LS-TaSC 4: Designing for the combination of impact, statics, and NVH

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Abstract

The projected subgradient method is major new methodology development for the topology optimization of huge, multi-disciplinary structural problems; for example, the combined impact, statics, and NVH design of a whole body in white. This paper accordingly discusses the projected subgradient method in LS-TaSC 4, with specific reference to the basic theory, the ability to combined impact and NVH load cases, and the performance for huge models. Also mentioned is how the method has been enhanced to handle generalized constraints using the multi-tensor numerical scheme.

Keywords LS-TaSC 4**;** projected subgradient method; impact design; huge models

1 Introduction

Generally, multi-disciplinary design optimization (MDO) is employed to seek for the synergism of several primary-mutually-interacted disciplines for improving the design of systems [1]. Due to the multi-fidelity and strong coupling between the multiple disciplines, the analysis and optimization solution time in MDO is typically much higher than the sum of the computational time of the constituent single-discipline optimization. Nevertheless, huge and complex models of the practical engineering structures and systems, such as automobiles and aerospace vehicles, make the burden of computational cost of MDO even more serious, because the MDO of these huge models are usually facilitated by a large number of design variables. With the rapid development of super-powerful computers over the past decade, interest in the MDO applications to sizing, shape, and topology optimization of huge and complex models has grown [2-3]. Multifidelity approximations and ensembles of metamodels have been effective and powerful means to alleviate the computational costs in the sizing and shape optimization of MDO problems [4-5]. However, they are not effective techniques for the topology optimization of MDO problems due to the accuracy and dimensionality issues of the metamodels. The topology optimization of MDO problems is a challenging task, especially if the MDO problem has a huge model saying with tens of millions of meshing elements (i.e., topological variables). In such a case, the choice of optimization approach obviously has enormous impact on the computational efficiency and convergence speed of topology optimization of huge MDO models.

In literature, the optimization approaches for addressing the topology optimization problems can be categorized into two major sets, including gradient- and non-gradient-based optimization approaches. The non-gradient-based optimization approaches generally refer to the heuristic approaches, such as genetic algorithm [6]. The non-gradient-based optimization approaches do a good job of spanning the vast majority of the design space because they have a good probability of attaining a global optimum solution. But the computational efficiency of the non-gradient-based optimization approach, which strongly depends on the number of design variables, is dragged down significantly due to numerous function evaluations within the design space. Apparently, the non-gradient-based optimization approaches are not options for solving topology optimization of huge MDO models. The gradient-based optimization approaches, such as the sequential quadratic programming [7], sequential linear programming [8], optimality criteria (OC) [9], and the method of moving asymptotes (MMA) [10] and its globally convergent version, the Globally Convergent Method of Moving Asymptotes (GCMMA) [11], are often used to search for the best material distribution of a structure in a manner of providing numerical material interpolation or geometric boundary representation schemes. Among the numerical schemes developed for topology optimization, the SIMP (Solid Isotropic Material with Penalization) method [12] is the most commonly used technique for building analytical relationships between the design variables and structural responses. With the analytical relationships by using SIMP, sensitivity analysis yields the derivatives of the structural responses with respect to the topological design variables in despite of huge MDO models. The gradient-based optimization approaches are efficient for solving problems with a large number of design variables using a limited number of function evaluations.

The choice of an algorithm for huge topology optimization poses some problems, specifically with respect to combining impact and NVH results and the huge number of variables. From our experience doing impact analysis it is clear that the most efficient method will involve projecting the problem onto the plane of constant mass and solving with an efficient Quasi-Newton method. Later in the project, the literature revealed that this approach actually belongs to a class of methodologies known as the subgradient methods developed by Shor [13] and other Soviet researchers. Although our implementation differ in some aspects, we adopted the existing name of projected subgradients because of the joint interest in properties such as (i) nondifferentiable functions, (ii) fixed step length, and (iii) jumps or increases in the function values which are properties of both the projected subgradient method [14] and the impact problems studied by us.

The current projected subgradient implementation is similar to the steepest descent method, but it is presented for topology optimization of constrained optimization problems. The main idea of the projected subgradient method is to project the update design along the decent direction on to the plane of the mass fraction constraint function, such that the mass constraint is satisfied automatically. Specifically, the combined impact, statics, and NVH design of a whole body in white with a mass constraint is targeted to solve for the huge MDO of an automobile. To the best of the authors' knowledge, the topology optimization of MDO problems for combined impact, statics, and NVH design of a whole system in white has not been touched yet in the field of topology optimization. The projected subgradient method is implemented in the latest version of LS-TaSC together with the high-fidelity analysis tool, LS-DYNA®. Three benchmark examples for NVH, multi-disciplinary design optimization considering NVH, impact and static load cases, and the ability to run huge models, are conducted to demonstrate the validity and computational efficiency of the proposed method.

2 The Projected Subgradient Method

2.1 Fundamentals

The projected subgradient method is proposed based on the steepest decent method while considering a structural constraint (e.g., mass constraint) and variable bound limits at the same time. In the steepest decent method, an update of the design is written as

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} - \alpha^{(k)}\boldsymbol{d}^{(k)} \tag{1}$$

where the upper subscript k represents the iteration number. $\boldsymbol{x}^{(k)}$ and $\boldsymbol{x}^{(k+1)}$ are the designs in the kth and (k+1)th iterations, respectively. $\alpha^{(k)}$ is the desired step size. $\boldsymbol{d}^{(k)}$ is the derivative vector of the objective function with respect to the design variables. The design search vector between two iterations is represented as $\Delta\boldsymbol{x} = -\alpha^{(k)}\boldsymbol{d}^{(k)}$.

In the projected subgradient method, the design search vector is projected onto the plane of an inequality structural constraint, so that the constraint function is satisfied with the update of the design. Assume that normal vector of the plane of the constraint function is presented as n. The design search vector projected onto the constraint plane can be expressed as,

$$\Delta\boldsymbol{x}^{P} = \Delta\boldsymbol{x} - \left(\frac{\Delta\boldsymbol{x}\cdot\boldsymbol{n}}{(|\boldsymbol{n}|)^2}\right)\boldsymbol{n} \tag{2}$$

where $\Delta\boldsymbol{x}^{P}$is the design change vector after the projection onto the constraint function.

Besides the constraint function, the side bounds on the design variable should be taken into consideration as well, as the side bounds on the design variables may cause the computations to be off. In order to correct for the effect of the side bounds of the design variables, a parameter λ is introduced to compensate the side bound effect on the updated design. The design search vector is thus expressed as,

$$\Delta\widetilde{\boldsymbol{x}}^{P} = \Delta\boldsymbol{x} - (1+\lambda)\left(\frac{\Delta\boldsymbol{x}\cdot\boldsymbol{n}}{(|\boldsymbol{n}|)^2}\right)\boldsymbol{n} \tag{3}$$

where $\Delta\widetilde{\boldsymbol{x}}^{P}$ is the design change vector after considering the side bound affects. In the above equation, the physical meaning of λ is to move the plane of the constraint function up and down, so that the constraint function is satisfied with updated designs within the side bounds. The value of λ can be positive, zero, or negative depending on how much the constraint plane should be moved. λ is typically found using a bisection algorithm such as to satisfy the constraint function.

Therefore, the updated design in the projected subgradient method is presented as,

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \Delta\widetilde{\boldsymbol{x}}^{P} \tag{4}$$

Due to the compensation of the side bound affects in the computation, the above updated design may exceed the range of the side bounds, $[x_{\min}, 1.0]$. Thus the updated design should be trimmed so that all the design variables have values within the range of the side bounds. The final updated design is obtained as

$$x_i^{(k+1)} = \begin{cases} x_{\min} & if\ x_i^{(k)} + \Delta\tilde{x}_i^P \leq x_{\min} \\ x_i^{(k)} + \Delta\tilde{x}_i^P & if\ x_{\min} < x_i^{(k)} + \Delta\tilde{x}_i^P < 1.0 \\ 1.0 & if\ x_i^{(k)} + \Delta\tilde{x}_i^P \geq 1.0 \end{cases} \tag{5}$$

Note that, without considering the constraint function, the updated design in Equation (1) will drive the structural property related to the constraint function to infinity. However, the updated design in Equation (5) can guarantee that the optimal design satisfies the constraint function all the time.

2.2 Step size and scaling of the gradients

The projected subgradient method differs from other methods in how the step size is computed. In most other methods, a line search is conducted to find the minimum function value in the search direction. In the projected subgradient method, however, no line search is required. Instead, a step size must be selected using the consideration of mass flow change between the iterative designs.

For the class of problems considered here, there is a natural choice of step size: the amount of material allowed to flow during an iteration. This means the step size depends on the mechanics of the problem and not the number of variables. Numerical concerns such as the mesh size therefore do not affect the step size.

To implement material flow as controlling the allowing step size requires that the difference between two sequential designs in computation should be considered. The material flow is a scaled version of the L_1 norm of the variable changes,

$$m_f = \frac{1}{N}\sum_{i=1}^{i=N}|\Delta x_i| \tag{6}$$

while the step size is the L_2 norm of the variable changes,

$$s = \sqrt{\sum_{i=1}^{i=N}(\Delta x_i)^2} \tag{7}$$

where N is the total number of elements in the structure. m_f represents the L_1-norm-based mass flow and s is the step size. The L_1 norm and L_2 norm of the variable changes are equal if all the variable changes have the same absolute value. In such a case, a material flow of $\overline{m_f}$requires an absolute change of $\overline{m_f}$ for each variable, given that the material flow is $m_f = \frac{1}{N}\sum_{i=1}^{i=N}|\Delta x_i| = \frac{1}{N}\sum_{i=1}^{i=N}\overline{m_f} = \overline{m_f}$. For this case, the step size is therefore computed as

$$s = \sqrt{\sum_{i=1}^{i=N}(\Delta x_i)^2} = \sqrt{\sum_{i=1}^{i=N}\overline{m_f}^2} = \sqrt{N}\overline{m_f} \tag{8}$$

Thus, different from the linear-search-based step sizes in the steepest decent method, a constant step size is used in the projected subgradient method.

In order to improve the computational stability, the value of *N* in Equation (8) must be adjusted to reflect the volume of the part being designed in the current iteration. Therefore, the value of *N* should be taken as the number of grey elements, N_{grey} in the design space, where the grey elements means the associated design variables of elements not at the side bounds. Besides the mass flow of the grey elements, the gradients of the objective must be scaled to its norm such that it matches the step size. Thus, the step size in the computation is obtained as,

$$\alpha^{(k)} = \frac{s^{(k)}}{|\boldsymbol{d}^{(k)}|} = \sqrt{{N_{grey}}^{(k)}}\frac{\overline{m_f}}{|\boldsymbol{d}^{(k)}|} \tag{9}$$

2.3 Constrained optimization

Most topology optimization problems have constraints on the structural responses; for example, a maximum displacement or required energy absorption. The current implementation uses the multi-tensor scheme as presented at the 2016 LS-DYNA conference [15]. This has been extended using continuous variables [16] as partially presented by Roux at the time and more fully at the 2017 USNCCM conference [17]. In addition, the projected subgradient method offers other possibilities.

3 Topology optimization of combined impact, statics, and NVH design

The typical goal of topology optimization is to obtain a structure with the best use of the material. Compliance and the strain energy density are the most commonly used objectives for linear static problems. For dynamic problems, like crashworthiness simulations, the structure needs to absorb the energy while maintaining the structural integrity and keeping the peak loads transmitted to the occupants low. In this paper, topology optimization of a structure with multiple load cases, specifically the combined impact, statics, and NVH load cases, is studied and solved by using the proposed method.
The optimization problem is formulated as

$$\min_{\boldsymbol{x}} \quad \sum_{j=1}^{L} w_j f_j(\boldsymbol{x}) \tag{10}$$

subject to

$$\sum_{i=1}^{N} \rho(x_i) V_i \leq M^* \tag{11}$$
$$x_{min} \leq x_i \leq 1.0 \quad (i = 1, \cdots, N; j = 1, \cdots L)$$

where the lower subscript i and j are the element number and load case indices, respectively. There are L load cases. $f_j(\boldsymbol{x})$ represents the objective function of the jth load case. w_j is the weight factor applied to the objective function of the jth load case. $\rho(x_i)$ is the density of the ith element, $V_i$is the volume of the ith element, and M* represents the allowable mass.

The objective for the NVH load case is to maximize the fundamental frequency, while for the impact and linear statics load cases the objective is to minimize the work done by the structure, which for linear structures is equivalent to minimizing the compliance or maximizing the stiffness. The search directions for the fundamental frequency is the derivative with respect to the element variables as described in the standard literature on design sensitivity analysis. The search directions for the impact and static load cases are computed from the internal energy densities of the elements. This history of using the internal energy density for design dates back at least to work by Venkayya et al. [18] in 1968 and is quite established as an optimality criteria.

The Projected Subgradient method uses a new stopping criteria called Solidification. It measures the discreteness of the optimized designs. A higher Solidification value indicates better topological designs. Assuming the total number of design variables as $N = N_{void} + N_{grey} + N_{solid}$, we define Solidification

$$M = \min(M_1,\ M_2) \tag{12}$$

where $M_1 = \frac{N_{void}+N_{solid}}{N}$, and $M_2 = 1 - \frac{\sum_{i=1}^{N} 4x_i(1-x_i)}{N}$. A default Solidification value of 0.95 is used for examples in this paper.

4 Examples

The examples show a comparison with the benchmark results for NVH, multi-disciplinary design optimization considering NVH, impact and static load cases, and the ability to run huge models.

4.1 Fundamental frequency and multidisciplinary problems

This example demonstrates the computational ability of the projected subgradient implementation for fundamental frequency and MDO problems for combined statics and NVH design. As shown in Fig. 1, an LS-DYNA model of a beam with dimensions of $8\ \mathrm{mm} \times 1\ \mathrm{mm} \times 0.5\ \mathrm{mm}$ is used as a benchmark problem to evaluate this. The material properties of the beam include density $\rho = 1.0$, Young's modulus $E = 10\ \mathrm{GPa}$, and Poisson's ration $v = 0.3$. A load of 10 units is applied at the center of the beam. Three boundary conditions are considered for the beam: 1) both ends of the beam are fixed, 2) left end of the beam is clamed and the right end is fixed, and 3) both ends of the beam are clamped. The beams with three boundary conditions are noted briefly as fixed-fixed, clamped-fixed, and clamped-clamped beams. A mass fraction of 0.5 is defined as the target mass fraction with element deletion switched on.

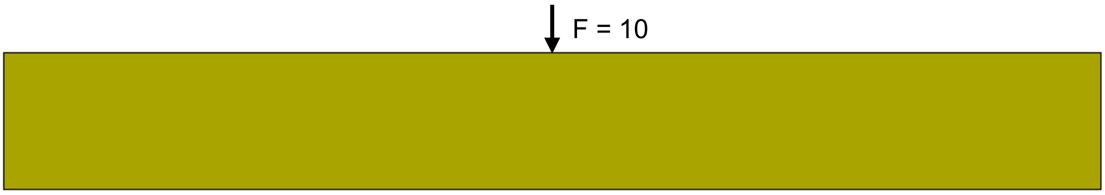
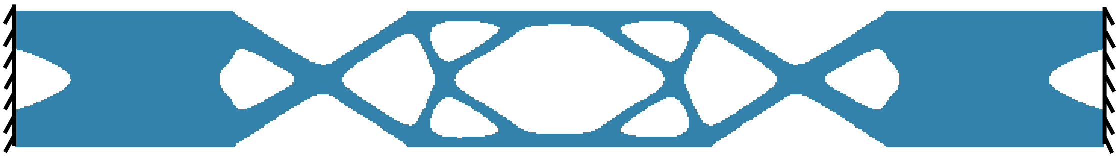


Fig.1: The geometry of a beam.

Firstly, topology optimization of the beam for fundamental frequency maximization is studied using LS-TaSC. Three optimization designs are conducted for the beam in terms of three boundary conditions. With the same optimization iterations, 100 iterations, three optimizations converge and the corresponding optimized configurations of the beam are shown in Fig. 2 a), b), and c) respectively. It can be seen that, the optimized results reflects the effects of the boundary conditions on the final configurations well. The optimized configurations show symmetric shapes on both sides of the beam because of the setting of boundary conditions. Three configurations of the beam in Fig. 2a), b) and c) yield maximal fundamental eigenfrequencies of 27.63 Hz, 44. 51 Hz, and 67.50 Hz, respectively.

a) Optimized geometry of the fixed-fixed beam

b) Optimized geometry of the clamped-fixed beam



c) Optimized geometry of the clamped-clamped beam

Fig.2: Optimized configurations of the beam with three boundary conditions for fundamental eigenfrequency maximization.

Secondly, topology optimization of the beam for multidisciplinary design of combined statics and NVH load cases are conducted by using the proposed method in LS-TaSC. The compliance and the first eigenfrequency of the beam are maximized simultaneously in the design. With respect to three boundary conditions, three MDO problems are solved. The convergence speeds are 40 iterations for the fixed-fixed beam, 46 iterations for the clamped-fixed beam, and 48 iterations for the clamped-clamped beam. The final configurations for three MDO problems of the beam with three boundary conditions are shown in Fig. 3 a), b), and c), respectively. Three configurations of the beam in Fig. 3a), b) and c) yield maximal fundamental eigenfrequencies of 27.16 Hz, 42.01 Hz, and 60.26 Hz, respectively.

a) Final structure for MDO of the fixed-fixed beam

b) Final structure for MDO of the clamped-fixed beam

c) Final structure for MDO of the clamped-clamped beam

Fig.3: Optimized configurations of the beam with three boundary conditions for combined statics and NVH design.

We can see from the above results that, the projected subgradient method can solve the MDO problems very well. In spite of the time for the finite element analysis in LS-DYNA, the convergence speeds of the MDO problems are much faster than that of the single-load-case problem of the fundamental eigenfrequency maximization.

4.2 Combined NVH, impact, and statics load case

This example shows MDO results and design contributions plots. The initial design together with the loading is shown in Fig. 4. It is designed for three load cases:

1. Fundamental frequency – the fundamental frequency of the structure is maximized
2. Impact load – an impactor hitting the structure as shown
3. Linear bending load case – a static load is applied.

The structure is designed is using 192,000 elements and a mass fraction of 0.1. All load cases are assigned an equal weight for the design. The material interpolation used is SIMP with continuation to final value of $p = 4$. The problem is optimized for 80 iterations with the resulting design as shown in Fig. 4.

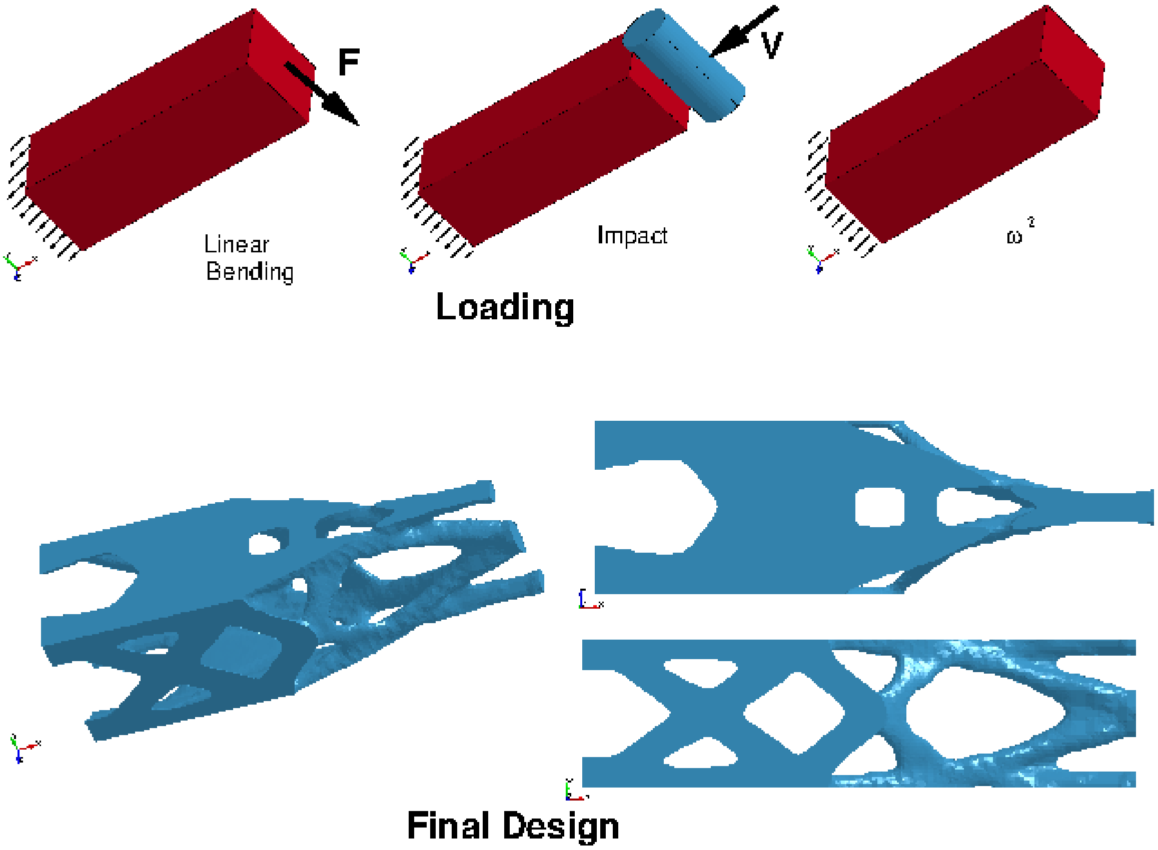


Fig.4: MDO topology design problem. The three load cases are shown at the top, while the bottom has different views of the final design.

The design contribution plot is shown in Fig. 5. The plot shows the load paths created by the different load cases. This is a new plot type available in the GUI for the analysts to study MDO designs.

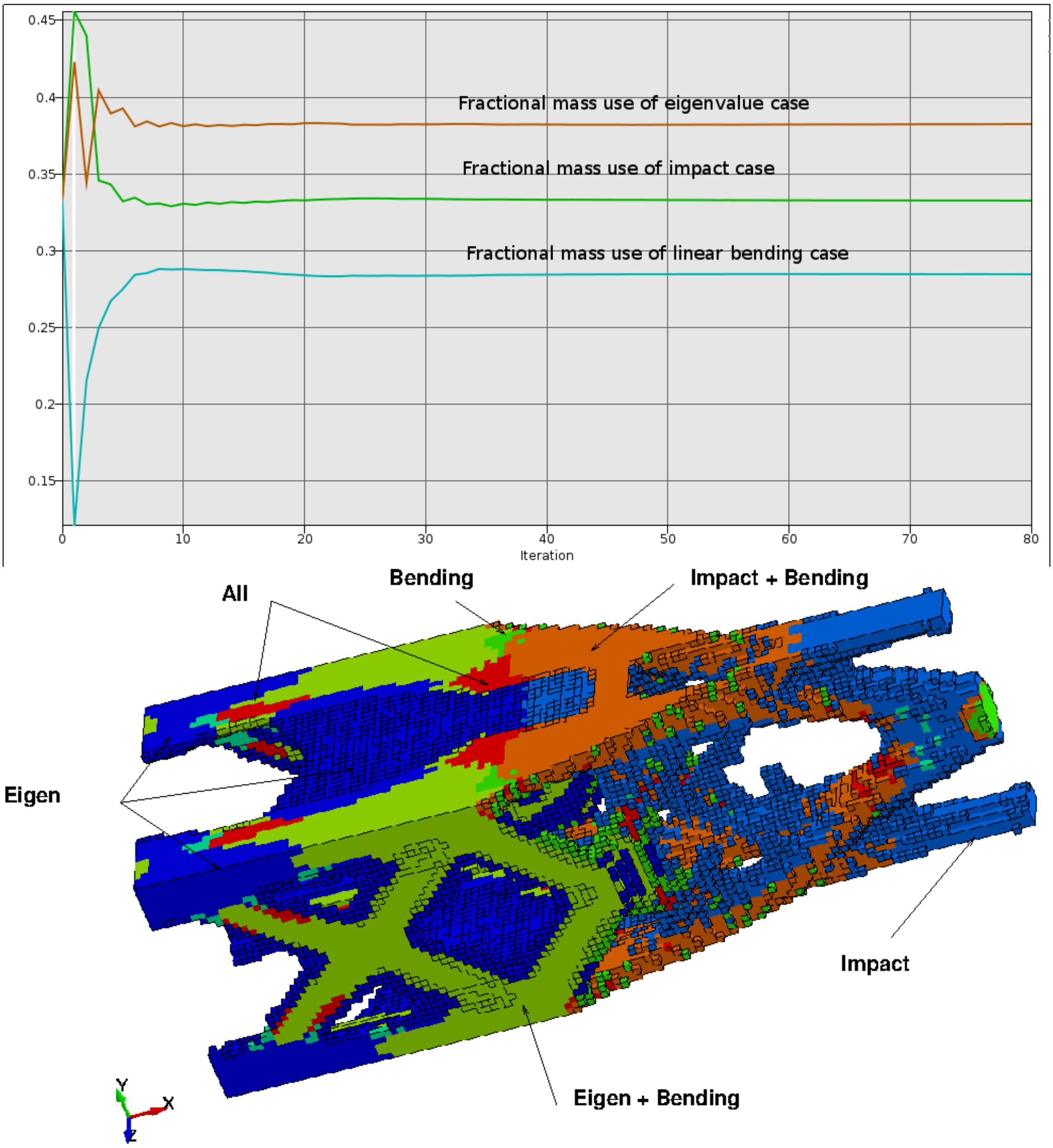


Fig.5: Contribution of the different load cases to the final structure. The history plot shows the fractional mass use of the various load cases, while the fringe plot shows which load case is using which piece of material. The fundamental frequency load case used most of the material.

It is interesting to note that very little of the structure is used purely for the linear bending load case. Most of the structure is used for the bending load case together with either the impact or the fundamental frequency load case. The impact and fundamental frequency load cases are dominating the use of certain sections of material.

4.3 Computational effort for a huge model (10+ million elements)

This example demonstrates the computational ability of the projected subgradient implementation for huge problems. A LS-DYNA model with 13.1 million elements is used to evaluate this.

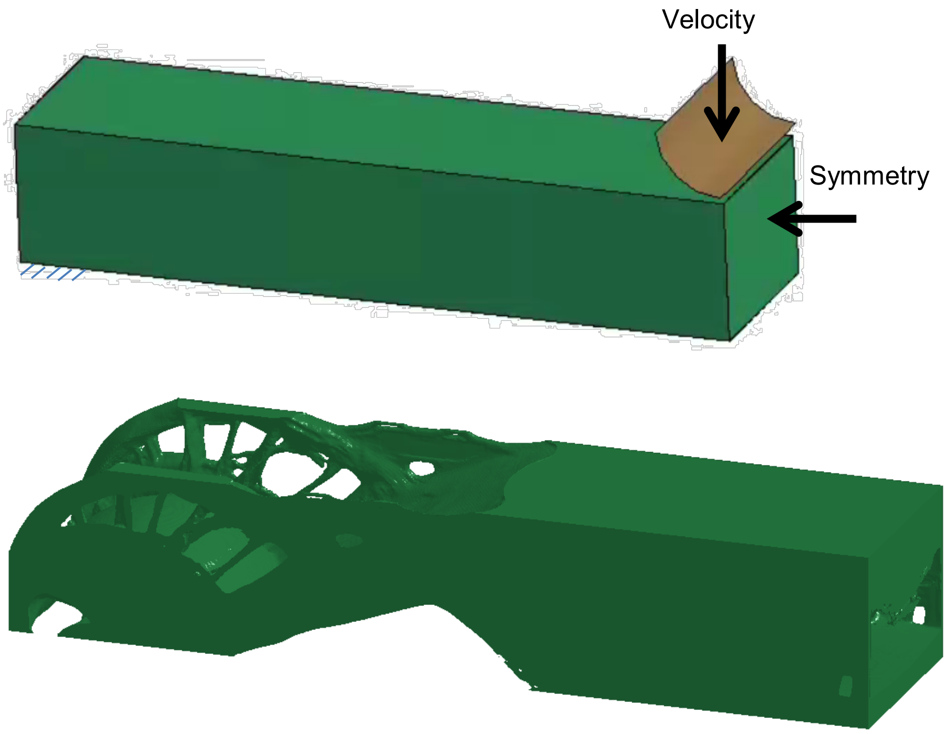


Fig.6: Symmetric beam problem. The top plot shows the initial design with the boundary conditions fixed at both ends and with the load at the center. The bottom plot is the design at the last iteration using an isosurface plot.

The beam model shown in Fig. 6 is considered as the design part for the topology optimization using LS-TaSC. It is a rectangular beam fixed at both ends with a pole impacting at the center at an initial velocity of 10 m/s. A mass fraction of 0.25 is defined as the target mass fraction with element deletion switched on. The optimization is done for a total of 30 iterations.

The following table summarizes the computational cost associated with running the topology optimization within LS-TaSC. Given that this is a huge problem, all the numbers look reasonable. Note that the projected subgradient algorithm ran on a desktop with on a single CPU utilizing about 15GB of memory, while the structural evaluation was offloaded to a cluster. The optimization algorithm takes longer for the design of the first design iterate, because it computes the optimal step size during the first iteration.

Number of elements in the model	13.1 million
Reading LS-DYNA input file	3.5 CPU minutes (1 CPU)
Generating topology filters	2.15 CPU minutes (1 CPU)
LS-DYNA analysis time for one iteration	600 CPU hours (5 hours using 120 CPUs on a remote cluster)
Extracting results from d3plot files for each iteration	46 CPU seconds (1 CPU)
Part design time – first iteration	25 CPU minutes (1 CPU)
Part design time – all other iterations	2 CPU minutes (1 CPU)
Peak memory use by LS-TaSC	15 GB

Table 1: Computational cost for the huge problem.

5 Conclusion

The projected subgradient method gives us the ability of doing multidisciplinary design optimization, specifically the ability to design for the triple crown of linear statics, impact, and NVH load cases together. In addition one is able to explore the resulting multidisciplinary design visually - examining both load paths in the structure as well as the amount of material dedicated to a certain use.

Huge FEA models pose no problem for the projected subgradient implementation. The initial version of the algorithm was tested using more than 10 million variables with no issues noted.

6 Literature

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