

# Simulations of Axial Impact of Composite Structures With a Coupled Damage-Plasticity Model

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## Abstract

*Under axial impact, a composite structure can split into pieces at the crush front while maintaining an apparent structural integrity in its uncrushed portion. In this way, the structure is capable of sustaining large deformation under a sufficiently high load. To simulate this behavior, the constitutive model must be able to describe the response of a substantially damaged composite under dynamic loading. The constitutive behavior beyond the peak loading is not well represented in common composite constitutive models. This paper presents the recent development of a coupled damage-plasticity model for composites and its applications in crush simulations.*

## 1. Introduction

Fiber reinforced polymer composites exhibit high Specific Energy Absorption (SEA) values under axial impact loading. They are the preferred materials for lightweight energy absorbing structures such as front rails in vehicles, airplane or helicopter subfloors, and landing gears. The high SEA value of composite structures is attributed to extensive failure, damage and fracture of the material. Fig.1 shows examples of failure morphologies of two types of fabric composite tubes and a steel tube under axial impact. The steel tube failed by buckling and folding without fracture. Composite tubes are crushed. The crush morphology was found to vary with the reinforcement architectures, types, etc. A composite structure can be crushed into debris or split into pieces at the crush front while maintaining an apparent structural integrity in its uncrushed portion. In this way, the structure is capable of sustaining large deformation under a sufficiently high load. To simulate the behavior of composites with continuous crush front, the constitutive model must be able to describe the response of a substantially damaged composite under loading and unloading conditions [1]. It has been shown that the continuum damage mechanics CDM based composite constitutive models cannot capture the behavior of damaged composites in unloading segment. Therefore, models with modifications to unloading segment [2,3] and coupled damage-plasticity models have been proposed [4]. This paper presents the recent development of a coupled damage-plasticity model for composites and its applications in crush simulations.

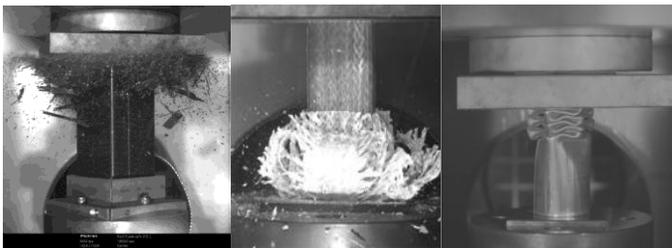


Figure1. Crushing morphology: (a) a woven carbon fiber/epoxy tube; (b) a tri-axial braided carbon/vinyl ester composite tube and (c) a dual phase steel tube, (courtesy of M.Starback).

## 2. Background

Following the notation of classic CDM [5,6], the modulus of elasticity of the undamaged material is denoted as  $E_0$ , the modulus of elasticity of the damaged material is denoted as  $E$ . The relationship between the two properties is

$$E = E_0(1 - d) \tag{1}$$

where  $d$  is a damage variable,  $d \in [0,1]$ . A damage value 0 to 1 corresponds to the undamaged state to a completely damaged state.

For an undamaged material, the stress-strain relation in the elastic range is given by the Hook's law

$$\sigma = \varepsilon E_0 \tag{2}$$

where  $\sigma$  is the stress and  $\varepsilon$  is the strain. The elastic stress-strain relation for a damaged material is obtained from Eq.1 and Eq.2

$$\sigma = \varepsilon E = \varepsilon E_0(1 - d) \tag{3}$$

Eq.3 indicates that the stress is always linearly proportional to the strain in terms of the elastic modulus of the damaged material. The stress-strain relation at any given point is uniquely determined if the elastic modulus of the damaged material is known.

In phenomenological CDM models, the damage variables are frequently expressed as functions of macroscopic measurable variables, such as strains [7], and generalized strains [8]. For example, an exponential damage evolution law as a function of strain was proposed as below [7]

$$d = 1 - \exp\left[-\frac{1}{me}\left(\frac{\varepsilon}{\varepsilon_f}\right)^m\right] \tag{4}$$

where  $\varepsilon_f$  is the nominal failure strain,  $m$  is a parameter and  $e$  is the Naperian logarithm base.

The damage evolution described by Eq.4 is shown in Fig. 2a. Fig. 2b presents a 1-D stress-strain curve of a material described by Eq.3 with the damage law in Fig. 2a.

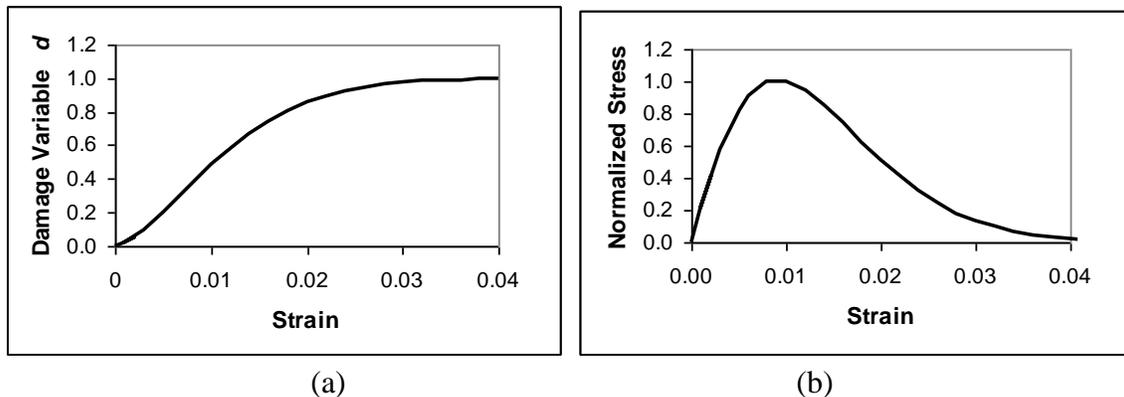


Figure 2. (a) Damage evolution described by equation (2.4) and (b) the stress-strain curve produced by CDM.

By representing the elastic modulus as a function of damage, the nonlinear stress-strain response including a post-peak softening behavior is generated with a linear stress-strain relation. This simple example illustrates the framework of the CDM constitutive models.

The main advantage of the CDM framework is its simplicity in representing strain softening. In CDM, the damaged material is treated as an elastic continuum. The deformation is determined solely by the damage state and does not depend on the loading history. It is relatively easy to implement CDM constitutive models in a finite element (FE) code. This simplification does not affect the computational results when the load increases monotonically. As long as the total strain is known, the stress value will be uniquely determined and the total energy absorption of the system will be correctly computed.

An obvious limitation of the CDM framework is the absence of irreversible strains. Irreversible deformation is inevitable in substantially damaged materials. When local load changes the sign from tensile to compression, the roughness of a cracked surface tends to prevent it from closing completely. Inversely, when local load changes its sign from compression to tensile, the fragmented matrix tends to block the way for the kinked or buckled fiber tows in compression damaged composites to straighten. Upon impact loading, brittle matrix becomes crumbled or fragmented which increases its occupied space. When the load is removed, the damaged material will not return to its original volume unless a load of opposite sign is applied.

When unloading occurs, neglecting the contribution of the irreversible strains can lead to error in the calculation of the total energy absorption. As an example, consider a displacement controlled uni-axial loading-unloading cycle, as depicted in Fig. 3a. The classic CDM model generates a stress-strain locus as shown in Fig. 3b. The internal energy of the material under deformation can be approximated by the area under the stress-strain curve. The area enclosed by OAB gives the total dissipated energy by the material for the entire loading-unloading cycle due to damage. However, if a portion of the total strain at point B were irreversible, the net energy absorption for this loading-unloading cycle would have been greater than the area enclosed by OAB.

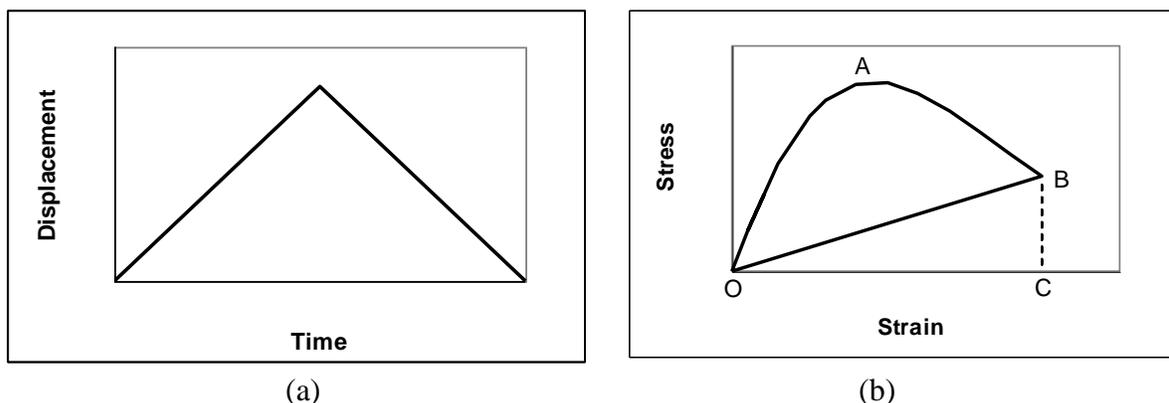


Figure 3. (a) A displacement controlled loading and unloading cycle. (b) The total energy absorption predicted by a CDM model is the area enclosed in OAB.

In structural analysis, even when the loading direction remains the same, some local areas can undergo unloading as a result of stress redistribution upon local damage and softening. Neglecting the irreversible deformation tends to result in an under predicted total energy absorption of the structure.

To describe the elastic softening and the irreversible strain, one has to use both CDM and plasticity theories. Coupling the two theories in a unified constitutive model for elastic, plastic, and damage behavior has been the subject of numerous studies. Most of these models were developed for metallic materials and formulated within the framework of plasticity, even when the anisotropy caused by damage was considered.

Among coupled damage-plasticity models, only Ladeveze model [9] was based on the framework of CDM. The damage-plasticity coupling was realized through the use of an elastic domain function, which was the summation of the effective stress and an isotropic hardening function. The total strain was decomposed as in a conventional elastic-plastic material law. The hardening function was expressed in the form of a power law of the accumulated plastic strain. Johnson et al. [10] implemented Ladeveze model in PAMCRASH.

### 3. A coupled damage-plasticity model

Based on the physical evidence of damage accumulation process in composites and the fundamentals of the two theories, a simple coupling method was proposed [4]. This method employs a perfect plastic flow rule within a CDM framework.

The main advantage of employing a perfect plastic flow rule to represent plasticity in CDM framework is the elimination of competition between elastic softening and work hardening. As depicted in Fig.4, a work hardening plastic flow rule describes a yield surface that expands, whereas, the failure surfaces in the sense of CDM shrinks with increasing strain. Using one surface with two potentials would create competition between elastic softening and work hardening and thus create complications. It not only requires iterations, but also hinders the model's ability to represent a strong softening response. Furthermore, a perfect plasticity rule requires minimal number of parameters. It satisfies the requirement of computational efficiency.

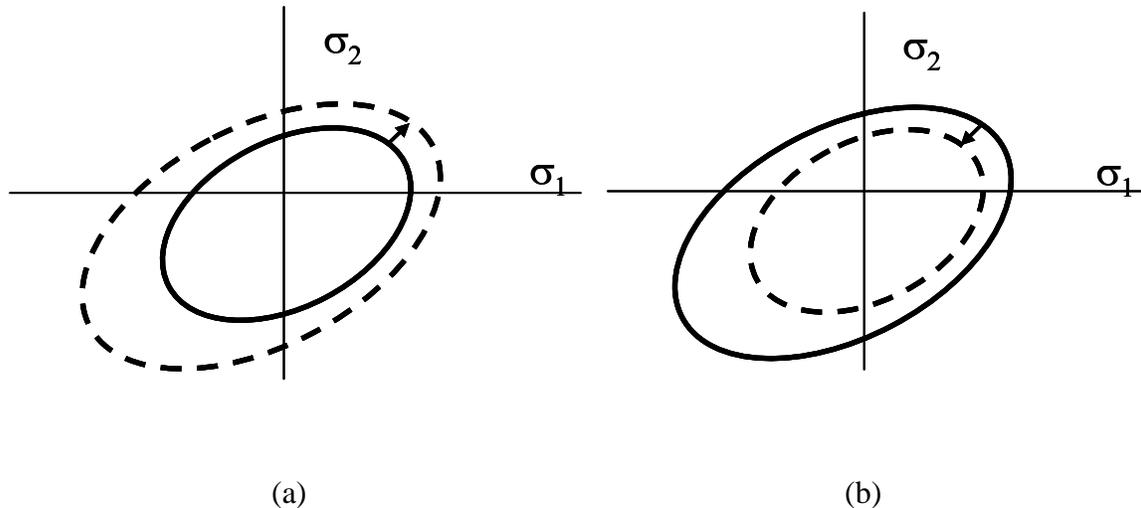


Figure 4. Schematic of the evolution of (a) the yield surface described by a work hardening plastic flow rule and (b) failure surface described by CDM.

The use of a perfect plastic flow rule also simplifies the strain decomposition. Because the plastic strain increment neither add nor reduce any resistance to deformation but simply to absorb the irreversible portion of it, the resistance to deformation can be attributed entirely to the elastic strain increment. This leads to a set of simple formulae as presented below.

### Strain partition

Considering that plastic deformation does not start at an infinite small strain, it is reasonable to assume a threshold strain value  $\varepsilon_0$ . For 1-D case, the relationship between the total strain increment  $\Delta\varepsilon$  and its components is written as:

$$\varepsilon \leq \varepsilon_0, \Delta\varepsilon = \Delta\varepsilon_e \quad (5)$$

$$\varepsilon > \varepsilon_0, \Delta\varepsilon = \Delta\varepsilon_e + \Delta\varepsilon_p \quad (6)$$

where  $\Delta\varepsilon_e$  is the elastic strain increment, and  $\Delta\varepsilon_p$  is the plastic strain increment.

A partition coefficient  $\alpha$  is introduced such that

$$\Delta\varepsilon_e = \alpha\Delta\varepsilon \quad (7)$$

$$\Delta\varepsilon_p = (1-\alpha)\Delta\varepsilon \quad (8)$$

$\alpha \in [0,1]$ .  $\alpha$  can be a scalar or a function. The partition coefficient can be determined by uniaxial loading and unloading experiment. For a 3-D composite, the above relation is applicable to every material direction. Anisotropy in plasticity can be accommodated by using different partition coefficients in different material directions.

### Stress increment

Assume that the plastic response of the material follows the description of a perfectly plastic material and the yield point of the material always coincides with the current failure surface. The stress increment, therefore, is solely related to the elastic strain increment

$$\Delta\sigma = \Delta\varepsilon_e(1-d)E_0 = \alpha\Delta\varepsilon(1-d)E_0 \quad (9)$$

Equations (8) and (9) allow the accumulation of plastic strain as the failure surface shrinks. Since plasticity does not contribute to the evolution of the failure surface, introducing plasticity in the CDM framework does not introduce iterations commonly needed in solving plasticity problems.

Figure 5 compares the 1-D stress-strain responses with unloading-reloading paths generated using a CDM model and a coupled plasticity-CDM model. In this study, the two models had the same damage function and the same parameters. The coupled model had a threshold strain corresponding to the strain at the peak stress and a partition coefficient  $\alpha=0.5$ .

As expected, the two curves are identical up to the peak stress. At the post-peak region, where the strain is greater than the threshold strain, the coupled model produced a slower softening response due to the accumulation of plastic strain.

With  $\alpha=0.5$ , the total strain is equally divided into elastic and plastic components, the total strain increment generated by the coupled model is twice of that by the CDM model. Because the stress increment is solely due to elastic strain, the elastic strains in the two models have the same value at the same stress level. This in turn results in parallel unloading and reloading paths between the two models when compared at the same stress level. If the coupled model uses a different set of damage parameters, the unloading slope would be different.

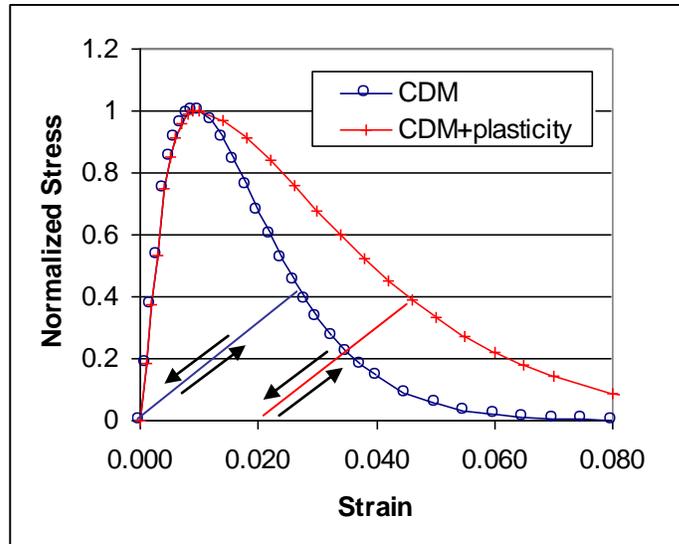


Figure 5. Schematic of the stress-strain curves generated by the CDM model and the coupled CDM-plasticity model. The unloading – reloading paths are shown.

#### 4. Implementation

To consider plasticity in an existing CDM model, two additional operations are required: strain partition and stress increment. Figure 6 presents an algorithm for inserting these two operations into a strain incremental step in a CDM framework for explicit FE implementation. It consists of three logical decisions (diamonds). From left to right, the first decision determines whether the plasticity is to activate or not. When the total strain is less than the threshold strain  $\varepsilon_0$ , the stress increment is calculated as in a CDM model. Otherwise, plasticity is considered. The second decision determines the loading conditions. The strain partition depends upon the loading conditions. When the loading condition is satisfied, the strain partition follows Eq. 7 and Eq. 8. When neutral loading or unloading conditions are satisfied, the material is in the elastic domain and the strain increment is entirely due to elastic deformation. The third decision determines whether the strain in the material is crossing the zero line from compression to tension. This decision is added to model the unique nonlinear response of composites reinforced with fabrics of large tow size.

With a perfect plastic flow rule, the incorporation of plasticity into the loading surfaces of the MLT model is achieved by replacing the strain components by the elastic strain components

$$\boldsymbol{\varepsilon}_e = (\varepsilon_{e11}, \varepsilon_{e22}, 2\varepsilon_{e12})^T \quad (10)$$

For 2-D case, the relationships between the total strain increment  $\Delta\varepsilon$  and its components for each strain components are similar to 1-D case as followings:

$$\varepsilon \leq \varepsilon_{0i}, \Delta\varepsilon_i = \Delta\varepsilon_{ei} \quad (11)$$

$$\varepsilon > \varepsilon_{0i}, \Delta\varepsilon = \Delta\varepsilon_{ei} + \Delta\varepsilon_{pi} \quad (12)$$

where  $i=11, 22$  and  $12$ .

Using the coupling method described above, the Matzenmiller-Lubliner-Taylor (MLT) model [7] was extended into a coupled damage-plasticity model. Details are provided in [4].

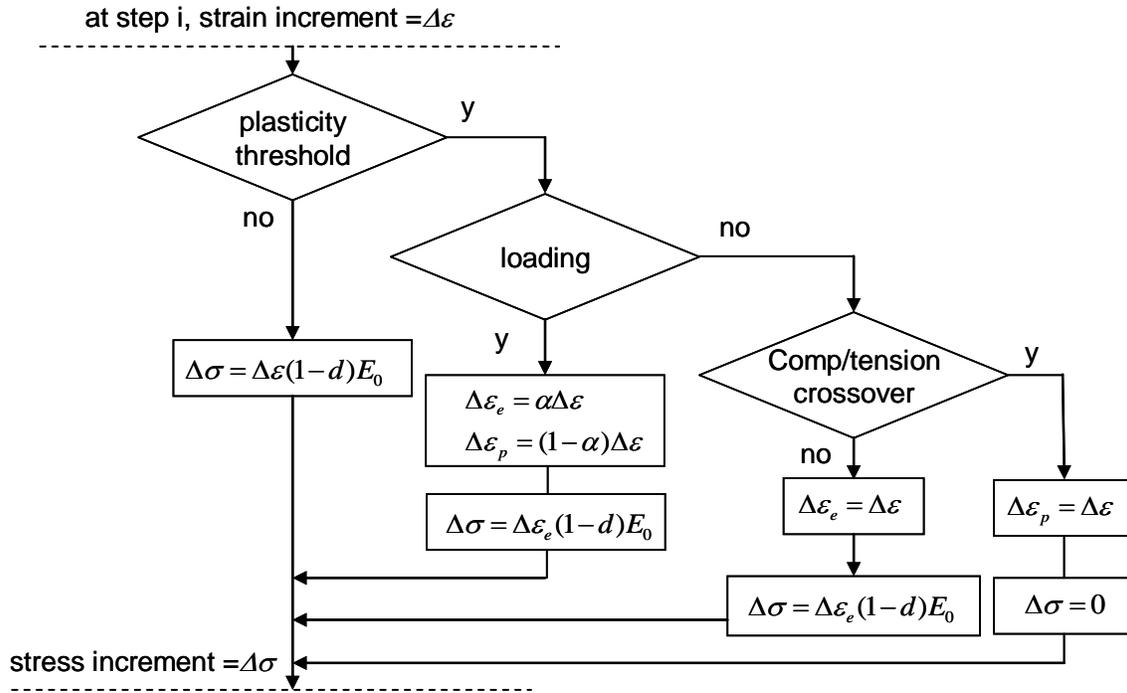


Figure 6. Flowchart for explicit finite element implementation.

## 5. Numerical Examples

### 5.1 A composite tube subjected to axial impact

Simulations were carried out for a composite tube reinforced with 1-ply Fortafil 80k/80k 0/30/-30 braid [1] at 7.1m/s. Figure 7 compares the deformation morphologies and the load-displacement responses obtained by MAT58 and the coupled model.

With MAT58, the crush front of the tube collapsed due to the excessive softening at large strain. With the coupled model, a more realistic crush morphology was obtained and the load-displacement response was closer to the experimental results.

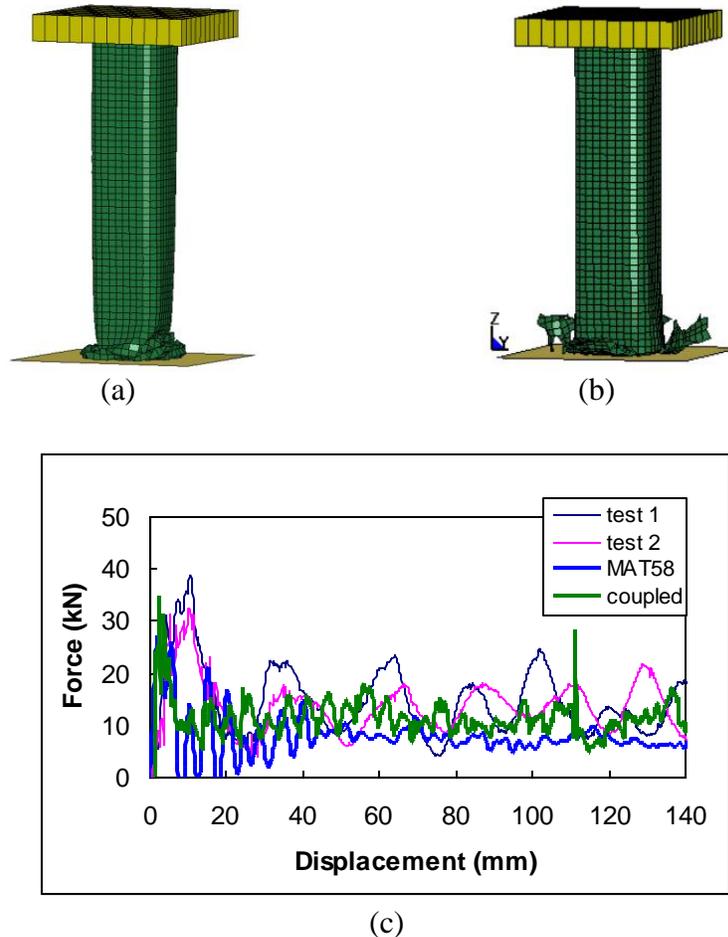


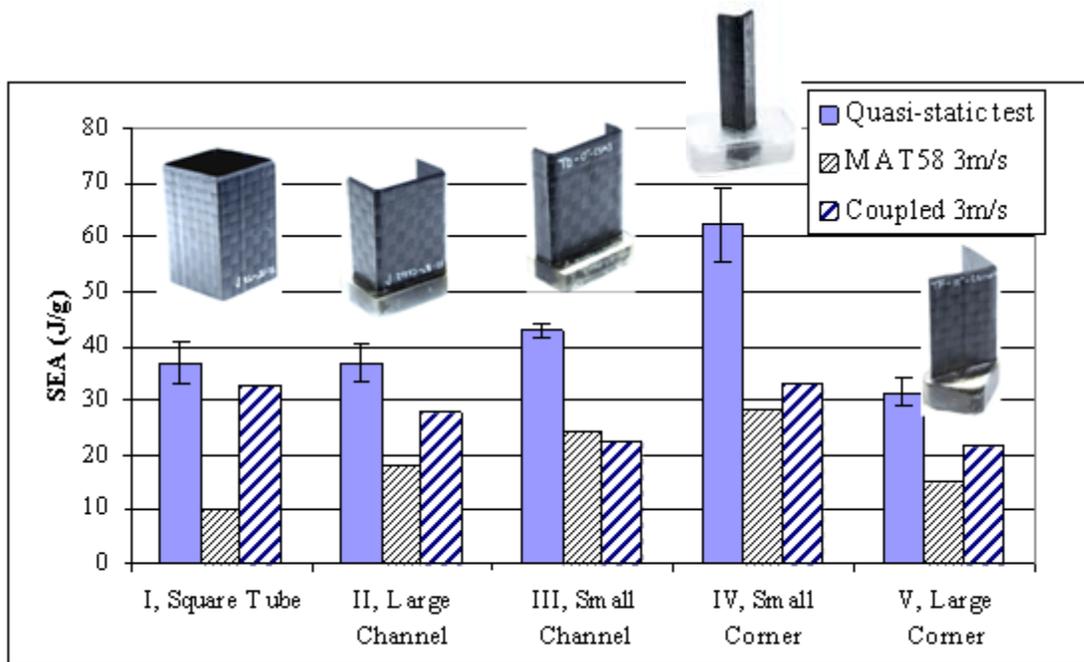
Figure 7 A composite tube reinforced with 1-ply tri-axial braid under axial impact. Simulation was performed using explicit FE code LS-DYNA with MAT58 and the coupled damage-plasticity model. Crush morphology obtained by (a) MAT 58 and (b) coupled model. (c) Comparison of force-displacement responses generated in simulation with the experimental curves.

### 5.2 Composite tube segments

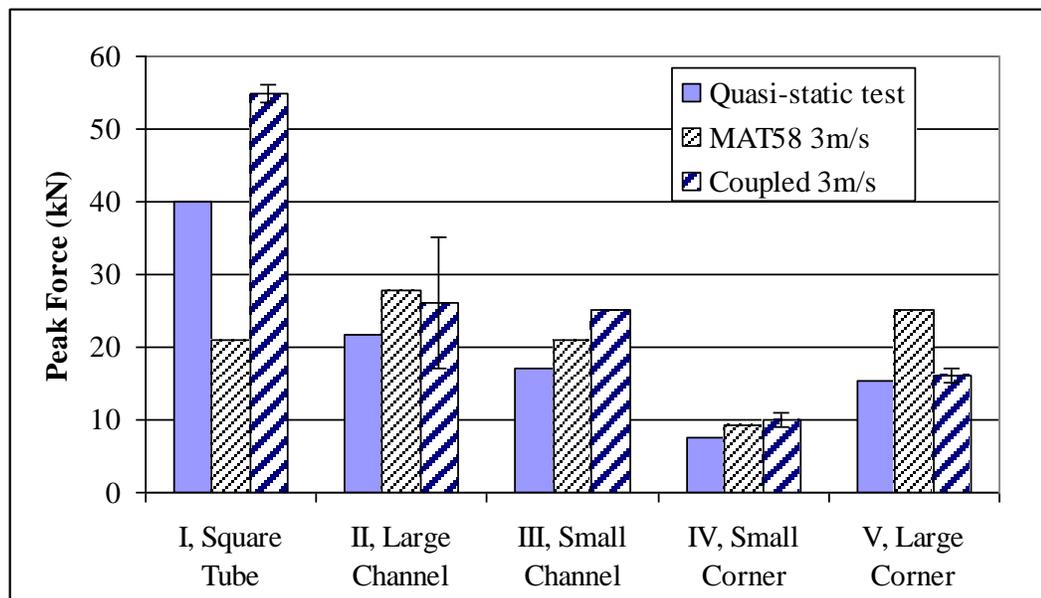
In a numerical round robin organized by the Composite Material Handbook CMH-17 crashworthiness working group, quasi-static crush experiments were performed for five different channel segments [11], Fig.8. The material was a carbon plain weave composite with large tow size. Simulations were carried out with MAT58 and the coupled models at 3m/s.

Fig.8a compares the specific energy absorption (SEA) values. It shows that both models yielded lower SEA values. This trend agrees with the experimental observations that the SEA values are lower in dynamic crush tests. Compared to MAT58, the coupled model predicted higher SEA and yielded a more reasonable prediction for shape I.

Fig.8b compares the peak force values. Both models yielded higher peak forces. The trend agrees with the experimental observations that the peak forces are higher in dynamic crush tests. In general, the simulations with the coupled model were more stable.



(a)



(b)

Figure 8 Comparison of quasi-static crush for tube and tube segments with simulations with MAT58 and the coupled model. (a) SEA. (b) The peak force.

## 6. Summary

Composite structures with large size fiber tows form continuous fronds in axial impact events. The material in fronds experiences unloading when moving out crush front. To correctly represent the stress-strain response of substantially damaged composites is an important requirement in modeling the energy absorption in crash events. The continuum damage mechanics (CDM) based composite models are not sufficient in modeling the behavior of composites with substantial damage. To consider the irreversible deformation due to damage, a coupled damage-plasticity model was developed. The coupled model improved the stability of the tube crush simulations and the predictions.

## 7. Acknowledgement

The simulations for CMH-17 round robin were performed by Ms. Azadeh Sheidaei.

## 8. References

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