Estimation of Stress Triaxiality from optically measured Strain Fields

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1 Introduction

Nowadays, strain fields can be experimentally measured with high accuracy through digital image correlation (DIC). This kind of measurement is becoming standard when it comes to physical testing of materials. The information from such measurements is then often used in the calibration and validation of material cards to be later used in LS-DYNA. Especially regarding the prediction of failure, the experimentally measured strain fields can be quite helpful. Among several methods for the calibration of material cards, one method relies on the direct use of such strains in the definition of the failure curve as a function of the stress triaxiality ratio. However, in such method, the triaxiality is usually estimated from the simulation of the specimens adopted in the physical tests or, sometimes, it is estimated from analytical calculations based on the loading type and on the geometry of the specimen. It is however widespread known that the triaxiality typically varies during experiments. Therefore, it would be interesting to observe the evolution of the triaxiality throughout the physical test. As mentioned before, the typical way of doing this is through the use of numerical simulation to perform this task. In this paper, we concentrate efforts in developing a method to estimate the triaxiality distribution and evolution using information directly from the DIC measurement. To that end, a plane stress state is assumed and the strain ratio is calculated from the measured strains. The stress triaxiality ratio is, in turn, a relation between the hydrostatic and the equivalent stress. Therefore, in order to calculate the triaxiality from the strain field, a relationship between the strain ratio and the triaxiality has to be defined. This is only possible through the consideration of a constitutive (i.e., material) model. Typically, the J2-based plasticity model (commonly known as the von Mises model, e.g., *MAT_024 in LS-DYNA [1]) is used for this kind of task. However, our research on the topic has shown that this assumption may lead to wrong triaxialities even in cases when the triaxiality is known beforehand, for instance, in a uniaxial tensile test before necking. This error can be significantly reduced if the anisotropy of the material is also taken into account. To that end, we use a Hill-based transversely anisotropic material law in order to consider the effect of the anisotropy. After some mathematical derivations under the assumption of plane stress, negligible elastic strains and proportional loading, it is possible to find a closed-form relation between the strain ratio and the triaxiality including the effect of the R value. The results for an aluminum sheet show that the triaxiality is much better predicted using the new formula. Using a software dedicated to the evaluation and visualization of optically measured strain fields, it should be possible to plot triaxiality fields from experimental data that can be later used either for the calibration or validation of a material card. Furthermore, this novel technique can also be employed on the development of new specimen geometries in order to better assess the stress triaxiality ratios obtained with the new geometry without having to first calibrate a material card for that.

2 The stress triaxiality ratio

The stress triaxiality ratio, or simply “triaxiality”, is a stress state indicator defined as the ratio between the pressure and the equivalent stress:

$$\eta = -\frac{p}{\sigma_{eq}} \qquad (1)$$

The triaxiality is a very useful measure of the stress state if one aims to characterize the fracture behavior of a metallic alloy. For plane stress and isotropic materials, the triaxiality alone is enough to define any possible stress state (but not its intensity) in respect to fracture characterization [2].

3 Triaxiality estimation based on the von Mises model

In the classical J2-based plasticity model, more commonly known as the von Mises model, the yield function is given as

$$\Phi(\boldsymbol{\sigma}) = \sqrt{3\,J_2(\boldsymbol{\sigma})} - \sigma_y\,(\varepsilon_{eq}^p) = 0 \tag{2}$$

In LS-DYNA, this model can be found for instance in *MAT_024 [1]. The yield function only determines whether a material point has reached the elastic limit for a certain strain increment. In order to determine how much of the strain increment is elastic and how much is plastic, a flow rule is needed, which in the present case reads

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\gamma}\,\frac{\partial\Phi(\boldsymbol{\sigma})}{\partial\boldsymbol{\sigma}} = \frac{3}{2}\dot{\gamma}\,\frac{\boldsymbol{s}}{\sigma_{eq}} \tag{3}$$

In the literature, the equation above is also known as the Prandtl-Reuss plasticity law [3].

Assuming principal directions and a plane stress condition, one has

$$\begin{bmatrix} \dot{\varepsilon}_1^p & 0 & 0 \\ 0 & \dot{\varepsilon}_2^p & 0 \\ 0 & 0 & \dot{\varepsilon}_3^p \end{bmatrix} = \frac{3}{2}\frac{\dot{\gamma}}{\sigma_{eq}} \begin{bmatrix} \sigma_1 + p & 0 & 0 \\ 0 & \sigma_2 + p & 0 \\ 0 & 0 & p \end{bmatrix} \tag{4}$$

Considering only the third component of the plastic strain rate tensor, straightforward algebraic manipulations and the use of Equation 1 lead to

$$\dot{\varepsilon}_3^p = \frac{3}{2}\dot{\gamma}\,\frac{p}{\sigma_{eq}} = -\frac{3}{2}\dot{\gamma}\eta \quad \rightarrow \quad \eta = -\frac{2}{3}\frac{\dot{\varepsilon}_3^p}{\dot{\gamma}} \tag{5}$$

Considering negligible elastic strains and that $\dot{\gamma}$ equals the equivalent strain (valid for the von Mises model), the (current) triaxiality can be approximated through

$$\eta_{cur} \approx -\frac{2}{3}\frac{\dot{\varepsilon}_3}{\dot{\varepsilon}_{eq}} \tag{6}$$

One should note that the original definition of the triaxiality is made by using stress rather than strain components (see Equation 1). However, in the general case, local stresses cannot be measured, or at least not without making some sort of restrictive assumption like, for instance, considering a constant stress distribution along the cross section of the specimen (a condition only met under special circumstances). Therefore, Equation 6 presents a very interesting manner of estimating the stress triaxiality relying on strain components only. In turn, strains can nowadays be accurately measured using optical measuring systems. This means that the stress triaxiality can be actually calculated directly from strain fields measured from the data obtained through DIC.

One important restriction has to be noted though: Equation 6 is only valid if, among other important assumptions, the physical material behaves close enough to what the von Mises model predicts. If not, the calculation is not accurate enough and might lead to misleading values of triaxiality. Obviously, the assumption of a plane stress condition also restricts accuracy but, from a practical point of view, this assumption may be sufficient for thin structures. It is also worth noting that the von Mises model assumes that the material behaves isotropically. Unfortunately, this is in practice rarely the case. At the begin of this study, the authors speculated that the error of assuming the material behavior to be isotropic would be probably very small for some typical metallic materials used in the industry. However, as will be shown later in this paper, it turns out that this effect is more significant than expected, or at least under uniaxial tension.

Another important point is that Equation 6 is formulated using strain rates, i.e., the variation of strain over time. This stems from the fact that the von Mises plasticity model is formulated in an incremental form. In a finite element framework, this means that the von Mises model has to be incrementally solved (this is how it is done in LS-DYNA). Therefore, the triaxiality value using Equation 6 should be interpreted as the *current* triaxiality value. However, if one assumes proportional loading (i.e., linear strain paths), Equation 6 can be rewritten as

$$\eta_{avg} \approx -\frac{2}{3}\frac{\varepsilon_3}{\varepsilon_{eq}} \tag{7}$$

In principle, the simplification given in Equation 7 should only be used for linear strain paths. However, if used in the general case of non-proportional loading, Equation 7 can be seen as an averaged value of stress triaxiality which can be quite helpful in practical applications.

Finally, it is important to remark that Equations 6 and 7 use the third strain component which, under the assumption of plane stress, would be the strain in thickness direction. The measurement of such strain can be quite challenging, especially for thin materials, requiring a special experimental setup. In order to avoid the measurement of the third strain component, an isochoric behavior can be assumed:

$$\dot{\varepsilon}_1 + \dot{\varepsilon}_2 + \dot{\varepsilon}_3 = 0 \quad \rightarrow \quad \dot{\varepsilon}_3 = -\dot{\varepsilon}_1 - \dot{\varepsilon}_2 \tag{8}$$

For metallic materials, this is realistic when the material is in the plastic regime. Using Eq. 8, the equations for the current and average triaxiality can be rewritten as

$$\eta_{cur} \approx \frac{2}{3}\frac{\dot{\varepsilon}_1+\dot{\varepsilon}_2}{\dot{\varepsilon}_{eq}}, \quad \eta_{avg} \approx \frac{2}{3}\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_{eq}} \tag{9}$$

4 New method: Triaxiality estimation based on a transversely anisotropic Hill model

As mentioned in the previous section, the estimation of triaxiality using Equation 9 requires that the physical material behave isotropically and also according to the von Mises model. As a matter of fact, the condition of isotropic material behavior is, in practice, rarely met. Typically, the Lankford parameter (or R value) is used to assess the level of anisotropy of metallic sheets. Assuming negligible elastic strains, linear strain paths and isochoric behavior, the Lankford parameter can be written as

$$R = \frac{\varepsilon_2}{\varepsilon_3} = -\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2} \tag{10}$$

If a material exhibits an R value smaller than 1.0, it means that the thinning (strain in the third principal direction) is more pronounced than the necking (strain in the second principal direction). Conversely, if R is larger than 1.0, the material exhibits more necking than thinning.

In order to include the effect of the R value in the determination of the stress triaxiality, a constitutive model which takes this effect into account has to be adopted. In this contribution, we consider a simplified orthotropic model based on the model proposed by Hill in 1948 [4]. Hill's original model can be simplified by considering a plane stress state and also only normal anisotropy (i.e., R_{00}= R_{45}=R_{90}=R). In fact, this simplified model is available in LS-DYNA in *MAT_037 [1], but could also be reproduced in *MAT_036 by setting R_{00}=R_{45}=R_{90}=R and m=2. The yield function is given by

$$\Phi(\boldsymbol{\sigma}, R) = \left[\sigma_{11}^2 + \sigma_{22}^2 - \frac{2R}{R+1}\sigma_{11}\sigma_{22} + 2\frac{2R}{R+1}\sigma_{12}^2\right]^{1/2} - \sigma_y(\varepsilon^p) = 0 \tag{11}$$

The simplified Hill model considers only a single R value, i.e., only a transversal anisotropy is considered. In practical terms, this means that only the amount of thinning can be described through the single R value. The assumption of a single R value contrasts with R values commonly measured. Typically, the R values are different for the different material orientations (i.e, R_{00}≠R_{45}≠R_{90}). From a simulation point of view, the use of a constitutive model which considers the effect of the different R values is indeed crucial for a general good description of the plastic behavior of the material. However, for the purpose of merely estimating the triaxiality from optically measured strain fields, this might not be as important, provided one uses the R value of the main loading direction in the experiment.

Similar to the von Mises model, an associated flow rule is adopted

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\gamma}\frac{\partial\Phi(\boldsymbol{\sigma},R)}{\partial\boldsymbol{\sigma}} \tag{12}$$

where the result of the derivative in Equation 12 is omitted here for the sake of simplicity. Although not as trivial as for the case of von Mises, it is also possible to derive a closed-form relationship between the triaxiality and local strains considering the simplified Hill model. The final equation is given by

$$\eta_{cur} = -\frac{2}{3}\sqrt{\frac{[(R+1)^2(1-2a+a^2)+(R+1)(2a-a^2-1)+a^2+a+1]}{3(1+a^2-a)}}\left(\frac{-\dot{\varepsilon}_1-\dot{\varepsilon}_2}{\dot{\varepsilon}_{eqv}}\right) \tag{13}$$

where

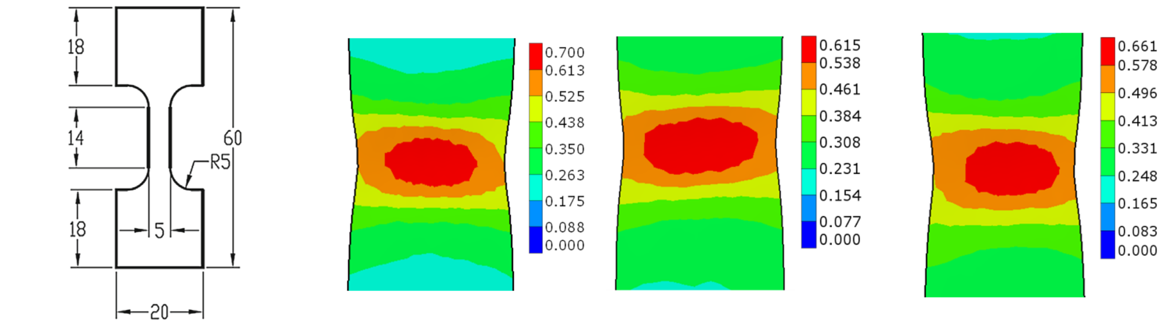
$$a = \frac{R+(R+1)b}{bR+R+1}, \qquad b = \frac{\dot{\varepsilon}_2}{\dot{\varepsilon}_1} \tag{14}$$

In the equations above, Equation 9a can be promptly recovered if R=1.0 (i.e., von Mises). Therefore, Equation 13 can be seen as a generalization of Equation 9a. For the “averaged” triaxiality based on the new equation, one simply has to use the total strains instead of using the strain increments in Equations 13 and 14.

As will be demonstrated in the next section, Equation 13 is able to estimate the triaxiality more accurately than Equation 9a.

5 Uniaxial tensile test

In this section, we assess the method described in the previous sections when used to estimate the triaxiality in a uniaxial tensile test. The material used was an aluminum sheet typically used in the automotive industry. The same material was approached by Andrade et al. in [5]. The geometry of the tensile test as well as the optically measured strain fields in the 0°, 45° and 90° directions is given in Fig. 1. The mean R values measured in the three directions were R_{00}=0.7, R_{45}=0.5 and R_{90}=0.8. A material card for *MAT_036E with HOSF=1 was also calibrated (see [5]). The material card will be later used in order to compare the estimated triaxiality with the triaxiality obtained in the simulation.



(a) Specimen dimensions *(b) 0°, R_{00}=0.7* *(c) 45°, R_{45}=0.5* *(d) 90°, R_{90}=0.8*

Fig.1: Tensile test: Geometry and optically measured strain fields close to failure (equivalent strain).

We use Equation 9a (von Mises based) and Equation 13 (Hill based) in order to estimate the triaxiality in the tensile test. A point at the center of the specimens in all three directions is chosen for all evaluations. From the DIC system, the strain components ε_1 and ε_2 over time are evaluated, exported to a file and then used as input of a simple script which evaluates Equations 9a and 13. Additional input information is the R value necessary for the Hill-based triaxiality estimation. After the script execution, the (current) triaxiality calculated from strain components ε_1 and ε_2 as well as the equivalent strain are saved in a new file.

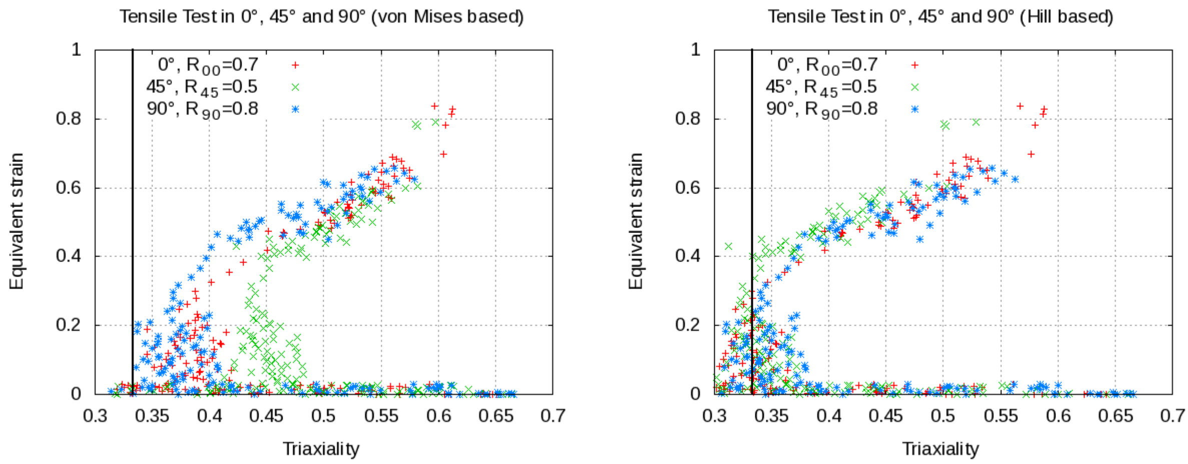
Figure 2 shows the strain-triaxiality paths for the central point in the tensile specimen in all three directions. The triaxiality used in Fig. 2a was estimated using Equation 9a, i.e., it was based on the von Mises model. At the beginning of the test, the strain increments measured with the optical system exhibit some oscillations which are directly reflected in the value of triaxiality calculated with Equations 9a and also 13. Therefore, points were used instead of lines in depicting the strain-triaxiality paths in the diagrams in order to facilitate comprehension. With increasing plastic deformation, the oscillations tend to decrease and a clearer strain-triaxiality path can be observed.

For the sake of reference, a line was added to the diagrams in Figure 2 for which the triaxiality of 1/3 is constant. In principle, a uniaxial tensile test should, up to the necking point, deliver exactly this value of triaxiality. If not, this would mean that the test was not perfectly uniaxial. As can be seen in Fig. 2a, the strain-triaxiality paths for the tensile test in the 0°, 45° and 90° directions do not start at exactly1/3.

Remarkably, the strain-triaxiality path for the 45° direction begins at a triaxiality around 0.45, a significant deviation with respect to the expected value of 1/3. Furthermore, Fig. 2a could lead to the conclusion that the tensile test was not perfectly uniaxial and that the different material orientations also have an impact on how "uniaxial" the test was. In fact, this conclusion is not correct. The reason for the deviations in triaxiality observed in Fig. 2a is related to the effect of the material anisotropy. Noteworthy is the fact that the 45° direction exhibits the largest deviation of the three directions. The 45° direction is also the orientation whose R value was the farthest from 1.0 (i.e., the isotropic case).

The observed deviations in the value of triaxiality can be corrected by using the Hill-based estimation of Equation 13. In the present evaluation, the mean R value in each direction was used for calculating the triaxiality. As can be seen in Fig. 2a, with the use of the Hill-base estimation, the initial triaxiality in all directions is around 1/3. The oscillation around the 1/3 triaxiality line can be explained by the oscillations observed in the strain increments measured with the optical system. In all three cases, the triaxiality starts to considerably increase after reaching a local true equivalent strain of around 0.2 which corresponds to the necking strain observed in the experiments. As expected, from the necking point on, the behavior is no longer homogeneous and the triaxiality increases.

At this point, it is important to emphasize that the consideration of the anisotropic coefficient was crucial in order to more correctly estimate the triaxiality in the physical test. It is also important to remark that Equation 13 is a closed-form equation which does not require any numerical solution. This makes its usage within a typical DIC software quite attractive.



(a) Von Mises based triaxiality estimation *(b) Hill based triaxiality estimation*

Fig.2: Strain-triaxiality paths based on the triaxiality estimation of the tensile test in three directions. All diagram data were calculated exclusively from the strain fields obtained experimentally.

Andrade et al. [5] presented an orthotropic material model which could reproduce strain fields of an aluminum sheet with high accuracy. This material model is implemented in LS-DYNA under *MAT_036E if the option HOSF is set to 1.0. For the present material, a material card was calibrated using this material model whose results are presented in more detail in [5]. In the present contribution, we used this material card to simulate the tensile test in different directions and evaluated the strain-triaxiality path in the simulation. Shell elements of type 16 with a mean element size of 0.5mm were used in the simulations. A comparison between the estimated and simulated strain-paths is given in Fig. 3. Notably, the agreement between simulation and estimation is very good.

It is worth noting though that the simulation was carried out with a single material card for *MAT_036E which is a more sophisticated material model than the transversal anisotropy Hill model adopted in Section 3. If one would adopt the formulation of *MAT_036E in order to estimate the triaxiality based on strain fields, a numerical solution would most likely be inevitable, making it less attractive in using in practical situations. However, the Hill-based estimation is able to deliver remarkably good estimations of the strain-triaxiality path for the three directions. One should note that, in the case of the estimation, a different R value had to be used for calculating the triaxiality for each load direction. This means that the current method requires that the loading direction with respect to the material orientation is known. The more the loading direction deviates from the material orientation, the less accurate the estimation of the strain-triaxiality path.

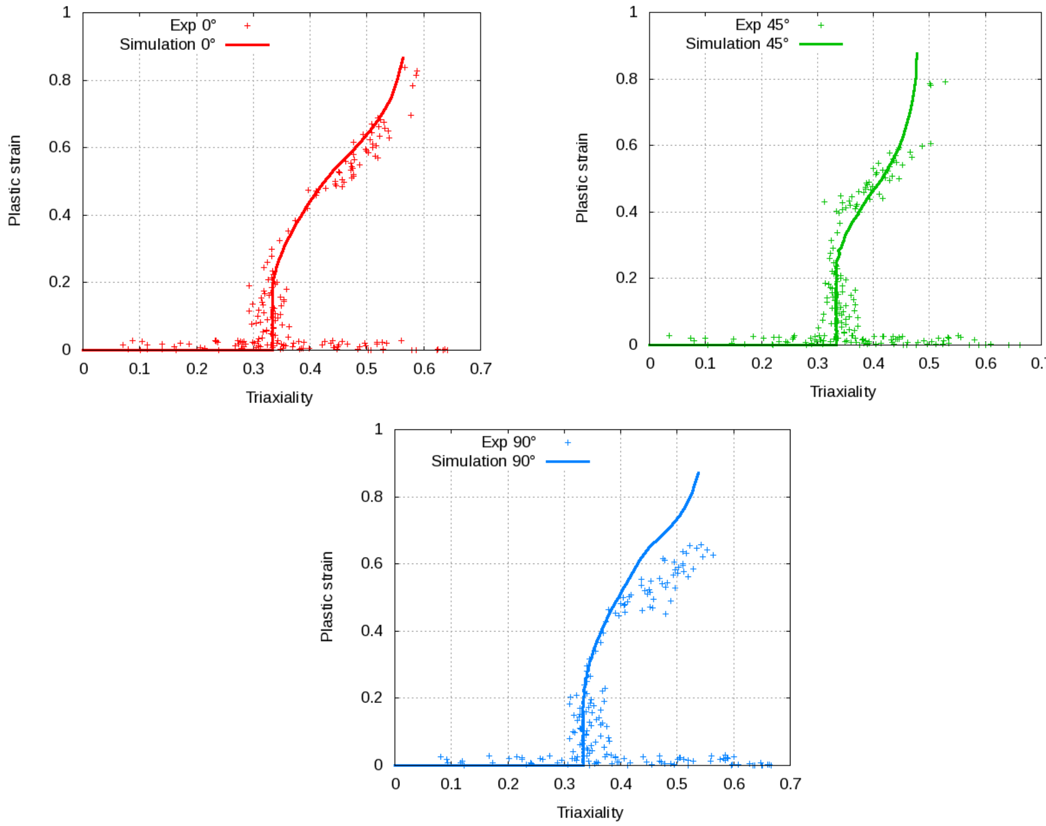


*Fig.3: Comparison of the strain-triaxiality paths between experimental estimation (Hill based) and simulated values with *MAT_036E with HOSF=1. Note that the experimentally estimated values of the triaxiality were calculated only using the strain fields from the optical measurement.*

6 Notched and shear specimens

In this section, we use the equations for the triaxiality estimation with the experimental data of a notched and a shear specimen for the same material than in Section 5. The dimensions of the specimens are given in Fig. 4 meanwhile the strain fields (equivalent strain) are given in Fig. 5.

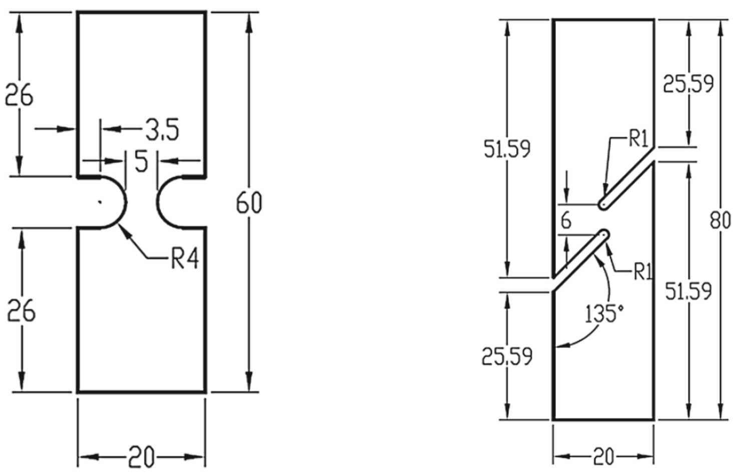


Fig.4: Dimensions of the notched and shear specimens.

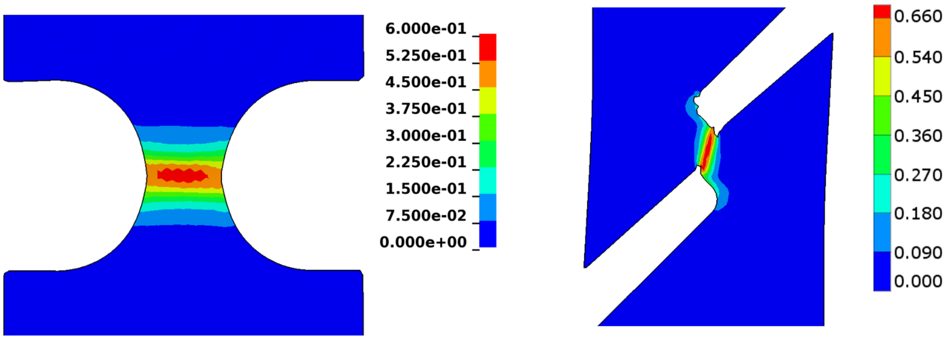


Fig.5: Strain fields optically measured for a notched and a shear specimen in 90° (equivalent strain).

In the current study, both the notched and shear specimens were tested only in the 90° direction. Fig. 6 has the strain-triaxiality paths estimated by using both the von Mises based and also the Hill based equations. Remarkably, both predictions are very similar. One possible explanation is the fact that these specimens were tested only in the 90° direction whose R value is around 0.8, i.e., not too far away from the isotropic case of R=1.0. Therefore, the differences between the von Mises based and the Hill based predictions are rather small.

Similar to the case of the tensile test, we also use the material card calibrated in [5] in order to simulate both specimens in LS-DYNA. The comparison between the estimated and the simulated strain-triaxiality paths can be seen in Fig. 7. A very good agreement is seen in the case of the notched test. However, in the case of the shear test, some deviation was observed where the estimation through Equation 13 gives a triaxiality much closer to 0.0 than the simulation. One possible reason for this mismatch is the relatively coarse discretization of the shear specimen. Fig. 8 shows a comparison of the strain fields in the simulation and the one measured with the optical system. As can be seen in this figure, there is a certain mismatch between the two strain fields. For instance, there is some excessive plastic deformation within a single element at the edge of the critical zone in the simulation. This straining was not observed in the experiment. A finer mesh might alleviate the problem or perhaps a yield curve for shear might be necessary, but further investigation still has to be pursued. Another possible reason for the mismatch between the estimated and the simulated strain-triaxiality path is a possible lack of accuracy of the Hill based estimation for certain cases.

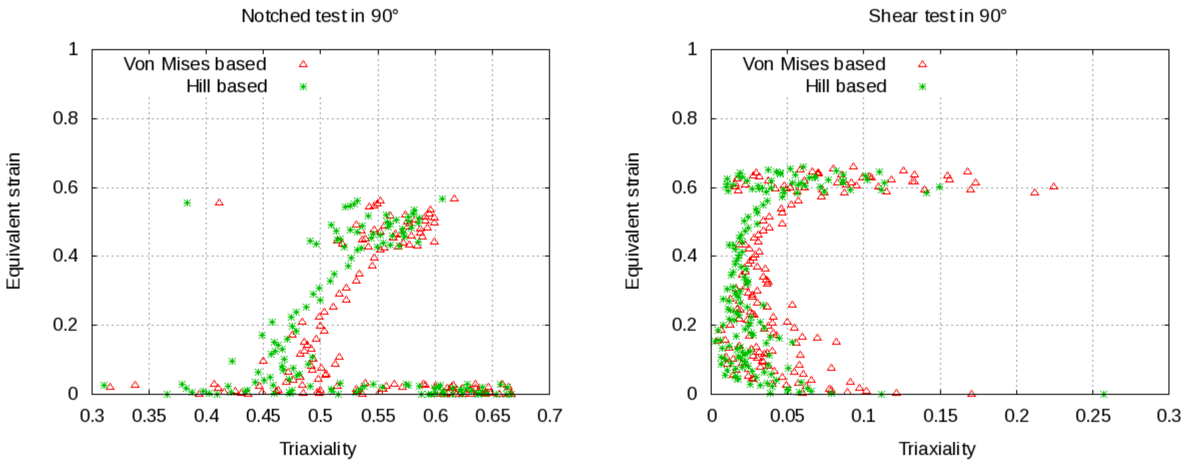


Fig.6: Strain-triaxiality paths from experimental data. The triaxiality was calculated using the von Mises (Equation 9a) and transversal Hill (Equation 13) based relations for the notched and shear test in the 90° direction.

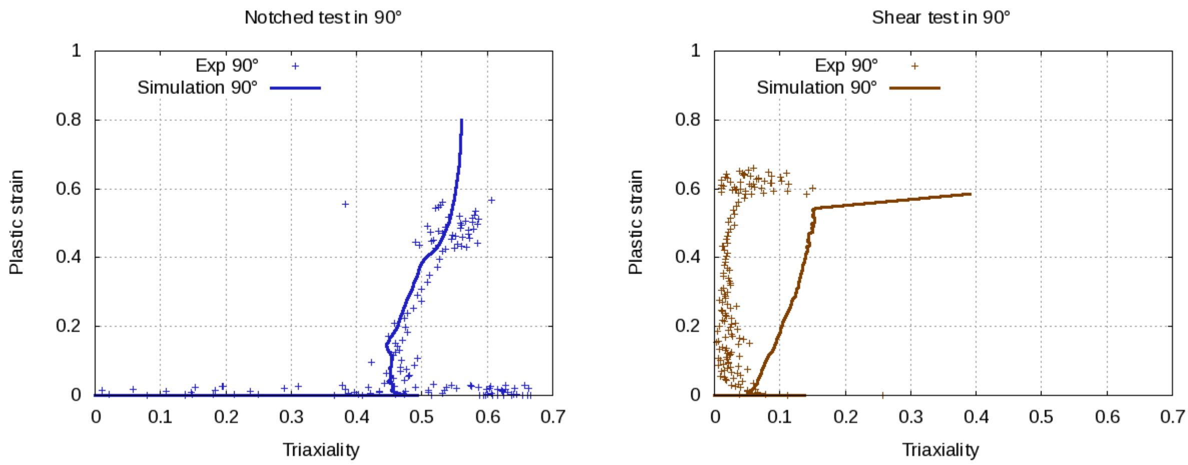


*Fig.7: Strain-triaxiality paths for the notched and shear specimens: Comparison between estimation (Hill-based) and simulation (*MAT_036E with HOSF=1).*

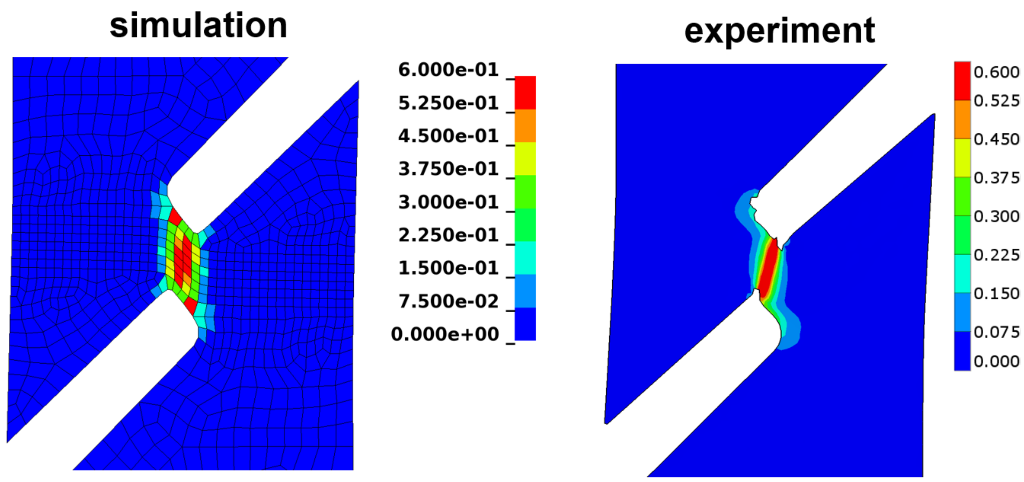


Fig.8: Comparison of the strain field (eq. strain) in the simulation and measured by the optical system

7 Summary

In this paper, we presented a method for estimating the triaxiality based solely on strain fields measured by optical systems. Using an estimation based only on von Mises might lead to incorrect results for the triaxiality. Therefore, we propose the use of a Hill based closed-form relation derived from a transversal Hill anisotropic plasticity model. The results show that the Hill based equation is able to deliver triaxialities quite close to the expected values of uniaxial tensile tests. The comparison with numerical simulation also shows very good agreement. This contribution only shows the first results of the new technique. Further investigation is however in progress.

8 Literature

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