

An LS-OPT[®] methodology for utilizing partial curve data for the calibration of material models

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1 Introduction

Parameter estimation is a considerably large application area of optimization. It fulfills the important purpose of characterizing materials based on models available in Finite Element analysis software such as LS-DYNA[®]. The development of special LS-OPT[®] features for parameter estimation using Digital Image Correlation and other experimental methods has been ongoing for a number of years. In earlier papers [1,2] some of the available similarity measures as well as the LS-OPT DIC methodology were discussed in broad detail and illustrated with examples.

The existing similarity measures available in LS-OPT are the Euclidean norm (historically known as Mean Squared Error) [7], Partial Curve Mapping (PCM) [3], Discrete Fréchet (DF) [2] and Dynamic Time Warping (DTW) [2]. PCM was introduced to address hysteresis and partial curves (in which the lengths are not comparable) with some success, except that noisy curves rendered the method ineffective. Subsequently, DF and DTW, which were invented to compare curves end to end, were implemented to address noise. They did so effectively as long as the curves being compared were of comparable length (with respect to the range of the ordinate and abscissa).

Unfortunately, in practical situations, curves are typically not of comparable length. While the test curve available from experimental data can be edited to any desirable length, the computational curves produced in the optimization cycle cannot, or can only be edited with difficulty. Examples are as follows:

1. A tensile test in which the coupon fractures at some point but then the analysis continues to a fixed termination time with the only response being the oscillation of the fractured remnants.
2. An example in which only a part, e.g. post-yield, of a curve needs to be matched.

While the PCM method is available for addressing curve partiality, the method has trouble with noisy curves because of its inherent property of using the arc length as a means of traversing the curve.

A search for matching methods able to address curve partiality, in combination with other properties such as noise and hysteresis, was recently incorporated as part of a Master's thesis study [5]. The study found that the Time-Warped Longest Common Subsequence (T-WLCS) method [6] is a prominent approach to partial curve comparison. Based on T-WLCS, the points of simulation and test curves are traversed and labeled as a match whenever the points are within an ε -range of each other. In doing so, a percentage of mismatch between the curves is calculated. The curve(s) with the lowest percentage of mismatch with respect to the experimental curve are determined and, if necessary, appropriately truncated.

As the percentage of mismatch is dependent on the matching threshold ε , more than one curve may have the lowest percentage of mismatch. To address this issue, a hybrid method was proposed [5] in which the second step consists of passing these (potentially truncated) matches to the DTW distance method in order to determine the (partial) computed curve that best matches the experimental data. The proposed hybrid measure does not require aligned x -values and allows matching one point from a curve to multiple points on the other. In contrast to measures such as DF and DTW, the proposed hybrid measure does not require all points on both curves to be matched, which yields an automated truncation, whenever there is noise or different curve lengths.

In addition to the hybrid method, a truncation method, which relies on knowledge of the application as well as a direct minimization of T-WLCS, is also investigated in this study.

The final part of the study is devoted to applying the theory to an example of a tensile test which includes noise and partiality as phenomena. The simulation data for this example is obtained by means of the finite element software LS-DYNA using the GISSMO damage model. Experimental data from a tensile test are used to identify seven material parameters of the model. The example exhibits both noise and mismatch of the curve length due to an uncertain termination point caused by oscillation after fracture. Three scenarios for addressing the partiality problem are investigated.

2 Theory of the Time-Warped Longest Common Subsequence method (T-WLCS)

The T-WLCS method is based on the Longest Common Subsequence (LCS) method, however it includes the time-warping property of DTW. As opposed to the original LCS, which was merely able to consider one-to-one or one-to-none matchings, this characteristic enables T-WLCS to additionally consider one-to-many matchings.

The basic idea is to traverse both curves and match those points which are within an ε -range of each other. In this manner, the method counts how many points of one curve could be matched, i.e. are similar, to the other. The reference and computational curves are first pre-processed by scaling both with respect to the reference (test) curve.

Mathematically, this number of matching points between two curves, say $A_n = (a_1, \dots, a_n)$ and $B_m = (b_1, \dots, b_m)$, is calculated via the following recursive formula, based on the papers by Guo and Siegelmann [6] and Mai et al. [9],

$$TWLCS_{\delta,\varepsilon}(A_n, B_m) = \begin{cases} 0 & \text{if } A_n \text{ or } B_m \text{ is empty,} \\ 1 + \max\{TWLCS_{\delta,\varepsilon}(A_{n-1}, B_{m-1}), TWLCS_{\delta,\varepsilon}(A_n, B_{m-1}), TWLCS_{\delta,\varepsilon}(A_{n-1}, B_m)\} & \\ & \text{if } \sqrt{(a_{x,n} - b_{x,m})^2 + (a_{y,n} - b_{y,m})^2} < \varepsilon \text{ and } |n - m| \leq \delta \\ \max\{TWLCS_{\delta,\varepsilon}(A_{n-1}, B_m), TWLCS_{\delta,\varepsilon}(A_n, B_{m-1})\} & \text{else.} \end{cases}$$

Assume A_n to be the experimental curve, which we wish to match. Upon obtaining the number of matching points, i.e. the length of the simultaneously calculated partial curve A_{partial} , a percentage of mismatch between the curves can be calculated via

$$\text{dist}(A_n, B_m) = 1 - \frac{\text{len}(A_{\text{partial}})}{\text{len}(A_n)}$$

The above recursive formula for $TWLCS_{\delta,\varepsilon}$ contains two variables, which require some further investigation, namely ε , also referred to as the *matching threshold*, and δ , also referred to as the *time constraint*. As is suggested in the literature [10], we want to use a fraction of the length (number of points) of the experimental curve for δ , i.e.

$$\delta = d \cdot \text{len}(A_n).$$

Here, d represents a user-definable δ -fraction.

For the matching threshold, which creates an ε -circle around the points to be matched, we want to consider the standard deviation of the curves on the x - and y -axis to obtain a reasonable radius. We base this threshold on the data of the experimental curve, resulting in a consistent value throughout the data set. We define ε as

$$\varepsilon = s \cdot \frac{\sigma_x + \sigma_y}{2}, \text{ where } \sigma_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (a_{x,i} - \bar{a}_x)^2}, \quad s \in [0,1].$$

In this formulation, we let a point in A_n be defined as $a_i = (a_{x,i}, a_{y,i})$ and let \bar{a}_x denote the mean over all x -values in A_n . Further, σ_y is the standard deviation over the y -values of the experimental curve, i.e. defined similarly to σ_x . By s we denote the ε -fraction, which may be adjusted by the user.

Figure 1 shows the effect of changing the ε -fraction value of the T-WLCS measure.

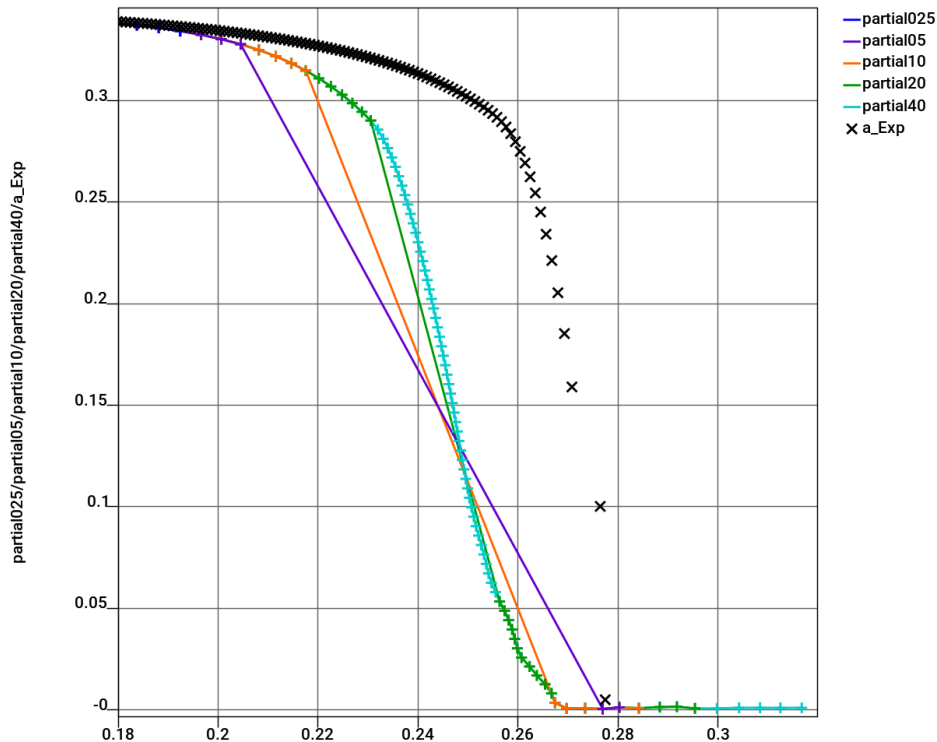


Fig.1: Truncated curves resulting from the T-WLCS mapping of a computational curve on to an experimental curve (x) for ε -fraction values 0.025, 0.05, 0.1, 0.2 and 0.4. The smooth, straight line edges represent shortcuts (or truncations) of the part of the curve which falls outside the ε range. All points fall within the 0.4 range of the ε -fraction (turquoise) (hence T-WLCS=0) but with a tightening of ε , the curves are progressively truncated. The resulting T-WLCS values are respectively 0.415, 0.39, 0.37, 0.275, 0 while the corresponding DTW distance values are 0.146, 0.0365, 0.0358, 0.0330, 0.0339. Note the shortening of the tail as the ε value is reduced. (LS-OPT).

3 Implementation

The T-WLCS function was implemented in LS-OPT as *TWLCSHistory* and *TWLCSMultiHistory* with the reference curve (from experimental data), computational curve (typically a crossplot) and the ε -fraction and δ -fraction as input. Using *TWLCSMultiHistory*, the methodology can also be applied to calibration when using Digital Image Correlation (DIC) [1]. The curves are normalized to the reference curve before applying the mapping. The resolution of the T-WLCS mapping can also be increased by specifying the number of points required. The methodology was implemented in a development version destined as LS-OPT Version 6.1.

4 Example

The example represents a tensile test of which experimental results are available and which was modelled using the GISSMO material model in LS-DYNA [8]. The response of the GISSMO model is noisy since, in combination with the element erosion feature in LS-DYNA, it constitutes a discrete method in which the elements typically fail progressively until the two ends of the coupon are detached. The oscillation of the detached ends can be seen in the first columns of Figures 2 and 3 and also at the top of Figure 4.

Three main approaches are followed using the distance measures Dynamic Time Warping (DTW) and Time-Warped Longest Common Subsequence (T-WLCS).

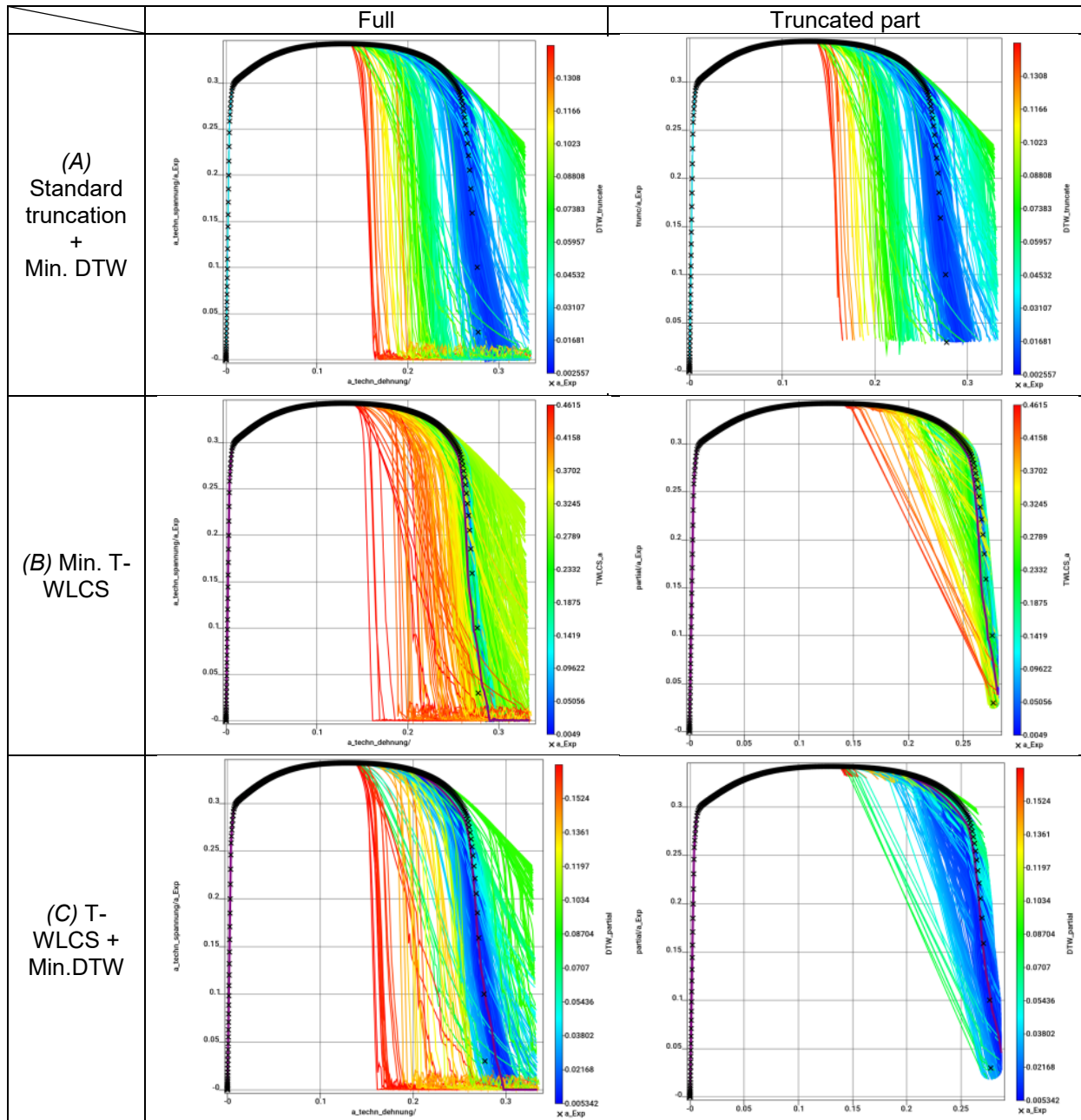


Fig.2: The table represents all the points (each with a unique set of parameter values) used in the optimization and compares computational curves with experimental results (x). The first row represents (A) the standard truncated curve at a specified level to which the DTW measure is applied, the second row represents (B) the minimization of the T-WLCS value and the third row represents (C) the proposed hybrid method in which the curve is truncated using the T-WLCS method followed by applying DTW to the truncated curve. The first column represents the full curves, up to termination of the simulation, whereas the second column represents the truncated curves as they were used to compute the DTW measure (only relevant in formulations A and C). The plots have been color-coded to represent the objective function for each formulation. (LS-OPT).

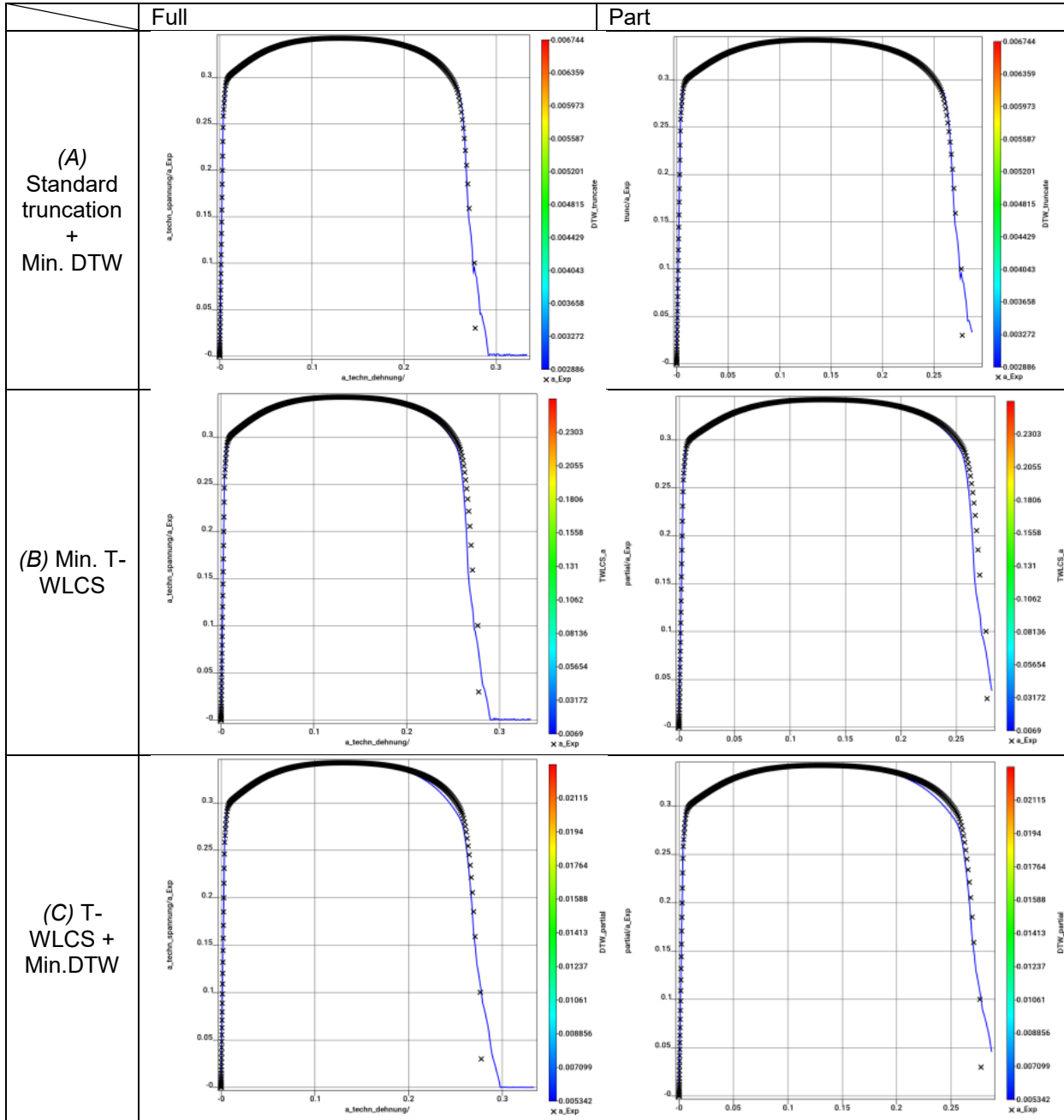


Fig.3: The table represents the final optimum point resulting from the optimization and compares the computational curve with the experimental result (x). The first row represents (A) the standard truncation formulation with minimization of the DTW measure, the second row represents (B) the minimization of the T-WLCS value and the third row represents (C) a hybrid method in which the curve is truncated using the T-WLCS method followed by applying DTW to the truncated curve. The first column displays the full curve, up to termination of the simulation, whereas the second column displays the final truncated curve as used to compute the DTW measure (only relevant in formulations A and C).

The main approaches are described as follows:

- (A) In this formulation the computational curve is truncated at a specified level (seen in the 2nd column of row 1, Figures 2 and 3) determined by the end of the experimental force-

displacement curve. DTW represents the distance between the truncated curve and the experimental curve. T-WLCS was not used in this formulation.

(B) The T-WLCS (ε -fraction = 0.05) value is minimized. The minimization forces the curve to be as complete (or full) as possible within the ε bounds. A tight value of the ε -fraction was chosen to force the computational curve as close as possible to the experimental curve. (An analysis with ε -fraction = 0.1 was investigated separately — see Figure 4).

(C) The computational curve is truncated by the T-WLCS (ε -fraction=0.1) method. DTW represents the distance between the truncated curve and the experimental curve.

The Genetic Algorithm with population size = 50 and generation limit = 100 was used to find the optimal curves. Figures 2 and 3 are explained in their respective captions.

Table 1 shows a comparison of the optimal parameters computed using the three formulations. Notable is that the standard truncation results in the smallest distance value, likely because the standard truncation is more accurate than the automated one using T-WLCS.

	(A) Standard truncation + min. DTW	(B) Min. T-WLCS (ε -fraction = 0.05)	(C) T-WLCS + min. DTW (ε -fraction = 0.1)
a	3.1	2.2	1.4
b	3.0	7.6	8.1
c	0.21	0.69	3.7
d	0.37	0.44	0.35
e	1.1	1.3	2.3
f	0.42	0.37	0.30
g	0.46	1.7	2.0
T-WLCS	0	0.0049	0.0155
DTW(truncated)	0.00256	0.00545	0.00556
DTW(T-WLCS)	0.00340	0.00506	0.00534
GA No. generations (10% repeat termination)	47	31	98

Table 1: Comparison of the optimal parameters and curve matching measures obtained using the three optimality formulations. The relevant objective function values are in bold italics.

To align with the ε -fraction value of 0.1 chosen for Formulation C, ε -fraction = 0.1 was also applied to Formulation B (min. T-WLCS value). This required only 3 GA generations (although 13 generations were required to establish convergence) which led to multiple feasible solutions being available as demonstrated in Figure 4. A more detailed description is given in the caption.

5 Observations

All three methods yielded good results with all optimal DTW values based on the partial curves less than 0.006. Hence a closer inspection is warranted.

When minimizing T-WLCS (B), it is clear from the purple band in Figure 4, that many feasible, near optimal, solutions can be obtained by choosing a large ε value. It is therefore a little surprising that the T-WLCS = 0 solution was not also obtained for the optimum in Case C. Apparently there are curves with non-zero T-WLCS which have smaller DTW values than curves with vanishing T-WLCS values. Further investigation into this aspect will be discussed in the presentation.

Assuming a small ε -fraction value, a successful match to a partial (T-WLCS) curve is expected to have a T-WLCS value close to 0 as well as a small DTW value. This correlation between T-WLCS and DTW was actually computed for the runs conducted for this example and found to be close to 0.9.

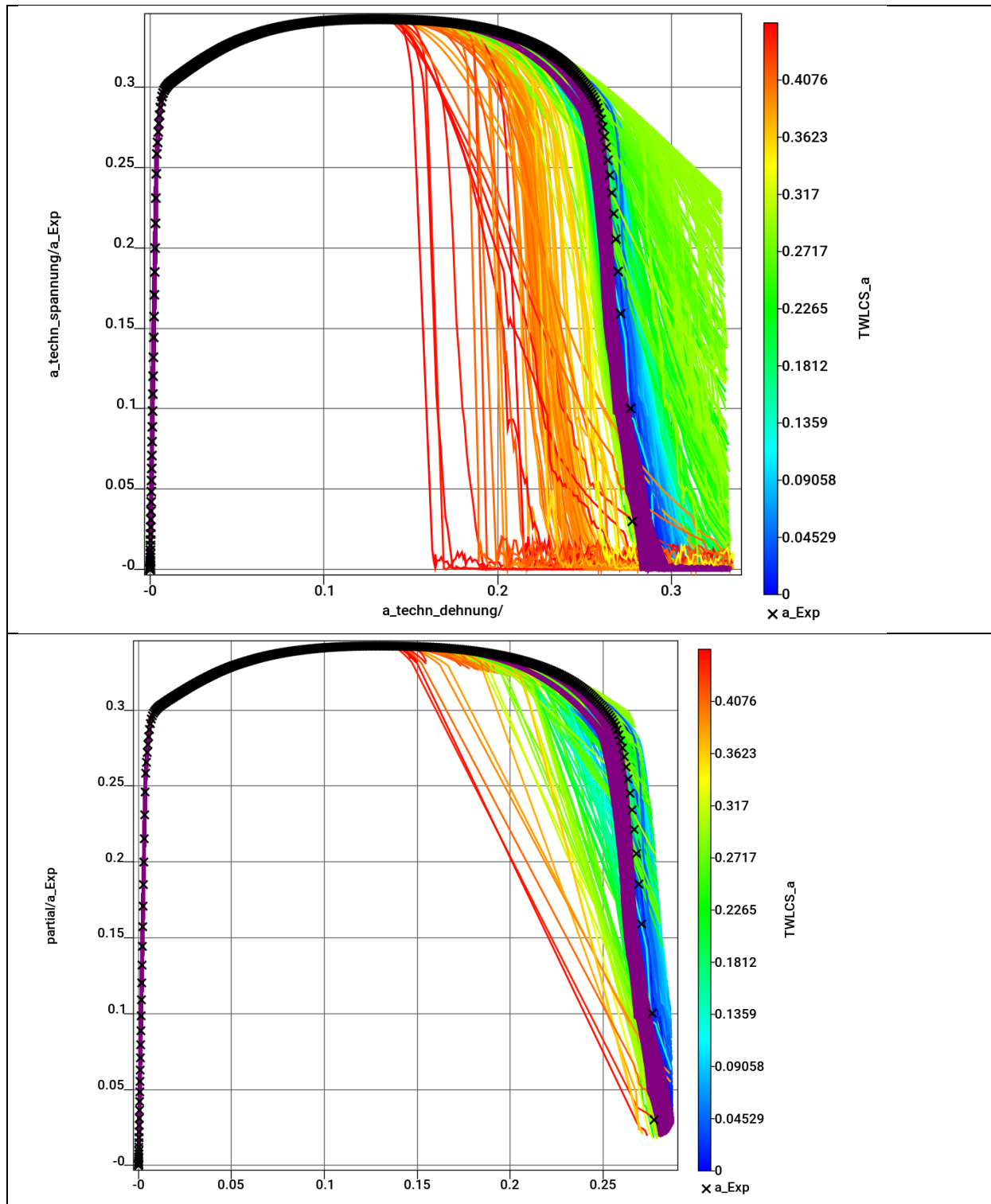


Fig.4: Formulation (B) (Min. T-WLCS) with ε -fraction = 0.1. The two figures represent all the points used in the optimization and compares computational curves with experimental results (x). The top row represents the full curves, up to termination of the simulation, whereas the bottom row represents the truncated curves. The color represents the T-WLCS value. The purple band represents all feasible solutions (T-WLCS = 0) for this formulation (Figures from LS-OPT).

6 Conclusions

After having addressed, and in many cases solved, the idiosyncrasies of curve comparison, such as hysteresis and noise, as part of earlier research efforts, a new study was conducted to explore an automated method for addressing the comparison of curves with significant differences in curve lengths. This is defined as the partial curve comparison problem and may occur due to distinct ranges of testing and computation or due to noise or spurious effects. The study is mainly motivated by the fact that the standard distance measures such as Discrete Fréchet and Dynamic Time Warping fail when applied to curves with significantly different lengths.

A pitfall of the T-WLCS minimization method (Formulation B) is the difficulty of choosing a suitable ε value. A large value of ε is bound to allow spurious data, such as the oscillating tail, to be partially ignored and may, as demonstrated, also find multiple solutions with the same zero T-WLCS value. Choosing a value which is too small may distort the actual curve due to gaps, thereby artificially inflating the DTW distance value. In spite of this uncertainty, this preliminary study seems to confirm that an ε -fraction value in the range [0.05,0.1] is a reasonable choice for this application. A parametric study, as well as a more comprehensive set of examples, would be required to further investigate this generalization.

The preliminary results presented using this example show that although the automated, T-WLCS-based method succeeds in identifying the relevant curve segment which best matches the experimental curve, a simple method in which the curves are truncated in a more conventional manner is also successful. In this case, the ability to map the ends of the experimental curve to the computational curve is required. This approach may be simple for this particular example, but may not always be as obvious, such as when experimental data is only available over a limited range. In that case the T-WLCS approach holds great promise to be effective.

Further work will include more comprehensive parametric studies, a variety of time-ranges of experimental data as well as newer and more complex applications such as Digital Image Correlation. Using T-WLCS with DIC data and multi-point histories (with the *TWLCSMultiHistory* function) is already possible with LS-OPT Version 6.1 (the current development version).

7 References

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