FEM-BEM Coupling with Ferromagnetic Materials

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1 FEM-BEM coupling for eddy current problems

Eddy current problems are typically modelled by a combination of Ampère's law, Ohm's law, the non-existence of magnetic monopoles and Faraday's law of induction. Using a magnetic vector potential **A**, such that the magnetic flux intensity is given by **B** = **curl A**, we end up with the equation $\sigma \partial_t \mathbf{A} + \mathbf{curl}\, \nu(\mathbf{A})\, \mathbf{curl}\, \mathbf{A} = \mathbf{J}_s$ [1]. Here, $\mathbf{J}_s$ are applied source currents, $\nu(\mathbf{A})$ the magnetic reluctivity whose dependence on **A** implies the possible non-linearity of the material behaviour, and σ is the electrical conductivity. In most applications, the domain of interest consists of conducting (e.g. metal parts) and non-conducting regions (e.g. air). In the non-conducting regions, the part $\sigma \partial_t \mathbf{A}$ is dropped from the equation and the model is that of magnetostatics. It has to be noted here that the underlying partial differential equations (eddy current and magnetostatics) imply the use of different function spaces as those commonly used for structural mechanics. Therefore, the FEM discretisation schemes are significantly more intricate and based on vector-valued shape functions with specific continuity properties. Additionally, in the non-conducting regions the resulting FEM system matrix is singular and thus special solution techniques are required. Here, we make use of a tailored Algebraic Multigrid Method [2].

It is nowadays standard to tackle the above partial differential equation by means of a FEM [3]. Nevertheless, there are several practical issues related to this approach: a volume discretisation of the surrounding air is needed and, despite the infinite extent of electromagnetic fields, the discretisation needs to be truncated somewhere. Moreover, in many applications individual parts of the analysed device move with respect to each other (e.g. actuators) and are prone to destroy the volume FE mesh. Therefore, we propose a different approach in which the surrounding air domain is represented by means of boundary integral equations. This idea is based on the fact that outside of conducting regions, the magnetic field **H** can be represented by means of a scalar potential (**H** = **grad** φ). Leaving technicalities such as the treatment of multiply connected domains aside, this approach leads to a potential problem in the exterior domain which can be tackled by a boundary element method. Finally, one arrives at a coupled system of equations with the magnetic vector potential and the tangential component of the surface magnetic field as unknowns [4].

Boundary element methods are very attractive because they only require a surface discretisation. This advantage comes at a price: singular integrals need to be computed and the resulting system matrix is fully-populated [5]. Whereas the numerical integration is highly involved, it can be handled in a robust way by means of sophisticated quadrature rules. The use of a fully-populated system matrix, on the other hand, leads to an inherent quadratic complexity and needs to be circumvented if problems of industrial relevance are to be analysed. There are several matrix approximation techniques on the market and we make use of the Fast Multipole Method [6], since it appears to give the best performance in terms of accuracy, robustness and computational speed. In a nutshell, the system matrix is decomposed into near- and far-field interactions and the latter are approximated based on analytic properties of the integration kernels in use. Finally, there will be no system matrix in a classic sense but only its application in a matrix-vector product. Therefore, iterative solution techniques are necessary which require careful preconditioning. Our implementation of the Fast Multipole Method together with FEM and BEM preconditioning techniques has an overall linear complexity in the number of unknowns.

2 Numerical results

The method as described in Section 1 has been implemented in C++ and applied to many benchmark problems. Most notably, several of the TEAM examples (https://www.compumag.org/wp/team) have been tackled. Figure 1 shows some results of the analysis of Problem 7 of that list. On the left, the geometry of the problem is displayed where a coil, subject to an alternating current, is placed above a conducting plate with a hole. In addition, the image shows the induced eddy-currents in that plate. The right of the figure shows the excellent agreement with measurement data of the induced currents that are provided in the benchmark database. Note that in this example, a finite element mesh is only needed for the conductor (and for the coil depending on the way the currents are applied).

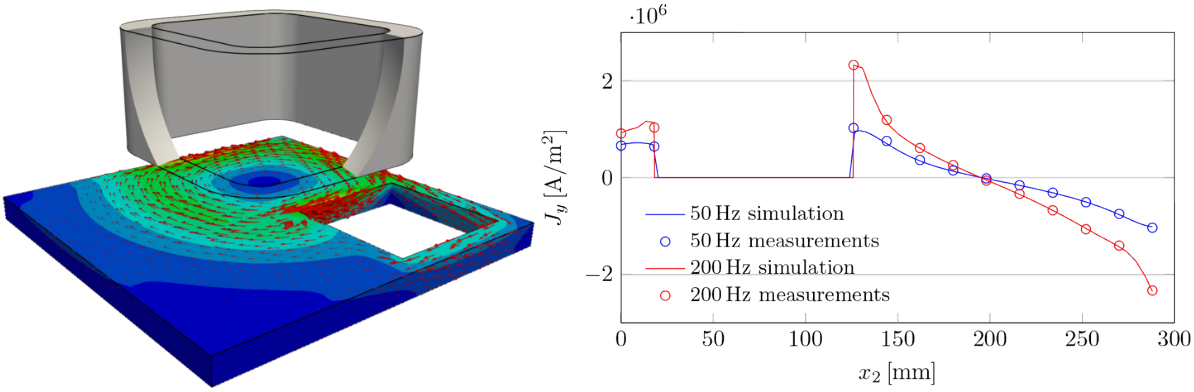

Fig.1: TEAM 7 problem: induced eddy currents (left) and comparison of numerical and measured data (right).

As a showcase example, we numerically analyse a classic experiment from physics: a magnet falling through a copper pipe. The puzzling observation is that the magnet decelerates extremely and floats at a constant, strikingly small velocity towards the bottom end of the pipe. The left image of Figure 2 illustrates the mechanism: the falling magnet creates a magnetic field in the pipe that varies with time and as such induces eddy currents in the copper pipe. These currents, in turn, create a magnetic field that opposes field of the magnet. This opposition in direction is commonly referred to as Lenz's law. The middle image of the figure shows a snapshot of the numerical simulation where one can see two circulating currents above and below the falling magnet. The right image shows the falling velocity and braking force of the magnet. One can see that after some initial deceleration phase, the magnet reaches a constant velocity and, correspondingly, the breaking force equals the gravitational force in magnitude. In this simulation, a finite element mesh only needs to be generated for the pipe and the magnet such that the modelling costs are minimal in comparison to a pure FEM mesh where either re-meshing or some sophisticated sliding cylinder in the mesh of the air region has to be generated.

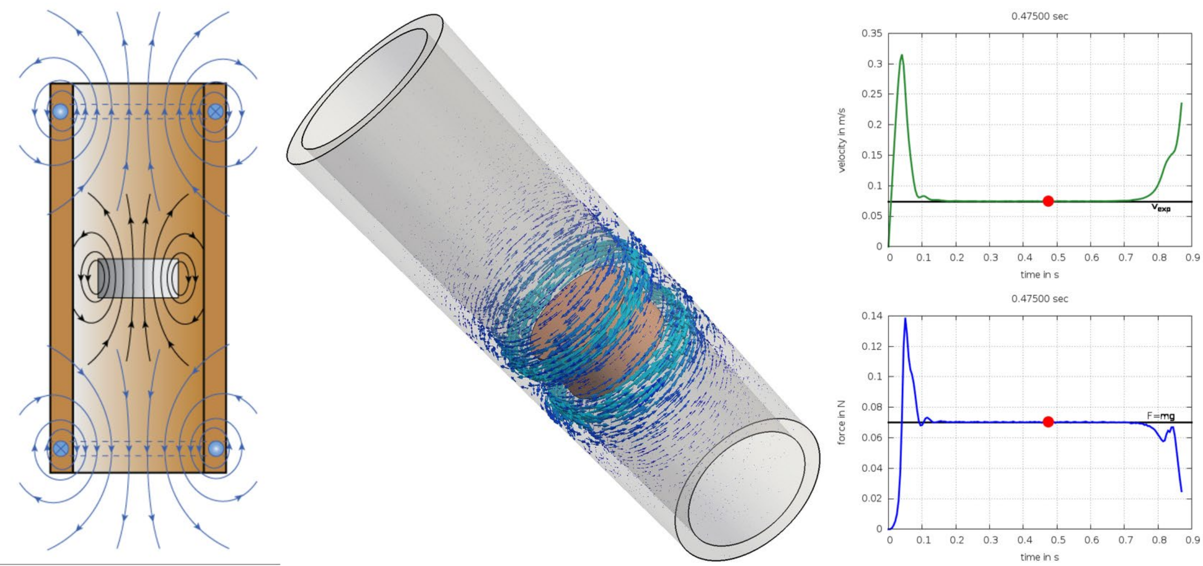

Fig.2: Magnet falling through a copper pipe: concept (left), computed eddy currents (middle), falling velocity and magnetic force (right).

Finally, the applicability of the proposed method to nonlinear material behaviour is demonstrated. More specifically, we consider a nonlinear relation between the magnetic **B**- and **H**-fields which is typically found in so-called B-H curves. Such behaviour is common for iron among others and found in a vast number of technical applications. As an example, here we demonstrate benchmark number 24 from the above mentioned list of TEAM problems. Figure 3 shows the geometry of the problem and a plot of the nonlinear material behaviour.

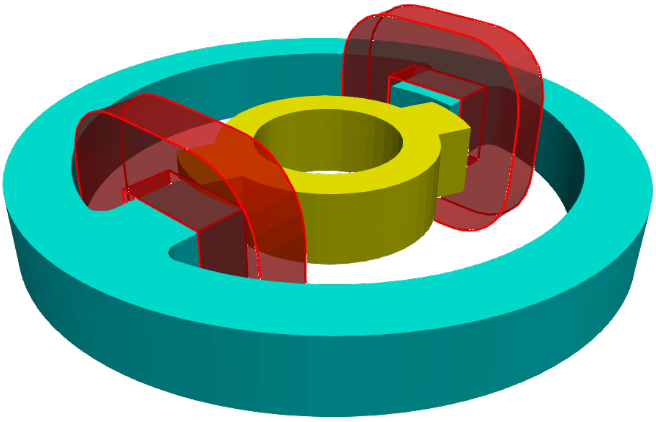
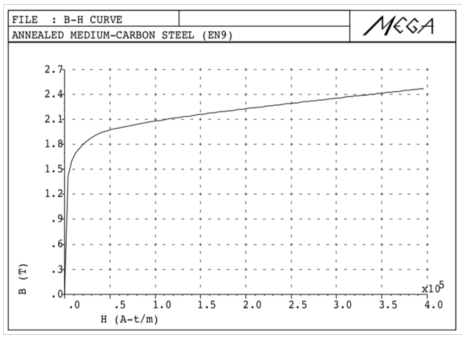

Fig.3: TEAM 24 benchmark problem: geometry (left) with coils (red), stator (blue) and rotor (yellow); nonlinear B-H curve (right).

In order to tackle this problem, a Newton method with line search method is employed [7]. The whole nonlinear solution algorithm with a Fast Multipole-based FEM-BEM coupling in each iteration is embedded into a time-integration scheme. Some results are given in Figure 4 where a colour plot of the magnitude of the **B**-field is shown together with the flux over time through a surface as specified in the TEAM problem descriptions. An excellent agreement with respect to published measurement data can be observed.

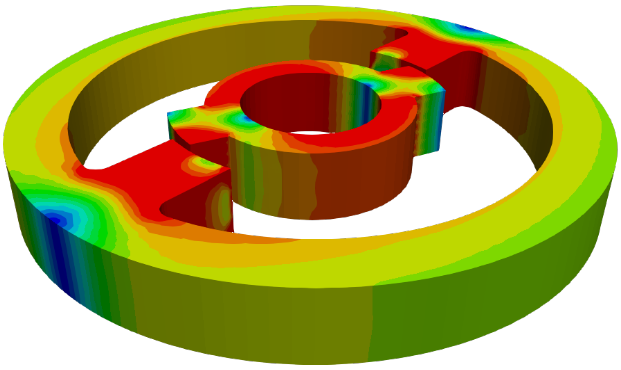
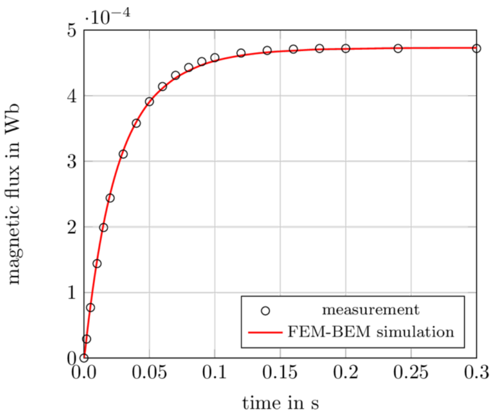

Fig.4: Numerical solution of *TEAM 24 benchmark problem: magnetic flux density at the final time step (left) and flux through a specified surface with measurement data (right).*

3 Summary

This paper presents a robust, efficient and accurate FEM-BEM coupling scheme tailored to magnetostatic and eddy-current problems with possible nonlinear material behaviour. Moreover, parts of this technology are currently being introduced to LS-DYNA. In a future release, with **`*EM_CONTROL`** and related keywords, users can make use of a FEM-BEM coupling based on a robust monolithic coupling scheme. Additionally, the handling of nonlinear BH-curves with Newton method and line search are being added to LS-DYNA.

4 Literature

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