Boundary Element Analysis of Muffler Transmission Loss With LS-DYNA®

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Abstract

This paper presents a case study of applying the boundary element method (BEM) in LS-DYNA for calculating transmission loss (TL) of mufflers. Both the three-point method and the four-pole transfer matrix method are used for calculating the transmission loss. The three-point method is easier to use, but it solves for the transmission loss only and nothing else. The four-pole method has the advantage of providing transfer matrix of the muffler, which contains important parameters when the muffler is connected to another muffler or other components in the silence system. Numerical predictions are examined by experimental results and theoretical results for all test cases. The results show that LS-DYNA can be used to perform muffler transmission loss analysis effectively.

Introduction

Muffler performance prediction is a good example of using BEM in industrial applications. Although the interior acoustic domain of a muffler is finite, the geometry inside the muffler can be very complicated. Internal components inside a typical muffler may include perforated tubes, thin bafflers, branched cavities, and extended inlet/outlet tubes. The BEM can provide an easy design tool because only the surfaces need to be modeled.

LS-DYNA is a widely used finite element code, intended to solve complex mechanical problems. One of the recent developments of the code is the addition of a vibro-acoustic solver [1], which enables users to perform a variety of vibro-acoustic simulations in the frequency domain. A new keyword ***FREQUENCY_DOMAIN_ACOUSTIC_BEM** has been implemented to LS-DYNA (ls971 R5). This new keyword allows user to run acoustic computation based on boundary element methods.

BEM Model for Acoustic System

In frequency domain, the acoustic wave propagation in an ideal fluid in absence of any volume acoustic source is governed by Helmholtz equation given as follows:

$$\Delta p + k^2 p = 0 \quad (1)$$

Where $k = \omega / c$ denotes the wave number, c is the sound velocity, $\omega = 2\pi f$ is the pulsation frequency and p is the pressure at any field point.

Equation (1) can be transformed into an integral equation by using Green's theorem. In this case, the pressure at any point in the fluid medium can be expressed as an integral of both pressure and velocity over a surface as given by the following equation:

$$Cp(P) = \int_{\Gamma} (G\frac{\partial p}{\partial n} - p\frac{\partial G}{\partial n}) d\Gamma \quad (2)$$

where $G = \frac{e^{-ikr}}{4\pi r}$ is the Green's function, n is the normal on the surface Γ , C is the jump term resulting from the treatment of the singular integral involving Green's function, and r is the distance between the field point P and surface integration point. The normal derivative of the pressure is related to the normal velocity by $\frac{\partial p}{\partial n} = -i\omega p v_n$.

The knowledge of pressure and velocity on the surface allows calculating the pressure of any field points. This constitutes the main idea of the integral equation theory. In practical cases, the problems are Neumann, Dirichlet or Robin ones. In Neumann problem, the velocity is prescribed on the surface while in Dirichlet case the pressure is imposed on the surface. Finally, for robin problems the acoustic impedance, which is a combination of velocity and pressure, is given on the surface. Hence, generally only half of the variables are known on the surface. By using the variational indirect method or the collocation method, a linear equation system can be established, which provides solution for the other half of the variables on the surface. Then the integral equation (2) can be used to calculate the acoustic pressure at any field points.

For the calculation of transmission loss of mufflers, two methods have been introduced and they will be discussed below.

Three-Point Method

The three-point method [2, 3] uses only one single BEM run to compute the TL at each frequency. The inlet is excited by a uniform velocity or pressure, while an anechoic termination (impedance equal to ρc, where ρ is density of the air) is used at the outlet end, as shown in Figure 1. The acoustic wave in the inlet tube contains an incoming wave as well as a reflected wave. The acoustic wave in the outlet tube contains only an outgoing wave due to the anechoic termination. Two points in the inlet tube are selected to extract the incoming wave. Let x_1 and x_2 be the longitudinal co-ordinates of the two selected points along the muffler axis, respectively. The corresponding sound pressures p_1 and p_2 at the two points can be written as:

$$p_1 = p_i e^{-ikx_1} + p_r e^{-ikx_1} \quad (3)$$

$$p_2 = p_i e^{-ikx_2} + p_r e^{-ikx_2} \quad (4)$$

Where p_i represents the incoming wave and p_r represents the reflected wave. Solving equations (3) and (4) for p_i gives:

$$p_i = \frac{1}{2i\sin[k(x_2 - x_1)]}(p_1 e^{ikx_2} - p_2 e^{ikx_1}) \quad (5)$$

Note that $\sin[k(x_2 - x_1)] \neq 0$. As shown in Figure 1, the third point can be placed anywhere in the outlet tube. The pressure at that point is p_3. Then, the TL of the muffler can be evaluated from (6):

$$TL = 20\log_{10}\left(\frac{|p_i|}{|p_3|}\right) + 10\log_{10}\left(\frac{S_i}{S_o}\right) \tag{6}$$

Where S_i, and S_o are the inlet and outlet tube areas, respectively.

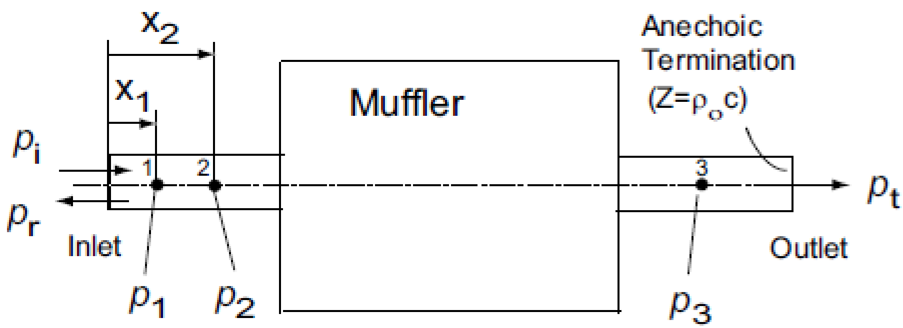


Figure 1. The three-point method

Compared to the four-pole method, the three point method is much faster for computing the TL, due to its single BEM run nature. However, the three point method does not produce the four pole transfer matrix. The four-pole method retains the transfer matrix of the muffler, which contains important parameters when the muffler is connected to another muffler or other components in the exhaust system.

Four-Pole Method

The TL of muffler can be computed by using the transfer matrix approach [4]. A muffler (refer to Figure 1) with an inlet and an outlet can be represented by a linear acoustic four pole transfer matrix:

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p_2 \\ -v_2 \end{bmatrix} \tag{7}$$

Where the p_1 and v_1 are the sound pressure and normal particle velocity at the inlet, respectively; p_2 and v_2 are the corresponding quantities at the outlet. A negative sign on v_2 is added because the normal vector at the outlet on the BEM model is opposite to the normal at the inlet. The four pole parameters, *A*, *B*, *C*, *D*, can be obtained from

$$A = p_1 / p_2 \,|\, v_2 = 0, v_1 = 1 \tag{8}$$

$$B = p_1 / -v_2 \,|\, p_2 = 0, v_1 = 1 \tag{9}$$

$$C = v_1 / p_2 \,|\, v_2 = 0, v_1 = 1 \tag{10}$$

$$D = v_1 / -v_2 \,|\, p_2 = 0, v_1 = 1 \tag{11}$$

Note that the velocity boundary condition at the inlet (v_1=1) in equations (8)-(11) may also be replaced by a pressure boundary condition. Two separate BEM runs are required to obtain the four pole parameters at each frequency. In the first BEM run, a zero velocity boundary condition is applied to the outlet end (v_2=0). This will produce parameters *A* and *C*. In the second BEM run, the sound pressure at the outlet end is set to zero (p_2=0). This will produce the remaining two parameters, *B* and *D*.

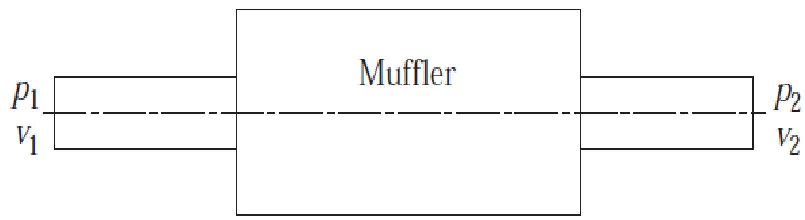


Figure 2. The four-pole method

Because of the two different types of boundary conditions applied at the outlet, the two BEM runs do not share the same coefficient matrix. That means that the matrix solver needs to be called twice at each frequency. This simply makes the conventional four pole method an impractical choice for computing the TL.

An Improved four pole method was introduced by Wu [5]. Rearrange equation (7) to get

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} A^* & B^* \\ C^* & D^* \end{bmatrix} \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix} \tag{12}$$

Where

$$A^* = p_1 \mid v_1 = 1, v_2 = 0 \tag{13}$$
$$B^* = p_1 \mid v_1 = 0, v_2 = -1 \tag{14}$$
$$C^* = p_2 \mid v_1 = 1, v_2 = 0 \tag{15}$$
$$D^* = p_2 \mid v_1 = 0, v_2 = -1 \tag{16}$$

Two BEM runs are still needed to get the above four parameters. The first BEM run produces A* and C*, while the second BEM run produces B* and D*. Nevertheless, only one BEM matrix needs to be solved at each frequency, because the two BEM runs share the same coefficient matrix. The second BEM run uses only a different velocity condition, and therefore, requires only a trivial back substitution procedure. Actually, the two BEM runs can be done simultaneously because the two right side vectors corresponding to the two different velocity boundary conditions may be formed at the same time.

The original four-pole parameters in equation (7) can be obtained by solving equation (12) for p_1 and v_1 in terms of p_2 and v_2.

$$A = \frac{A^*}{C^*}, B = B^* - \frac{A^* D^*}{C^*}, C = \frac{1}{C^*}, D = -\frac{D^*}{C^*} \tag{17}$$

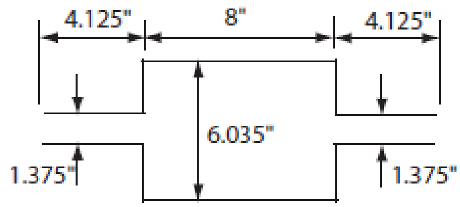
The TL of the muffler can be evaluated by

$$TL = 20\log_{10}\left(\frac{1}{2}\left|A + B\left(\frac{1}{\rho c}\right) + C\rho c + D\right|\right) + 10\log_{10}\left(\frac{S_i}{S_o}\right) \tag{18}$$

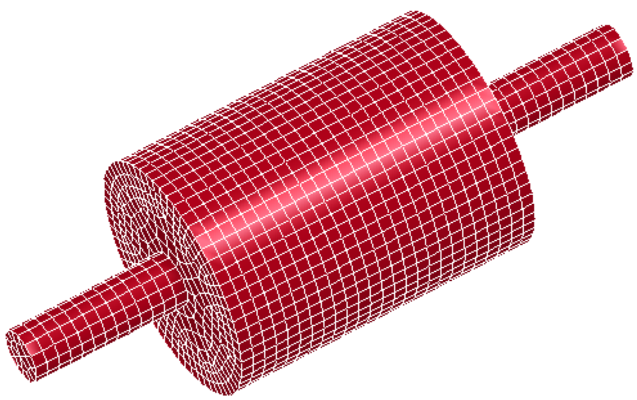
Where S_i, and S_o are the inlet and outlet tube areas, respectively.
The major advantage of the improved method is that it not only provides a very fast method for computing the TL, but also produces the four pole parameters.

Numerical Results

In this example we model a simple expansion chamber and compute the transmission loss. The BEM solutions are compared with the experiment results extracted from the publication by Tao [6] and the analytical solution calculated based on the plane wave theory [7].



(a)



(b)

Figure 3. Simple expansion chamber silencers (a) dimensions, (b) BEM mesh

The theoretical TL from [7] is

$$TL = 10\log_{10}\left(\frac{1}{4}\left[4\cos^2 kL + (m + 1/m)^2 \sin^2 kL\right]\right) \tag{18}$$

Where L is length of central chamber, $m = S_o / S_i$, S_c is the area of central chamber, and S_i is the area of the inlet pipe (here, $S_i = S_o$).

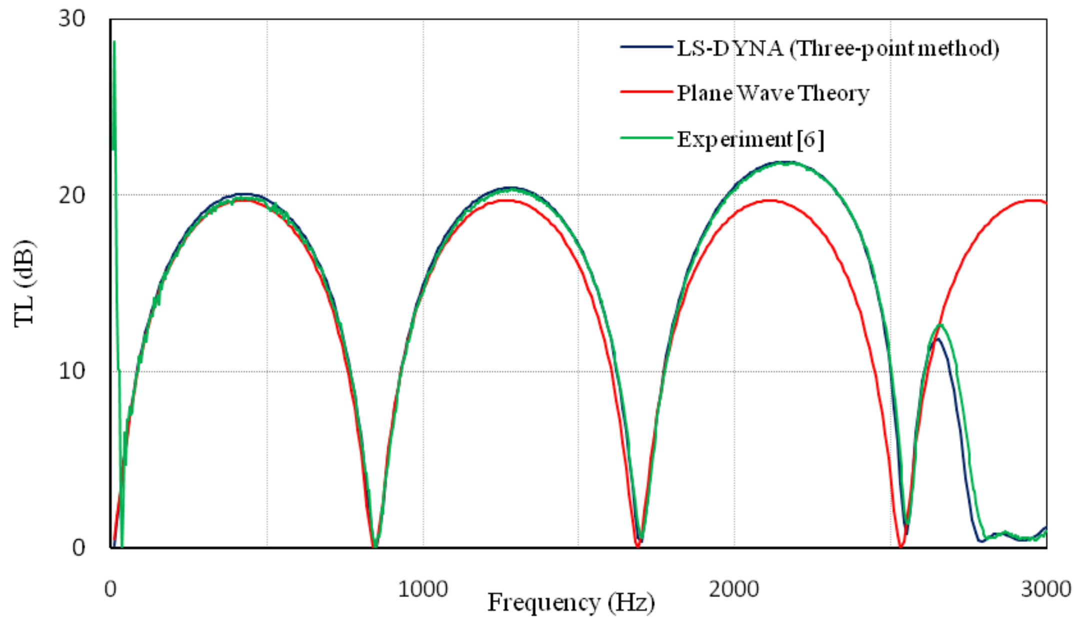


Figure 4. Transmission Loss Comparisons (Three point method)

Figure 4 shows the transmission loss comparisons between LS-DYNA (using three-point method), plane wave theory and the measured data. In three-point method, the three field point sound pressures are computed to get TL. The field point 1 is arbitrarily selected to be 0.3 inches away from the inlet (that is, x_1=0.3), point 2 is 2.3 inches away from the inlet (x_2=2.3), and point 3 is at the distance of 0.3 inches from the outlet.

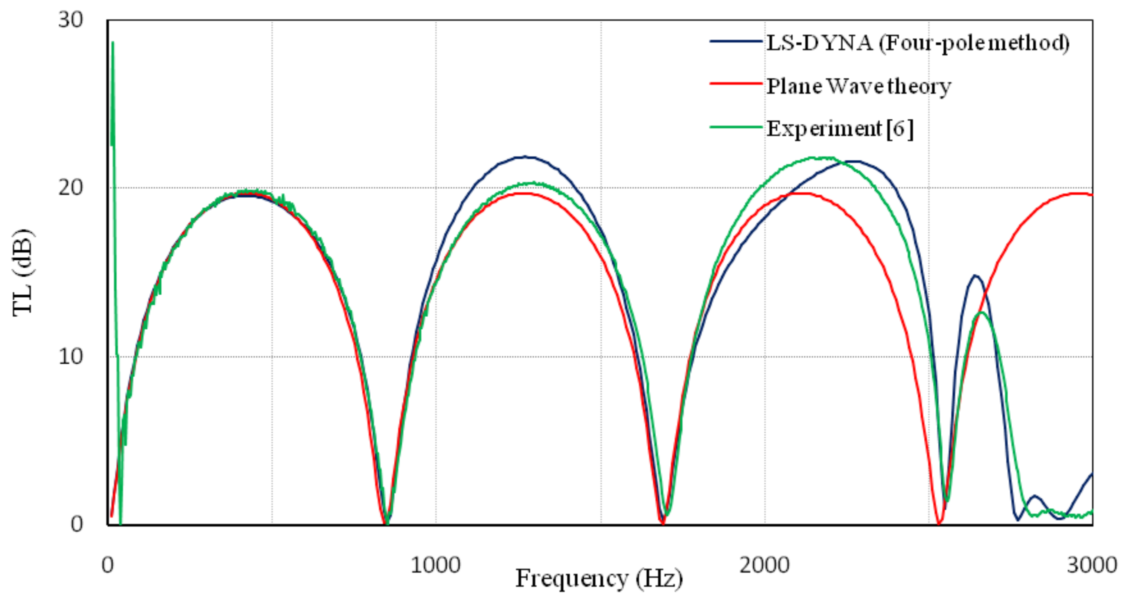


Figure 5. Transmission Loss Comparisons (Four pole method)

Figure 5 shows the transmission loss comparisons between LS-DYNA (four-pole method), plane wave theory and the measured data. Two field point sound pressures are computed. The field

points are arbitrarily selected to be 0.3 inches away from the inlet and outlet within the muffler. The four-pole parameters are evaluated from the field point pressures from run 1(v_1=1, v_2=0) and run 2 (v_1=0, v_2=-1).
The plane wave theory is not valid for frequencies above cutoff frequency. From [7] the cutoff frequency is $c/2d$ (d is diameter). In this case, the lowest cutoff frequency is 1119 Hz. For f >1119 Hz, the wave transmission in the expansion chamber is not a plane wave, and becomes more complex. The LS-DYNA results match well with the published measurement results over the entire frequency range.

Conclusions

In this paper, BEM implemented in LS-DYNA is used to predict the transmission loss of mufflers. A muffler of simple expansion chamber is employed as a case study. Both the three-point method and the four-pole matrix method are used to calculate the transmission loss. The numerical results are examined with the plane wave theory and the experiment results. It shows that LS-DYNA can provide an effective tool for computing the transmission loss of mufflers.

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