On Mooney-Rivlin Constants for Elastomers

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Abstract

The Mooney-Rivlin constitutive equation for rubber is

$$W = C_1(I_1 - 3) + C_2(I_2 - 3)$$

where material constants C_1 *and* C_2 *must be determined through tests. The constant* C_1 *can be determined by uniaxial tension or compression tests; however,* C_2 *cannot be determined accurately by uniaxial tension or compression tests. In order to determine* C_2 *, biaxial tests must be performed. A biaxial test, inflation of a circular membrane, is presented in detail here for determining both* C_1 *and* C_2 *.*

Introduction

For Mooney-Rivlin materials the strain-energy density equation is:

$$W = C_1(I_1 - 3) + C_2(I_2 - 3) = C_1[(I_1 - 3) + \alpha(I_2 - 3)] \quad (1)$$

where C_1 and C_2 are material constants and $\alpha = C_2 / C_1$. The strain invariants I_1 and I_2 are written in terms of the principal stretch ratios λ_1, λ_2 and λ_3

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$$
$$I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2 \quad (2)$$

An incompressibility condition is assumed in the Mooney-Rivlin material constitutive equation so that

$$\lambda_1\lambda_2\lambda_3 = 1 \quad (3)$$

For uniaxial tension or compression, the stress (force per unit undeformed area) σ is related to the uniaxial stretch ratio λ

$$\sigma = 2C_1\left(\lambda - \frac{1}{\lambda^2}\right)\left(1 + \frac{\alpha}{\lambda}\right) \tag{4}$$

The dimensionless uniaxial stress $\dfrac{\sigma}{C_1}$ for various values of α vs. uniaxial stretch ratio λ is shown in Figure 1.

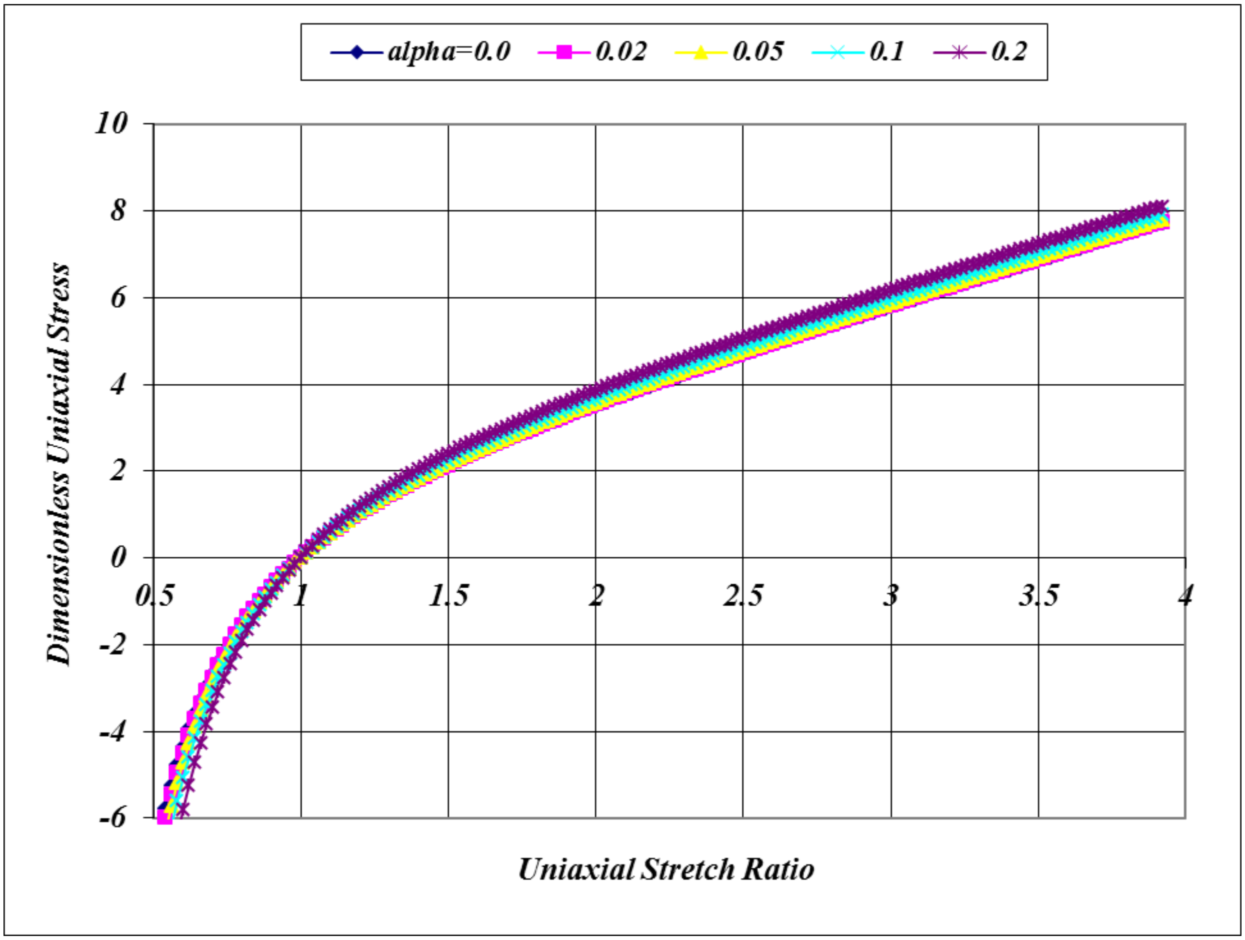

Figure 1, Uniaxial stress-strain curves

For homogenous biaxial tension or compression,

$$\lambda_1 = \lambda_2 = \lambda\,. \tag{5}$$

The stress (force per unit undeformed area) σ is then related to the stretch ratio λ

$$\sigma = \frac{2C_1}{\lambda}\left(\lambda^2 - \frac{1}{\lambda^4}\right)\left(1 + \alpha\lambda^2\right) \tag{6}$$

The dimensionless homogenous biaxial stress $\frac{\sigma}{C_1}$ for various α vs. biaxial stretch ratio λ is shown in Figure 2.

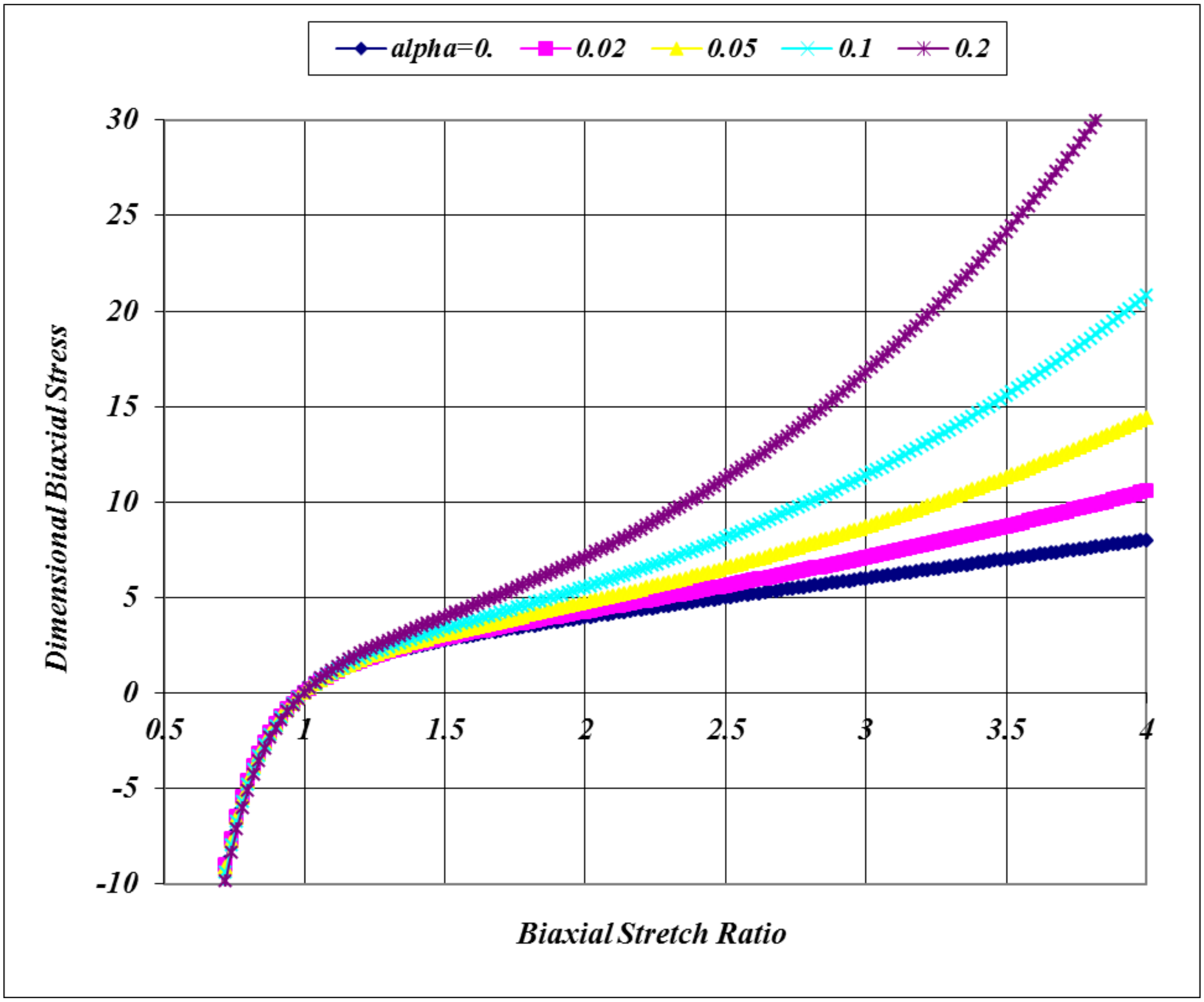

Figure 2, Homogenous biaxial stress-strain curves

From Figures 1 and 2, it is clear that uniaxial tests cannot be used to determine α, and biaxial tests must be performed. A biaxial test, inflating of a plane circular membrane, is presented here.

A Biaxial Test

In the experiment a flat circular membrane of rubber was clamped between two plates as shown in Figure 3. The membrane is inflated by air or liquid from a reservoir at a constant temperature. The height of the deformed membrane at the pole is measured by a LVDT or a laser beam. The pressure is measured with a pressure transducer. A data acquisition system and a computer gather the data from the LVDT and pressure transducer and are shown in Figure 4.

The pressure-height relationship is measured. During loading, at the pole, the deformation is in a uniform biaxial stress state.

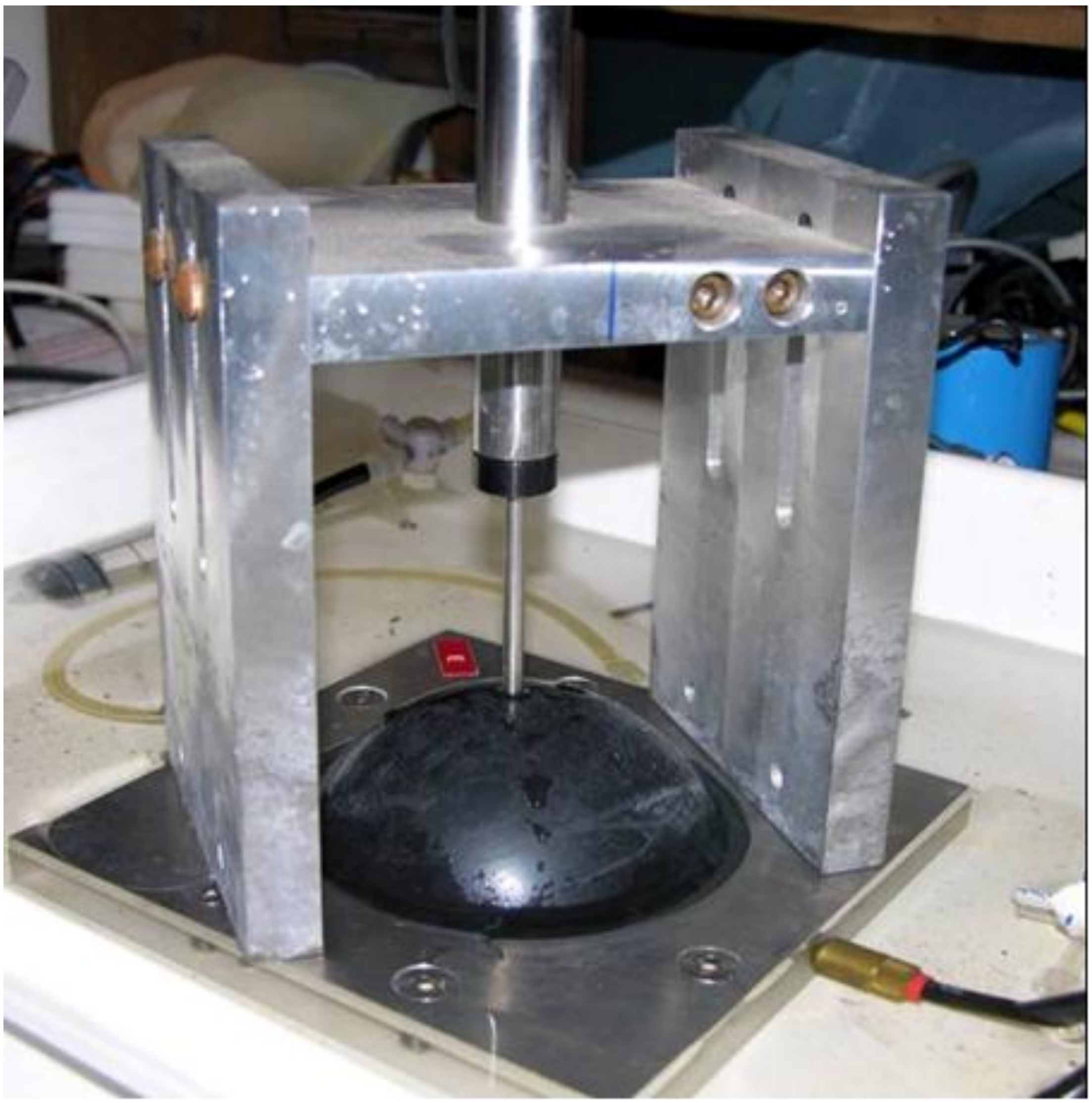

Figure 3, The apparatus

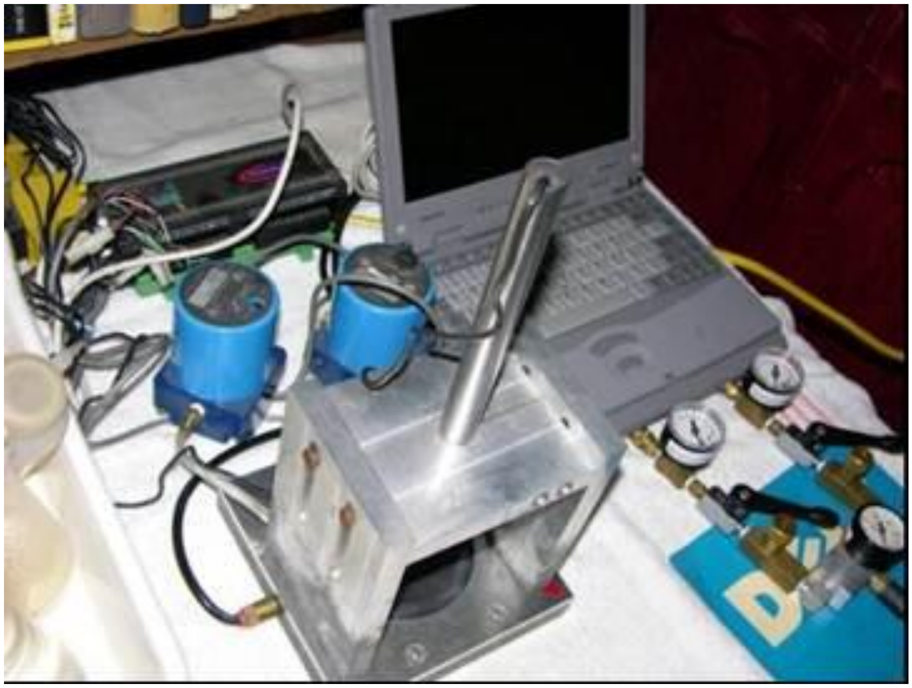

Figure 4, Laboratory set-up for the biaxial test

Approximate Solution for Inflating a Circular Membrane

Numerical solution for studying the inflation of a thin circular membrane has been obtained by Yang and Feng [1] and Feng [2]. However, using the numerical method for determining the material constants will be cumbersome. Here we used the approximate solution of inflating pressure and deformation at the pole, by Christensen and Feng [3], to determine the material constants C_1 and α. The result of the approximate solution by Christensen and Feng [3] is outlined here. The approximate relationship between the inflating pressure (P) and the deformation at the pole (Δ) is

$$P = \frac{4C_1 H}{R}\left\{\frac{2}{\left(\frac{\Delta}{R}+\frac{R}{\Delta}\right)}\left[\left(1-\frac{1}{\lambda^6}\right)+\alpha\left(\lambda^2-\frac{1}{\lambda^4}\right)\right]\right\} \tag{7}$$

The relationship between λ and Δ is

$$\lambda = \frac{\left(\frac{\Delta}{R}+\frac{R}{\Delta}\right)}{2}\sin^{-1}\left(\frac{2}{\left(\frac{\Delta}{R}+\frac{R}{\Delta}\right)}\right) \tag{8}$$

where R is the initial radius of the circular membrane and H is the initial thickness of the circular membrane. The dimensionless pressure ($\tilde{P}$) and the dimensionless displacement at the pole ($\tilde{\Delta}$) are

$$\tilde{P} = \frac{RP}{4C_1 H} \quad \text{and} \quad \tilde{\Delta} = \frac{\Delta}{R} \tag{9}$$

Hence,

$$\tilde{P} = \frac{2}{\left(\tilde{\Delta} + \frac{1}{\tilde{\Delta}}\right)} \left[\left(1 - \frac{1}{\lambda^6}\right) + \alpha\left(\lambda^2 - \frac{1}{\lambda^4}\right) \right] \tag{10}$$

$$\lambda = \frac{\left(\tilde{\Delta} + \frac{1}{\tilde{\Delta}}\right)}{2} \sin^{-1}\left(\frac{2}{\left(\tilde{\Delta} + \frac{1}{\tilde{\Delta}}\right)} \right) \tag{11}$$

When $\tilde{\Delta} \le 1$, the $\sin^{-1}()$ takes the value in the first quadrant. When $\tilde{\Delta} \ge 1$, the $\sin^{-1}()$ takes the value in the second quadrant. The approximate solution obtained from the above equation, the exact numerical solution by Feng [2], and the test data for inflating a circular thin membrane are shown in Figure 6 for a Mooney material with $\alpha = 0.02$.

The approximate solution is very good.

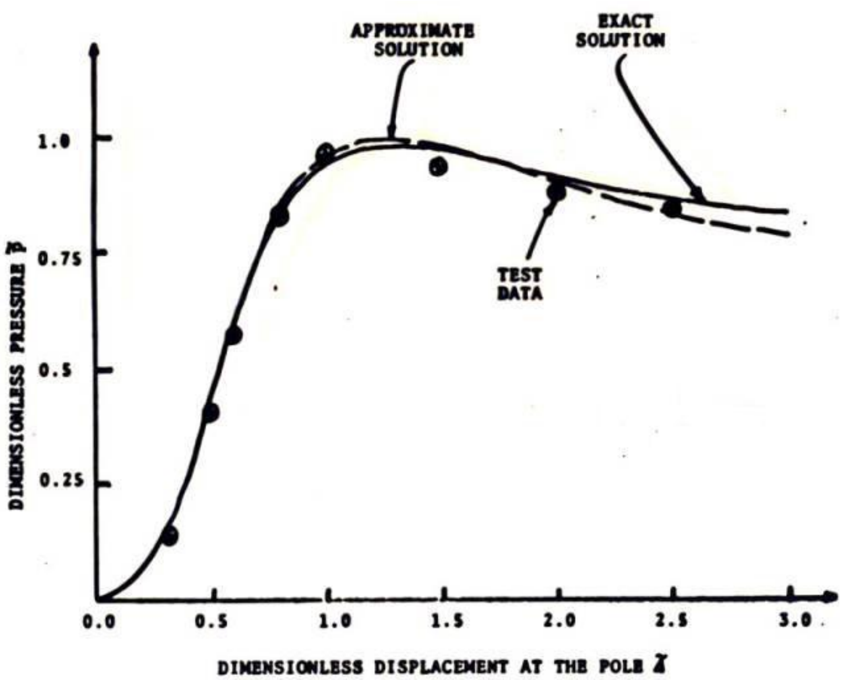

Figure 5, The approximate solution, exact solution and test data for an inflated membrane.

Determination of C_1 and C_2

The approximate solutions for an inflated membrane with various α are shown in Figure 6. From the inflating pressure–displacement at the pole test curve, we can use the shape to determine α and the dimensionless pressure to determine C_1. With α and C_1, C_2 is determined. One may also write a simple computer program to use the least square fit to determine these two constants.

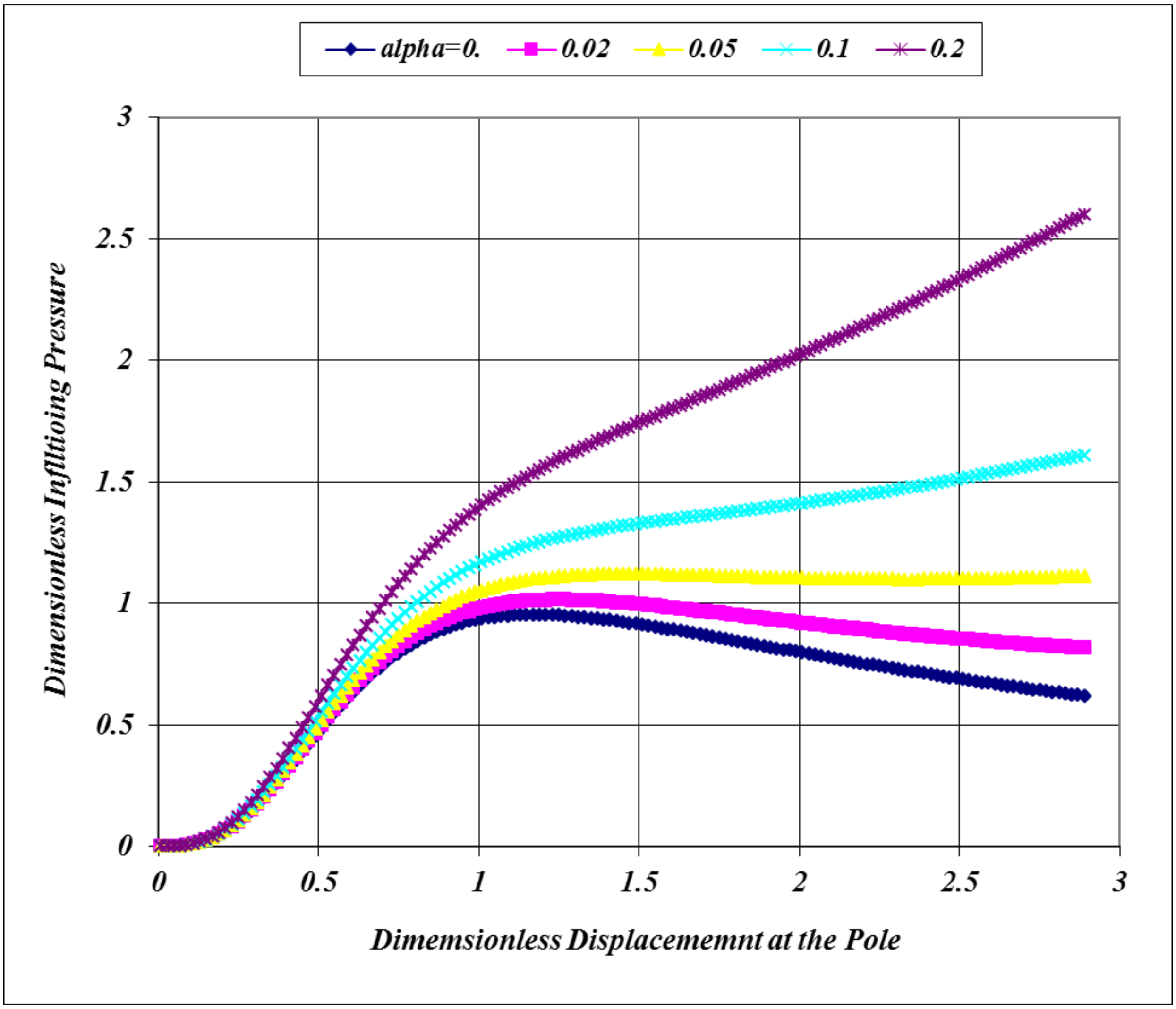

Figure 6, The dimensionless inflation pressure and the dimensionless displacement at the pole for the inflation of an initial flat circular membrane of various α.

In order to determine α accurately, the membrane should be inflated such that $\tilde{\Delta}$ is greater than 1.5.

A typical deformed membrane during a test is shown in Figure 7. The initial specimen geometry is given by $R = 1.0$ inch, $H = 0.01$ inches. The rate of data acquisition is one datum per second and the total test takes about 3 minutes. The pressure-deformation at the pole curve is shown in Figure 8. It is seen that the theoretical from provides a very satisfactory model of the data. It has the material constant values of $C_1 = 72$ psi and $\alpha = 0.105$; hence, $C_1 = 72$ psi and $C_2 = 7.56$ psi.

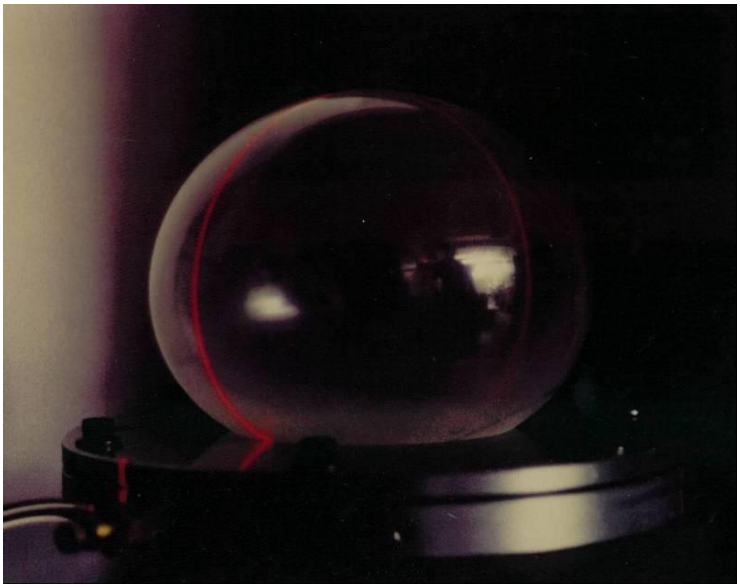

Figure 7, A membrane during test

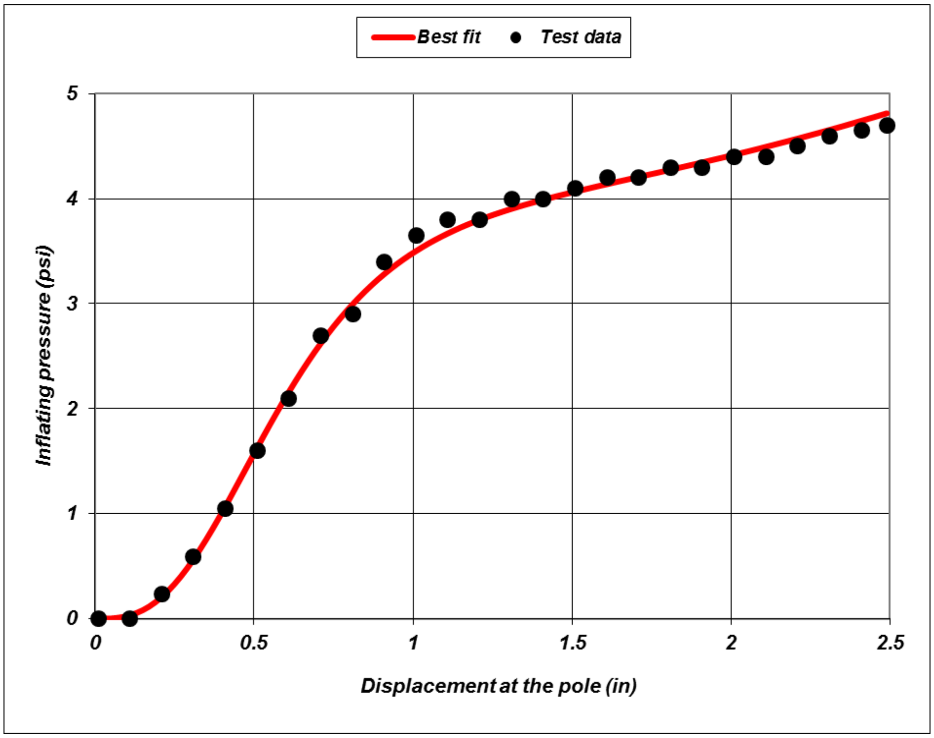

Figure 8, Test data and analysis

Remarks

It is noted that the same test apparatus can be used to determine material constants based on other constitutive equations. The test apparatus can also be used to provide test data for determining the material functions for viscoelastic behavior [4], the Mulllins effect [5] and the aging of elastomers [6].

References

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