

where $E[D]$ is the expected damage ratio, n_i is the number of cycles at stress level S_i, and N_i is the number of cycles for failure at stress level S_i, given by material's S-N curve.

To get n_i from the PSD of the random stress response and further compute $E[D]$, a variety of approaches have been proposed. They are briefly introduced in this section.

Dirlik method

The Dirlik method was developed during the 1980's [3]. This method was found to have wider applications than other methods and to be very accurate comparing with other methods. The method uses an empirical closed-form expression for the Probability Density Functions (PDF) of stress amplitude, based on the Monte Carlo technology. The method consists of a series of calculations which are based on the moments of the PSD functions.

The n-th moment of the PSD stress is computed as

$$m_n = \int_0^\infty f^n \, G(f) \, df \tag{4}$$

where f is the frequency and $G(f)$ is the PSD stress at frequency f.

Based on the moments of the PSD stress, some useful parameters can be calculated:

$$E[0] = \sqrt{\frac{m_2}{m_0}} \tag{5}$$

is the number of upward zero crossing;

$$E[P] = \sqrt{\frac{m_4}{m_2}} \tag{6}$$

is the number of peaks;

$$\gamma = \frac{E[0]}{E[P]} = \frac{m_2}{\sqrt{m_0 \cdot m_4}} \tag{7}$$

is the irregularity factor. The irregularity factor, which varies between 0 and 1, is a useful term when interpreting the type of the random stress signal. It approaches 1 as the stress signal approaches narrow band (e.g. a single sine wave for $\gamma = 1$). It approaches 0 as the stress signal approaches wide band (e.g. for $\gamma = 0$, the stress signal is white noise).

Another useful parameter is calculated as

$$\lambda = \sqrt{1-\gamma^2} \tag{8}$$

which is called the bandwidth parameter, an alternative version of the irregularity factor.

Then, the cumulative damage ratio is given as

$$E(D) = \sum_i \frac{n_i}{N_i} = \sum_i \frac{p[S_i]\, E[P]\, T\, dS}{N_i} \tag{9}$$

where T is the exposure time and the PDF function is expressed by

$$p(S) = \frac{\frac{D_1}{Q} e^{\frac{-Z}{Q}} + \frac{D_2}{R^2} e^{\frac{-Z^2}{2}} + D_3 Z\, e^{\frac{-Z^2}{2}}}{2\sqrt{m_0}} \tag{10}$$

Where

$$Z = \frac{S}{2\sqrt{m_0}}; \quad D_1 = \frac{2\,(X_m - \gamma^2)}{1+\gamma^2}; \quad D_2 = \frac{1-\gamma-D_1+D_1^{\,2}}{1-R}; \quad D_3 = 1 - D_1 - D_2;$$

$$R = \frac{\gamma - X_m - D_1^{\,2}}{1-\gamma-D_1+D_1^{\,2}}; \; Q = \frac{5\cdot(\gamma - D_1 - D_2 R)}{4D_1}; \quad X_m = \frac{m_1}{m_0}\sqrt{\frac{m_2}{m_4}}$$

As can be seen from the equations, X_m, D_1, D_2, D_3, Q and R are all functions of the PSD moments m_0, m_1, m_2 and m_4.

Steinberg's three band method

For Steinberg's method [4], the expected damage ratio is given as

$$E(D) = E[0]\,T\,\left[0.683\,(2\sqrt{m_0})^m + 0.271\,(4\sqrt{m_0})^m + 0.043\,(6\sqrt{m_0})^m\right]/\,a \tag{11}$$

where $E(0)$ is given by equation (5).

The solution is based on the assumption that stress levels occur for 68.3% at $2\sigma_{RMS}$ (=$2\sqrt{m_0}$), 27.1% at $4\sigma_{RMS}$, and 4.3% at $6\sigma_{RMS}$.

The Steinberg's approach leads to a very simple solution based on the assumption that no stress cycles occur with ranges greater than 6 σ_{RMS}. The method is used for testing electronic equipment in USA.

Narrow band method

The narrow band method was presented by Bendat [5]. Bendat showed that the PDF of peaks for a narrow band signal tended towards Rayleigh distributions as the bandwidth reduced. According to Bendat's theory, the expected cumulative damage can be written as

$$E[D] = \frac{E[P]T}{a}\int S^m p(S)dS \tag{12}$$

Or $$E[D] = \frac{E[P]T}{a} S_{eqv}{}^{m} \quad (13)$$

where the equivalent stress is defined as $S_{eqv} = \left[\int S^m p(S) dS\right]^{1/m}$ (14)

For narrow band process,

$$p(S) = \frac{S}{4m_0} e^{\frac{-S^2}{8m_0}} \quad (15)$$

and equation (13) can be simplified as

$$E[D] = \frac{E[P]\,T}{a} \left(\sqrt{2m_0}\right)^m \Gamma(1 + m/2) \quad (16)$$

where, Γ(.) is the gamma function.

Many expressions have been proposed to correct the conservatism associated with this solution. Most were developed with reference to offshore industry. The solutions of Wirsching, Chaudhury and Dover, Tunna and Hancock were all derived using this approach. They are all expressed in terms of the spectral moments up to m_4.

Wirsching method

Wirsching's equation is given as [6]:

$$E[D]_W = \zeta_W \; E[D]_{NB} \quad (17)$$

where, ζ_W is the rain-flow correction factor. It is an empirical factor derived from extensive Monte Carlo simulations that include a variety of spectral density functions. It is expressed as follows

$$\zeta_W = a_W + (1 - a_W)(1 - \lambda)^{b_W} \quad (18)$$

where a_W and b_W are best fitting parameters expressed as

$$a_W = 0.926 - 0.033\, m \quad (19)$$

$$b_W = 1.587\, m - 2.323 \quad (20)$$

Chaudhury and Dover method

For Chaudhury and Dover method [7], the expected damage ratio is expressed by equation (13), with the equivalent stress given as

$$S_{eqv} = 2\sqrt{2m_0} \left[\frac{\lambda^{m+2}}{2\sqrt{\pi}} \Gamma\left(\frac{m+1}{2}\right) + \frac{\gamma}{2} \Gamma\left(\frac{m+2}{2}\right) + erf(\gamma) \cdot \frac{\gamma}{2} \Gamma\left(\frac{m+2}{2}\right) \right]^{1/m} \quad (21)$$

Where,

$$erf(\gamma) = 0.3012\gamma + 0.4916\gamma^2 + 0.9181\gamma^3 - 2.354\gamma^4 - 3.3307\gamma^5 + 15.6524\gamma^6 - 10.7846\gamma^7 \quad (22)$$

Tunna method

The procedure for Tunna method is similar to that for Dirlik method. The PDF is given as [8]

$$p(S) = \frac{S}{4\gamma\, m_0} \exp\left(\frac{-S^2}{8\gamma\, m_0} \right) \tag{23}$$

For $\gamma = 1.0$, this formula becomes the narrow band formula given earlier (equation (15)). Tunna's equation was developed with specific reference to the railway industry.

Hancock method

For Hancock method, the expected damage ratio is expressed by equation (13), with the equivalent stress given as [9]

$$S_{eqv} = 2\sqrt{2m_0}\,\left[\gamma\, \Gamma(1+m/2)\right]^{1/m} \tag{24}$$

Numerical examples

Several examples are presented in this section. For the first one, a panel bracket structure is considered and the Steinberg's three-band method is used to calculate the cumulative damage ratio; for the second one, an aluminum beam given in the literature [10] is studied, for which the theoretically estimated fatigue life is compared with the results given by experiments; the third example under consideration is an industrial model: an antenna support mounted on a bogie frame of railway vehicle. It is subjected to acceleration PSD defined by the International Standard for railway applications IEC61373 [11]. The numerical results given by LS-DYNA are compared with the observation.

Example 1: a panel bracket

Consider a panel bracket structure in Figure 1. Material properties are given as density $\rho = 2800$ kg/m^3, Young's modulus $E = 72.4 \times 10^9$ Pa, Poisson's ratio $\gamma = 0.33$. Shell element type 18 (Fully integrated linear DK quadrilateral/triangular shell) is adopted. Totally 1972 nodes and 1865 shell elements are used. The structure is fixed to shaker table. Constant base acceleration PSD 2.0 g^2/Hz for the range 100-2000 Hz is applied. The structure is exposed to the random vibration environment for 4 hours (14400 seconds).

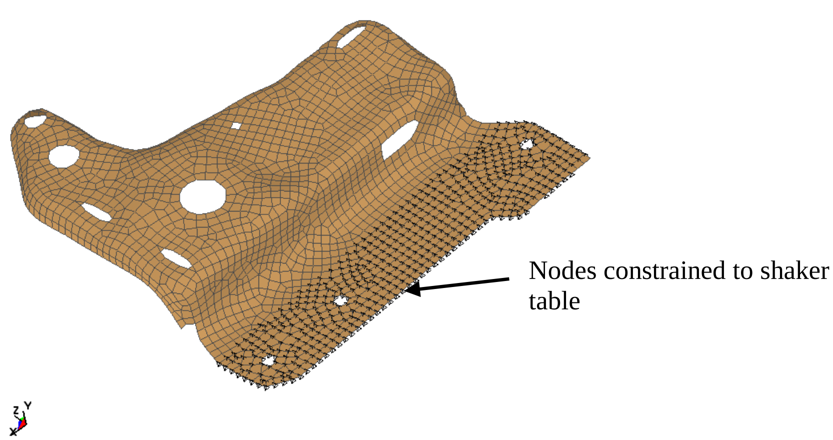


Figure 1 – A rectangular plate with free boundaries

The material's S-N fatigue curve is given as Figure 2.

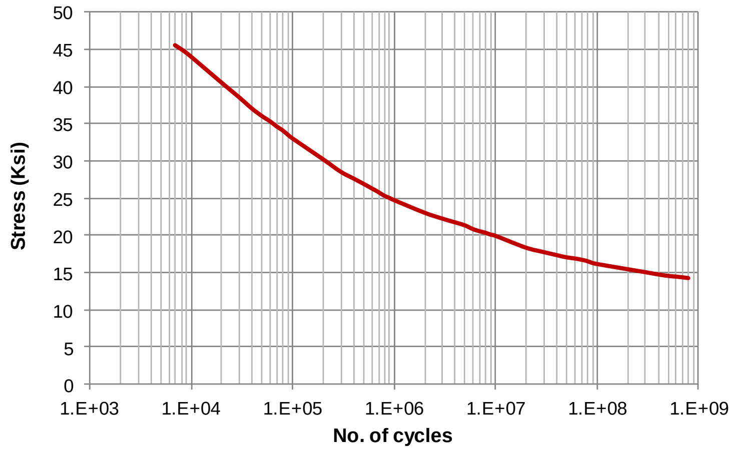
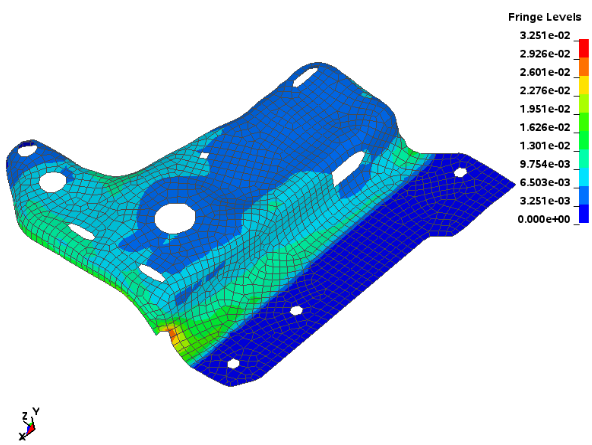


Figure 2 – S-N fatigue curve

Steinberg's three-band method is used for this problem. The RMS of Von-Mises stress is given in Figure 3 (d3rms) and the cumulative damage ratio is given in Figure 4 (d3ftg). It is seen that the highest cumulative damage ratio (0.7282) takes place at the same location as the highest RMS Von-Mises stress, which is on the edge with sharp curvature. Meanwhile, the expected fatigue life for the structure can be computed as 14400 / 0.7282 = 19774.79 seconds.



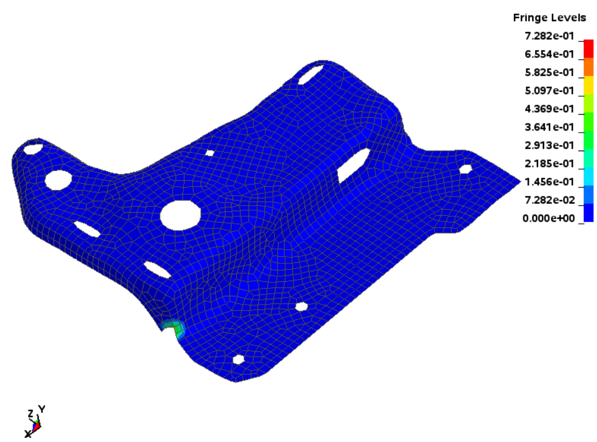


Figure 3 – RMS of Von-Mises stress

Figure 4 – cumulative damage ratio

Example 2: aluminum beam

A simple cantilever aluminum beam [10] subjected to base accelerations is considered. The numerical values are compared with the experimental results. Different fatigue failure theories are used to predict fatigue life.

The model is composed of 2205 nodes and 2039 4-node type 18 shell element (fully integrated linear DK quadrilateral and triangular shell). Aluminum alloy 5754 is adopted for the material model and its properties are given as density $\rho = 2700$ kg/m^3, Young's modulus E = 70,000 MPa and Poisson's ratio ν=0.33. The model is shown in Figure 5. The beam is designed such that failure occurs at a predetermined location where failure can be observed manually.

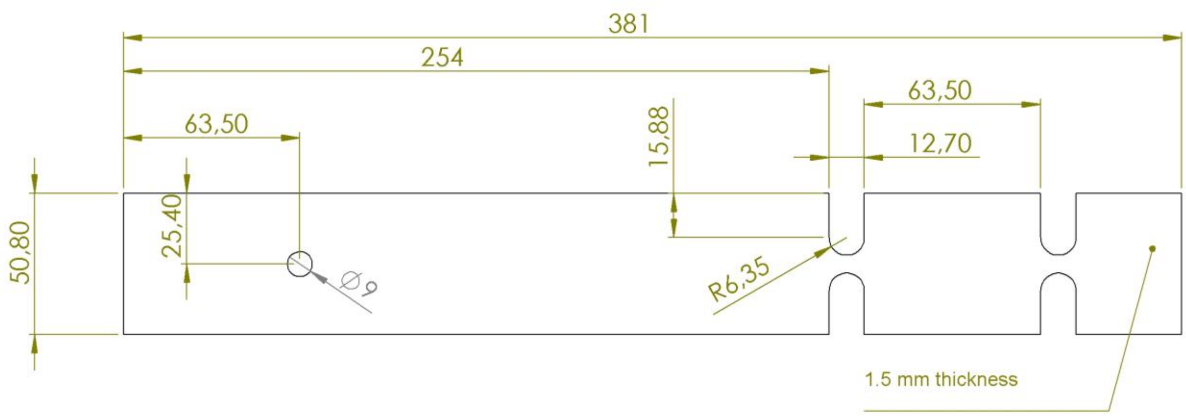


Figure 5 – aluminum beam using for the shaker table experiment

(all dimensions are in millimeter)

The attachment point of the beam is subjected to base Acceleration Spectral Density (ASD) for the range of frequency 10-300 Hz, shown in Figure 6. A constant modal damping ratio 0.04 is adopted. The beam is exposed to the random vibration load for 30 minutes (1800 seconds).

The fatigue assessment of the notch area is suggested by the European Standard Eurocode 9 [12]. According to this code the fatigue strength of plain material is described by the detail category FAT100 calculated for a probability of survival equal to 97.7%, a reference fatigue strength $\Delta\sigma$ = 100 MPa at 2.10^6 cycles and a single inverse constant slope m=7.0.

The first 10 natural modes are required for the eigenvalue analysis. To get stress on shell surface it is necessary to use Lobatto's integration rule with three integration points through the thickness of the aluminum sheet (INTGRD=1 in the keyword *CONTROL_SHELL).

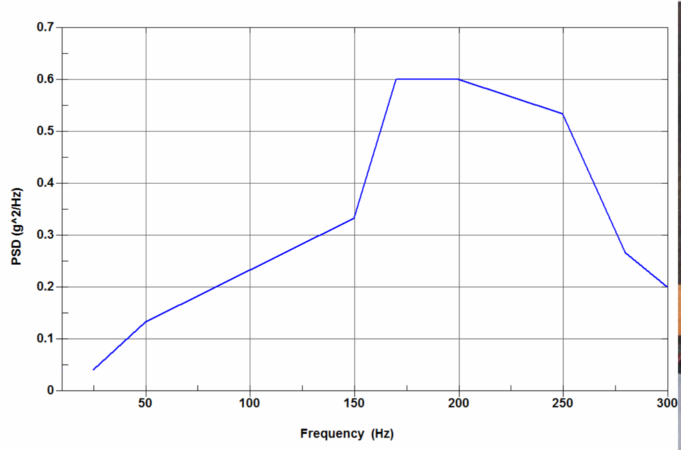


Figure 6 - input acceleration PSD

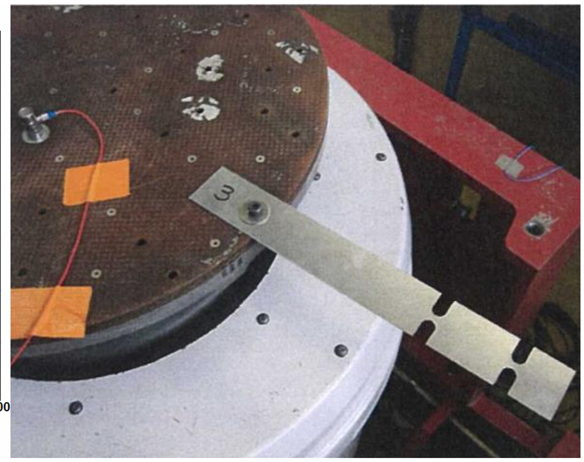


Figure 7 – beam on the shaker table

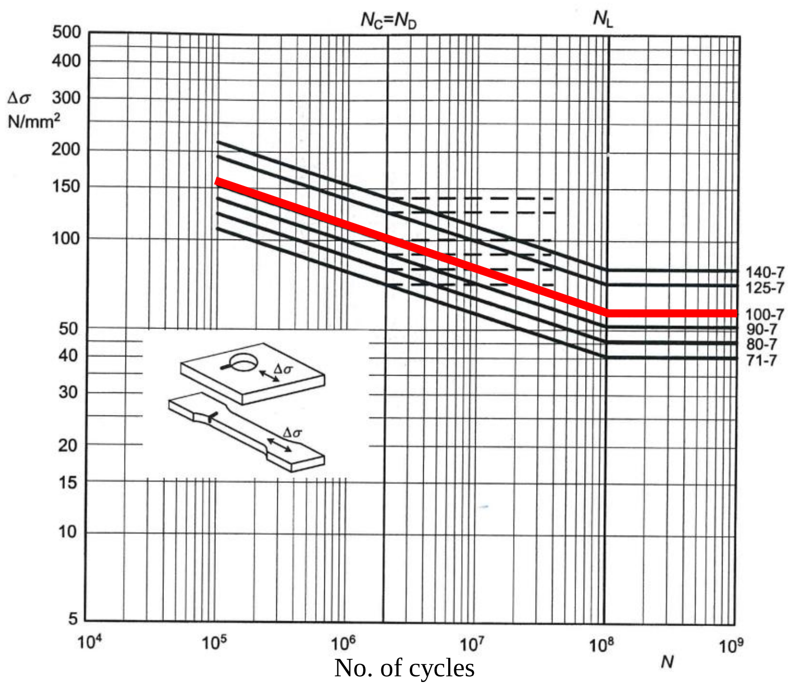


Figure 8 – S-N fatigue curve used for the notched aluminum beam

The response stress PSD measured at the critical point shows that several natural frequencies are excited by the input loading in the range 10-300 Hz (Figure 10). The natural frequencies of the beam, i.e. 13 Hz, 64 Hz and 185 Hz, obtained with LS-DYNA, match reasonably well with the experimental results: 12Hz, 58 Hz and 180 Hz. Three samples of the aluminum beam are tested in the experiments. The average fatigue life value observed is 7 mn 25 s with a minimum of 5 mn and a maximum of 10 mn 30 s. Table 1 summarizes the fatigue life observed in the experiment and the numerical predictions archived with LS-DYNA.

The results obtained provide satisfactory match with the experimental results, although the results depend on the method used to interpret the RMS results. Dirlik, Wirsching and Chaudhury & Dover methods give the best theoretical results. Steinberg gives conservative results while Tunna's predictions are completely off. These different observations and results are entirely consistent with those found in the literature on the same subject. Also, the same problem has been simulated with ANSYS® and RADIOSS® BULK and the maximum RMS stress S_x computed by LS-DYNA (35.0 MPa) is in good agreement with the results by the others commercial software : 33.5 MPa for the first and 35.7 MPa for the second.

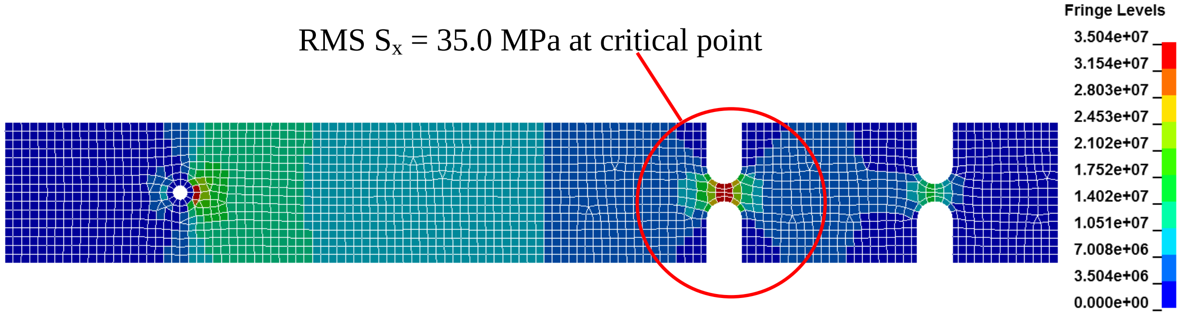


Figure 9 – RMS of Sx stress in the local element axis

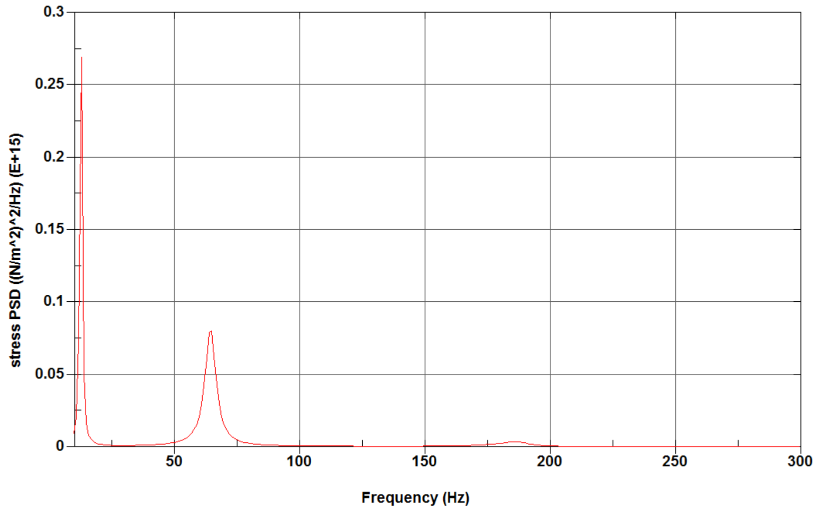


Figure 10 – Stress PSD at critical point

Theoretical methods	Fatigue life (mn, s)	Theoretical fatigue damage
Experiment	*7mn 25s*	-
Steinberg	4mn 10s	7.19
Dirlik	5mn 25s	5.54
Narrow Band	2mn 05s	14.41
Wirsching	5mn 45s	5.08
Chaudhury and Dover	6mn 03s	6.03
Hancock	4mn 06s	7.31
Tunna	22mn 18s	1.35

Table 1 – Experimental and theoretical fatigue life

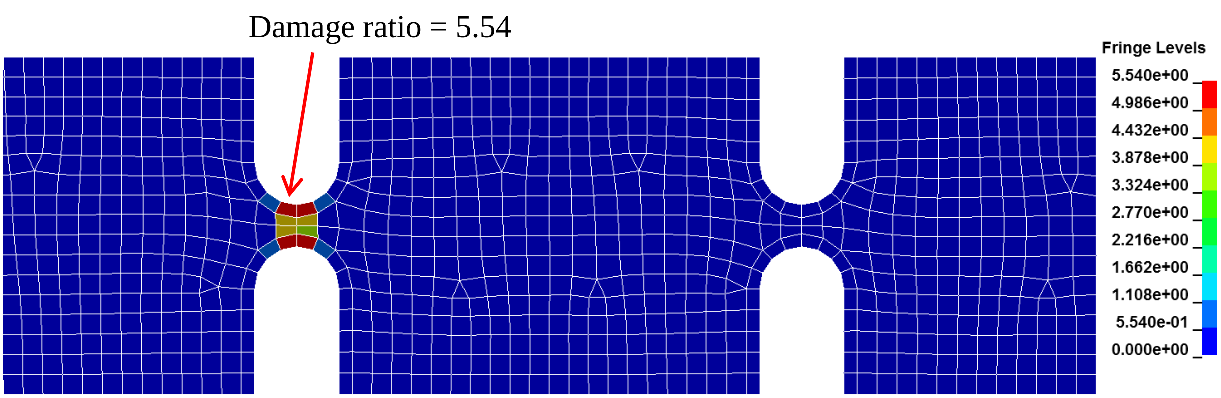


Figure 11 - Cumulative damage ratio by Dirlik method

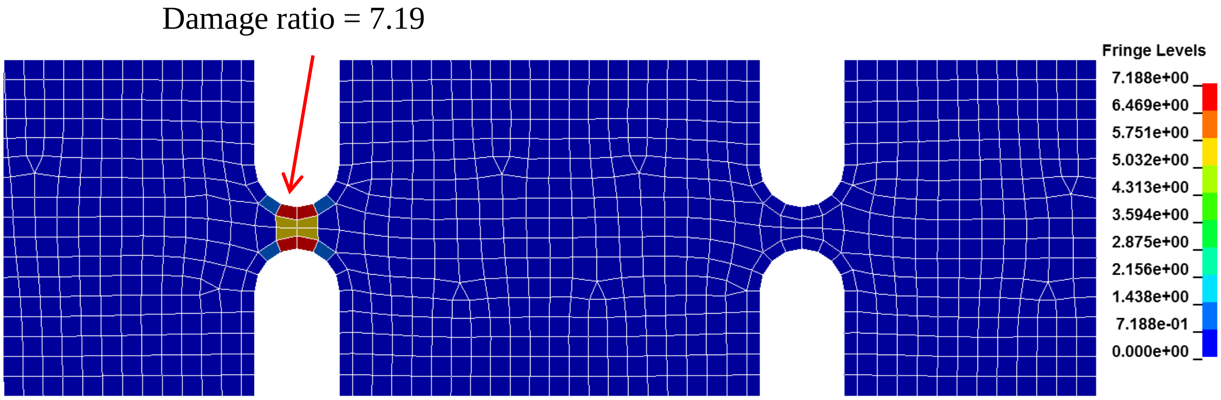


Figure 12 - Cumulative damage ratio by Steinberg's method

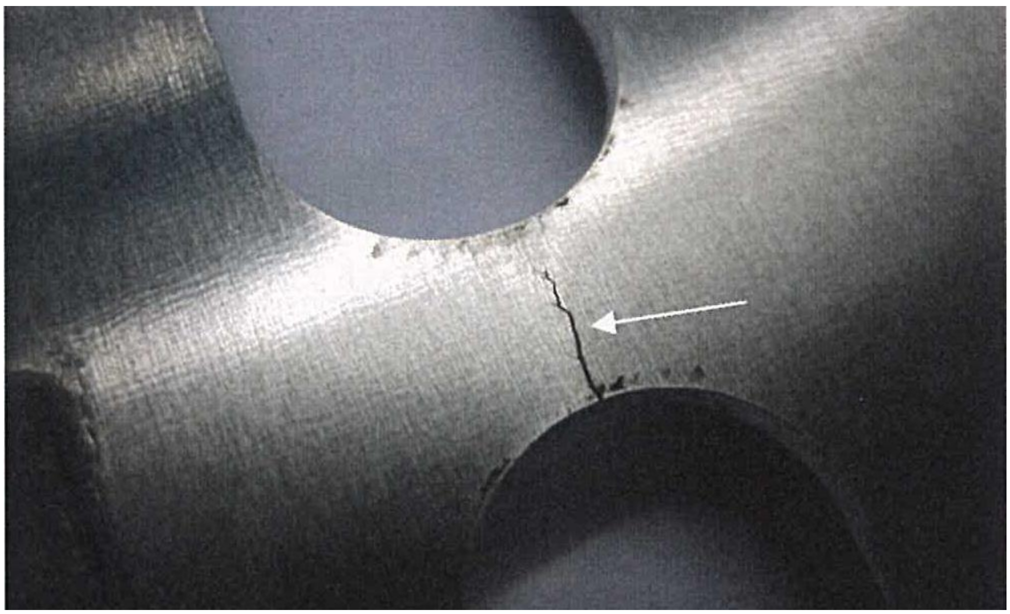


Figure 13 - Failure at the notched point in experiment

Example 3: support antenna

The antenna support is mounted on a bogie frame of a railway vehicle and required a random fatigue analysis to determine the fatigue life before its failure. The model is shows in Figures 14 (a) and (b) and it's composed of 38,199 nodes and 14,993 4-node shell element type 20 (fully integrated linear assumed C^0 shell). This element is based on thick plate theory and is recommended for thin and thick plates. The antenna is built up using 76,051 tetrahedron elements with an element solid formation type 10 (one point tetrahedron). Its mass is 2.7 Kg. The total mass of the support is 23.7 Kg. The material properties of P275NL1 grade steel are given as density ρ = 7850 kg/m^3, Young's modulus E = 210,000 MPa and Poisson's ratio ν = 0.30.

The material failed due to dynamic fatigue loading after only a few years in service. Before proposing a new design it is necessary to estimate the safe life of the actual configuration. The crack started around a mechanically fastened joint between support and transverse stop and spread through the support to break it (Figure 15).

The structure is subjected to base acceleration defined by the International Standard IEC61373 to simulate the long-life test. This standard intends to highlight any weakness which may result in problem as a consequence of operation under environment where vibrations are known to occur in service on a railway vehicle. Acceleration levels depend only upon the equipment's location within the vehicle. Component which is to be mounted on the bogie will be tested as category 2 with acceleration spectral density 6.12 $(m/s^2)^2/Hz$ for the range of frequency 5-250 Hz. The test duration is 5 hours (18,000 seconds). Constant modal damping ratio 0.02 is adopted and the first 15 natural modes are employed in the modal superposition.

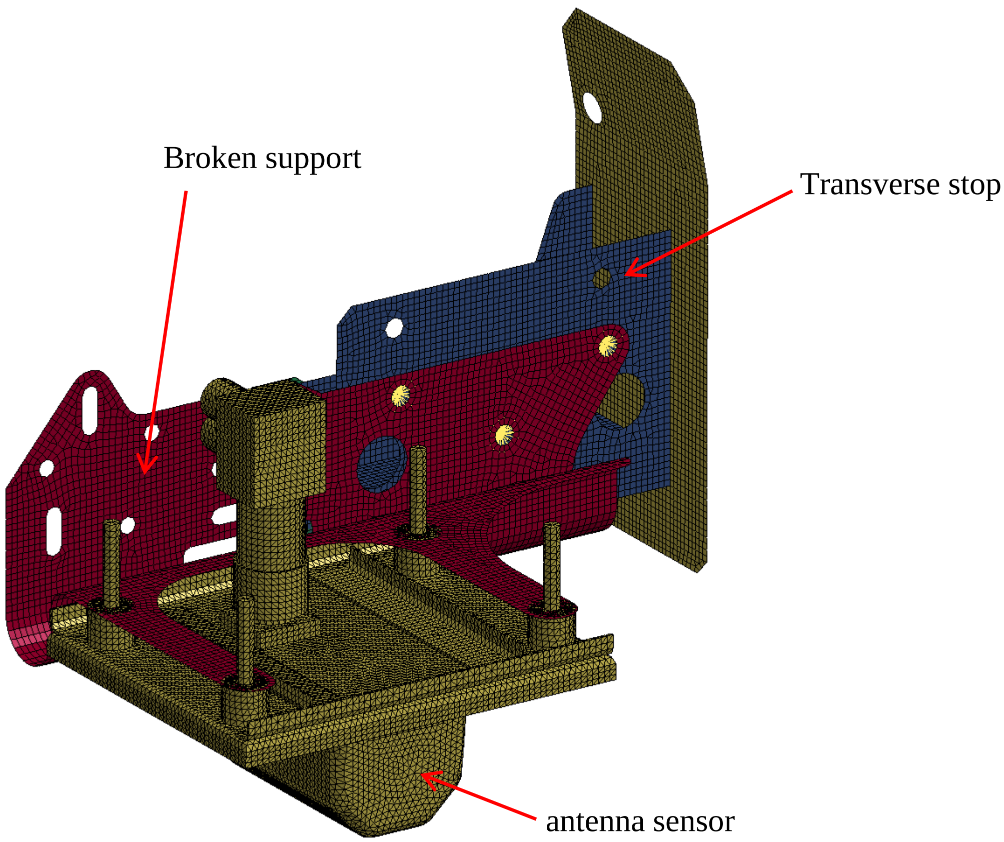


Figure 14 (a) – global view of support antenna model

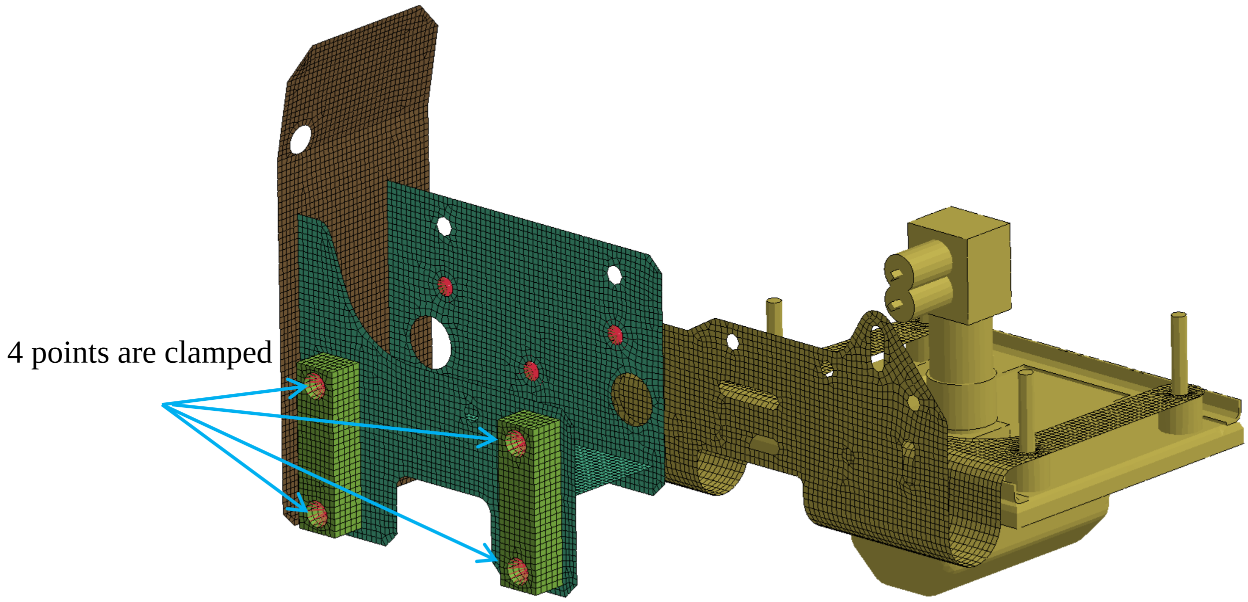


Figure 14 (b) – back view of the support antenna model

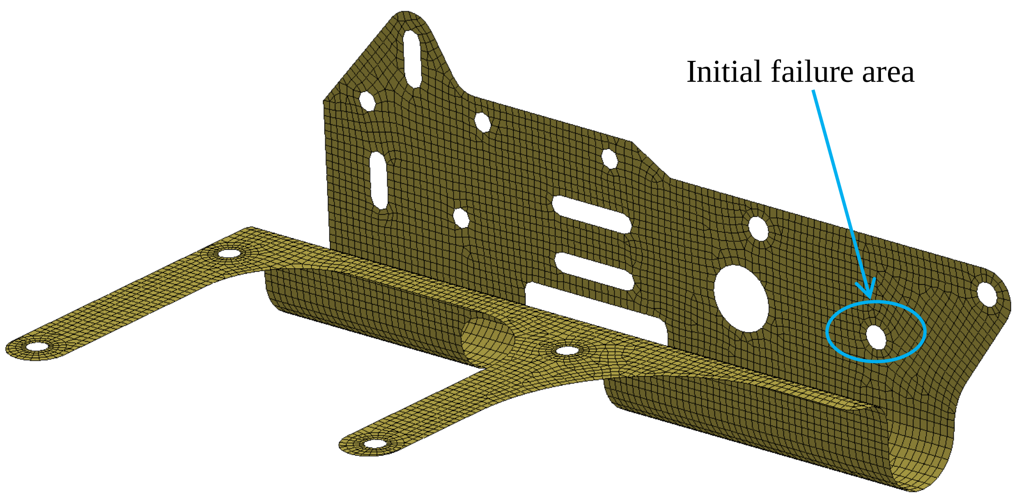


Figure 15 – broken support

The fatigue life calculation should be based on the use of the European Standard Eurocode 3 [13] which defines the acceptable fatigue strength. Mechanically fastened joints are assumed to be described by the detail category FAT112: probability of survival equal to 97.7%, reference fatigue strength $\Delta\sigma$ = 112 MPa at 2×10^6 cycles and $\Delta\sigma$ = 45.3 MPa for cut-off limit at 10^8 cycles. The fatigue analysis is based on the RMS of Von-Mises stress and the Steinberg's method.

Figure 16 (a) and Figure 16 (b) provide respectively contour plot of the cumulative damage ratio observed on the support and a photograph of the broken support. Results given by LS-DYNA are in good agreement with the expected results.

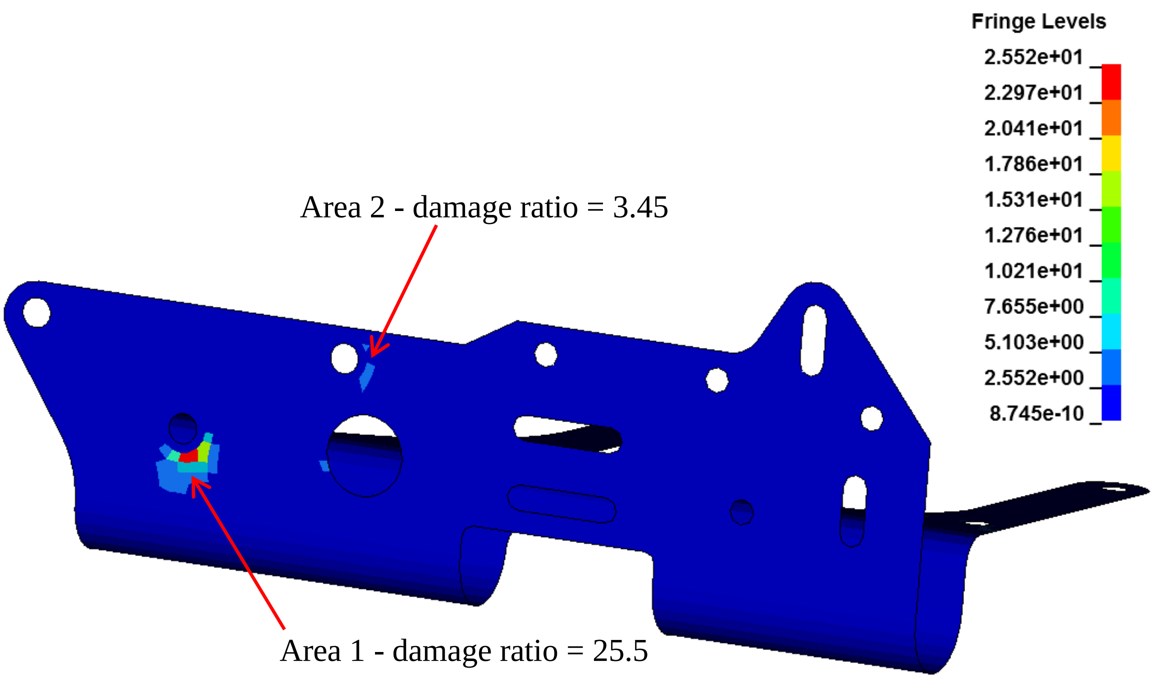


Figure 16 (a) – cumulative damage according Steinberg's method

Figure 16 (b) – broken support

Conclusion

A new feature of LS-DYNA, random vibration fatigue analysis is introduced in the paper. The feature provides cumulative damage ratio calculation and also fatigue life prediction for structures subjected to random vibration excitations, based on various theories. The feature has wide application in durability analysis for various industries.

Three examples, including one from industry, are adopted to demonstrate the effectiveness of the new feature. The numerical prediction matches reasonably well with experimental results, or observations. Different methods of fatigue analysis are discussed.

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