Discrete Element Modelling of a metamaterial for launcher tanks dynamics experiments

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1 Abstract

Experimental vibration certification of launcher cryogenic tanks is an important issue in the aerospace industry. Liquid hydrogen LH2 dynamic vibratory behaviour in tanks experiments cause some scientific and technical problems. Indeed, the security, transport and conditioning problematics still remain critical.

Because of its demonstrative character, the space industry, particularly the launchers applications, have been chosen in order to show the effectiveness of the Discrete Element Method (DEM) recently implemented in LS-DYNA software. This project, financed by the French "Occitanie region" (FEDER), aims at understanding and reproducing the dynamic behaviour of a light fluid (LH2) in a tank thanks to a granular media.

This metamaterial will be composed of hollow spheres represented by Discrete Element Sphere (DES). Its aim is to represent the LH2 dynamic behaviour within a tank, without presenting the same flammability, explosiveness and fugacity risks as the LH2. This new material permits to simplify and secure cryotechnical launcher tanks qualification.

In order to reach this objective, a new methodology is developed by combining a numerical, an analytical and an experimental methodology. Thus, the project is based on these three main approaches:

- Thanks to homogenized models, the analytical approach permits to effectively establish the link between the homogenized granular media mechanical properties and the dynamic response of the fully filled tank. These simplified analytical calculations are used to delimit the research in term of material type (E, ρ, v) and frequency range.
- Thanks to the DEM, the numerical approach permits to make the link between the properties of one hollow sphere, of an equivalent full sphere and of the granular media made by these spheres. The dynamic study is also performed, thanks to the entire and fully filled tank modelling.
- Thanks to the coupling between the manufacturing, the numerical tests and the experimental ones, a procedure to produce and characterize the granular media properties will be establish in order to control its dynamic (vibratory) behaviour in a tank.

This article focuses on the numerical approach. The first part of the article especially deals with the numerical granular media properties establishment. Several experimental tests are reproduced in order to establish the main hypothesis that have to be taken into account to correctly reproduce the media characteristics (mechanical properties, friction, organization, dispersion...).

The second part deals with the study of the vibrating tank (empty and fully filled with spheres), and particularly focuses on the methodology used to obtain the required frequencies. This methodology consists in applying a defined pressure on the granular material confined in the cylinder in order to modify its density and its stiffness. Both properties have important impacts on the tank vibratory behaviour.

2 Introduction

Liquid hydrogen is widely used in space launchers. In spite of its energy capacities, its use represents a big issue because of its explosiveness and fugacity. Experimental vibration certification of launcher cryogenic tanks faces some challenges in the aerospace industry and causes some scientific and technical problems.

Nowadays it is then complicated to perform a dynamic experimental qualification of cryogenic tanks filled with liquid hydrogen. This is the reason why substitute materials, like water [1], are currently used to characterize the tanks dynamic behaviour. Unfortunately, the density difference between water and liquid hydrogen does not permit to measure all required information (eigenshapes, damping value...).

In order to safely perform those dynamic experiments and to obtain the missing information, the objective is to find an alternative metamaterial with the similar dynamic behaviour as the liquid hydrogen in the tank. This metamaterial will be composed of hollow spheres represented by Discrete Element Spheres (DES). Thus, this launchers applications have been chosen in order to show the effectiveness of the Discrete Element Method (DEM) recently implemented in LS-DYNA software.

This project, financed by the French "Occitanie region" (FEDER), aims at understanding and reproducing the dynamic behaviour of a light fluid (LH2) in a tank thanks to a granular media. To reach this objective, 3 approaches are jointly used to specify the equivalent metamaterial dynamic and vibratory properties:

- i. Thanks to homogenized models, an analytical approach permits to effectively establish the link between the homogenized granular media mechanical properties and the dynamic response of the fully filled tank [2]. These simplified analytical calculations are used to delimit the research in terms of material type (E, ρ , v) and frequency range.
- ii. Thanks to the DEM [3], a numerical approach permits to make the link between the properties of one hollow sphere, of an equivalent full sphere and of the granular media made by these spheres. The dynamic study is also performed thanks to the entire and fully filled tank modelling [4].
- iii. Thanks to the coupling between the manufacturing, the numerical tests and the experimental ones [5], a procedure to produce and characterize the granular media properties will be established in order to control its dynamic (vibratory) behaviour in a tank.

This article focuses on the numerical approach. The numerical approach is presented in two parts:

- The first part of the article especially deals with the numerical granular media properties establishment. Several experimental tests are reproduced in order to establish the main hypothesis that have to be taken into account to correctly reproduce the media characteristics (mechanical properties, friction, organization, dispersion...).
- The other one deals with the study of the vibrating tank (empty and fully filled with spheres), and particularly focuses on the methodology used to obtain the required frequencies. This method enables to identify modes while applying a white noise to the structure.

3 Metamaterial study

In order to establish a metamaterial that should have the same vibratory behaviour than the liquid hydrogen, multiple hypothesis have been established (described in [6]) and a study material has been chosen. The study material, used for this first step, has been chosen mainly because of its availability but its mechanical properties do not correspond to those permitting to reproduce tank filled with liquid hydrogen vibratory behaviour.

As a first step, the granular medium behaviour has to be investigated and understood. It is crucial to be able to characterize the granular media mechanical properties. Since there are links between the spheres properties (mechanics and tribology), the media ones and its vibratory behaviour, two different scales are studied in this part: the sphere level and the media one.

3.1 Sphere scale

3.1.1 Mechanical properties and sphere geometry

A sphere is characterized by its geometry, its density, its Young's modulus and its Poisson ratio. In the case of hollow spheres, two scales can be distinguished: the scale of the shell constituting the sphere (Fig. 1 (a) and (b)) and the scale of the hollow sphere, considered as a homogeneous equivalent material (Fig. 1 (c)).



Fig.1: (a) experimental picture of the used hollow spheres (cut), (b) FEM and (c) DEM



Fig.2: (a) experimental picture (compressive test), (b) zoom on the elastic domain

An average curve is calculated from the mass measurement and a spheres set diameters average. Compression tests are performed for one sphere to get back this given spheres properties (Fig. 2) and a numerical methodology is setting up, permitting to easily obtain the equivalent Young modulus and the Poisson ratio corresponding to the elastic domain:

- i. First, a finite element model (FEM) using some known sphere shell properties (e, E_{shell}, v_{shell}) is used to deduce the homogeneous sphere Poisson ratio (v_{DES}) from the hollow sphere model [6]. As long as only the hollow sphere elastic behaviour will be considered in further study, the behaviour of the shell is managed by a linear elastic law. The thickness and radius irregularities (Fig. 1 (a)) are not taken into account here, and spheres are supposed to be perfectly spherical with a homogeneous shell thickness (Fig. 1 (b)).
- ii. Then, a discrete element model (DEM) [7] is used to find a homogeneous equivalent material (since a full sphere is then considered) which properties correspond to the hollow sphere ones (E_{DES}, v_{DES}). Considering a given density p_{DES}, the equivalent contact stiffness k_{DES} is calculated, depending on the couple formed by the Young's modulus E_{DES} and the Poisson ratio v_{DES} (Fig. 1 (c)). The aim is to fit the experimental elastic stiffness k_{exp}, measured during a compressive test (Fig. 2).

3.1.2 Tribological properties

In the Discrete Element Method, some other interaction types between spheres and between spheres and structures exist and have to be considered: tangential stiffness, damping coefficients, static and rolling friction coefficients, etc.

To estimate the friction between the spheres, a grain flow test through a funnel is performed [7]. The flow rate and angle of repose are measured and the friction coefficient can be deduced.



Fig.3: friction determination device

The friction coefficient between the spheres and the tank material has been determined thanks to the numerical fitting of an experimental test. This test corresponds to the measurement of the distance travelled by a sphere subjected to an initial velocity on a plane made with the same material as the tank one.

The tangential stiffness is chosen very low and as a function of the normal one [8]. Damping is not implemented in this work.

3.2 Media scale: mechanical properties

Using the mechanical properties determined in the sphere scale, the aim is to find the mechanical properties of the granular media made by those spheres. Several experimental compression tests are performed on this granular media to determine its stiffness. The configuration is shown on the Fig. 4.



Fig.4: Numerical model (b) experimental model

A first compression test is made on the granular media until spheres failure (Fig. 5 (a)). Those tests are numerically fitted to make the link between a sphere properties and the media ones. The Fig. 5 (a) shows that those tests are not perfectly reproducible, but an order of magnitude can be considered.

As long as spheres are supposed to remain in their elastic domain during future vibratory tests, the attention focuses on the elastic domain of this curve (Fig. 5 (b)). Considering the stress levels, a high dispersion appears. This dispersion is probably linked to granular media mechanical behaviour, which is partly managed by the spheres initial positions and its compactness regarding this kind of loading. Those initial position and compactness dependencies have been numerically observed.

Multiple charge-discharge compression tests were then performed on this domain. The Fig. 6 shows a fairly good correlation between the various experimental tests average curves and the numerical one.

Some discrepancies are observed between the numerical and the experimental results. Some physical parameters were simplified, like the diameter dispersion, and the experimental spheres non-perfect sphericity. Furthermore, the results seem relevant and correlate the experimental order of magnitude. So, the previously determined spheres properties are considered like sufficiently appropriated to represent the material at the media scale. Those properties are used to characterize spheres in the vibrating tank (fourth section).







Fig.6: Experimental charge-discharge compression tests average curves and numerical test

4 Identification method on an empty tank

Once the spheres properties are characterized, the next step is to characterize the dynamic behaviour of a vibrating cylinder. As a first step, the empty cylinder is studied, in order to establish the numerical and experimental modal identification procedure.

The aim of this part is first to verify that the excitation of researched modes with the available experimental device is possible. So at the beginning tests are performed on an empty cylinder.

4.1 Experimental test

To model the cryogenic tank, the experimental tank device is composed of a closed cylinder filled or not filled by spheres clamped on a shaker (Fig. 7 and 8). The stiffness of the granular material is driven by the value of pre-stress imposed during the vibration tests. This pre-stress can be checked thanks to pressure sensors located on the cylinder. Three related load cells are fixed to the clamping system (Fig. 7 (b) and Fig. 8 (a)).

The cylinder material is a translucent polycarbonate, which thickness is 3mm, radius of 100mm and length of 602mm. The cylinder lid, made of aluminium, is free (six degrees of freedom) and is fixed to the cylinder by screws. The bottom is clamped to a vibrating table by clamping jaws (Fig. 7 (c) and (d)).

Twelve accelerometers are disposed around the cylinder cross section at a high of about one fifth of the cylinder one. The accelerometers purpose is to track transversal displacements (lobe mode shapes). Seven accelerometers are considered for tracking the longitudinal displacements in the vibrations direction (direction for which the bending is fostered) (Fig. 8 (b) and (c)).

The applied transversal loading corresponds to a vibrating table frequency scanning over a defined frequency range (up to 3kHz). This process permits to immediately extract the tank frequency spectrum in order to analyse which of the scanned frequencies excite natural modes.



Fig.7: Main steps for preparing the vibration test of the tank fully filled by spheres (from (a) to (e))



Fig.8: (a) Experimental embedding and load cells, (b) experimental vibrating table device and (c) accelerometers positioning and displacements rebuilding during the vibrating phase

4.2 Numerical test



The numerical device is a simplification of the experimental one because the lid, the compression plate and the three force transducers are considered as rigid parts. The screws fixing the lid to the cylinder are modelled by additional discrete masses. Moreover the clamping system is not explicitly reproduced and only the cylinder bottom nodes are clamped. The cylinder behaviour is considered fully elastic (the cylinder is only submitted to elastic deformations) and is modelled with shells. The tank mesh size has been chosen in accordance to the spheres diameter [4] (Fig. 9).

Fig.9: Numerical device

Since DEM cannot be used in implicit computations, the objective of this part is to find ways to be able to identify the tank modes using the explicit solver. Since, reproducing the experimental test leads to too high computation durations, two other explicit identification alternatives have been established and validated thanks to the implicit solver (exclusively on the empty tank model):

The first one mimics the experimental configuration (Fig. 10 (a)). It consists in vibrating the cylinder base via a table vibrating at a specific frequency. So, to obtain the cylinder behaviour on a frequency range, multiple vibratory computations have to successively be performed (one frequency per computation). Even if this modelling method is efficient, it is really time consuming because of the high number of computed simulations.

The second one, called "pulse method", consists in applying a local and punctual pulse on the middle section area of the cylinder (Fig. 10 (b)). This kind of loading permits to excite all the natural modes of the structure: it is a white noise. This method is the most appropriated one because it is efficient and less time consuming than the numerical vibrating table one.



Fig.10: Load illustration for numerical mode detection: (a) transversal vibration, (b) pulse with the chosen identification points (green)

The aim of using the pulse method is to quickly identify the natural frequencies and shapes of the tank. The natural frequencies can easily be obtained, but the mode shapes identification requires a transformation from the frequency domain to the time domain. Thus, six steps permits to find the modal frequencies and the modal shapes:

- To apply a pulse to the cylinder with LS-DYNA. This permits to excite all the structure modes;
- To extract the displacements time history of chosen nodes (green points on the Fig. 10 (b));
- To compute a Fast Fourrier Transform (FFT) to the extracted displacements of each node in each direction. This permits to identify the modal frequencies. Then the associated mode shapes have to be identified.
- To isolate a mode by multiplying the previous FFT to a Hamming filter centred on the frequency of the isolated mode.
- To apply an inverse FFT to the new signal. This step permits to find the nodal displacements associated to the isolated mode.
- To build the surface made by nodes on the cross section whose displacements are those calculated before.

This process has to be performed for each mode shape that has to be identified.

4.3 Results: eigenmode on the empty tank

The Fig. 11 (a) shows a part of the experimental device frequency spectrum of three accelerometers compared to the numerical one of one point (Fig. 11 (b)). The transducers positions have been chosen in order to measure precise modes:

- The upper one measures the bending modes displacements;
- The two next ones, situated on a cylinder section measure the bending modes and the transversal ones (Fig. 11 (c)).



Fig.11: (a) Experimental frequency spectrum, (b) experimental accelerometers positioning, (c) numerical frequency spectrum and (d) numerical transducers positioning, on the empty tanks

The numerical natural frequencies are in good accordance with the experimental ones (the average error is around 3% on the 9 first modes frequencies). This validates that the numerical applied load excites all the tank modes and that its natural frequencies can be determined thanks to the FFT of its displacements. However, the numerical graph shows more natural frequencies that the experimental one. This is because the numerical loading is able to excite more natural modes than the experimental one. The Fig. 11 (a) shows that the transversal modes have a magnitude more than ten times less than the fisrt bending mode one, and that they are always coupled. This coupling probably comes from the chosen load because the numerical spectrum (Fig. 11 (c)) does not show this kind of coupling and present high frequencies for transversal modes too.

The second part of the numerical identification method consists in comparing the experimental mode shapes to the reconstructed numerical ones. The Fig. 12 compares the three first experimental and numerical mode shapes corresponding to each natural frequency.



Fig. 12: Comparison of the three first experimental and numerical mode shapes

The mode shapes match very well. However there exists a quite significant difference in frequency between the natural frequencies of the oval modes. Two main reasons that could explain this difference have been identified:

- The imprecision of the numerical model (shell formulation, sampling frequency);
- The experimental modes coupling. On the Fig. 12 there is a coupling between the oval mode and the three lobes one. Moreover, since the experimental loading is a transversal one privileging the bending modes, all the lobe modes are more or less coupled to bending modes which can slightly modify the structure response. The numerical loading is not a transversal one, which can partly explain the difference between the experimental and numerical natural frequencies.

The numerical mode shapes study confirmed the previous expectation that the numerical loading permits to excite more natural modes than the experimental process. Those tests permit to validate the numerical identification method on an empty tank. The next section deals with the experimental modal identification tests on the tank filled with spheres and the application of the numerical modes identification method.

5 Application to a tank filled with spheres

5.1 Experimental and numerical protocols

The experimental and numerical protocol is composed of the same three main steps:

- i. the tank filling;
- ii. the pre-stressing of the granular material and the locking of the system;
- iii. the shaking.

The experimental and numerical device is the same as the previous one where spheres are put in the tank (Fig. 13 (a) and (b)). Hollow spheres are the same as the one whose characteristics were established in the first section.



Fig.13: (a) Experimental tank filled with hollow spheres, (b) numerical tank filled with equivalent full spheres

The main difficulty of the protocol is the tank filling. A real spheres organization problem occurs during experimental and numerical tests. Indeed, during experimental and numerical tests, the vibrations cause movements between spheres and so reorganization. The effect is a material compaction and then the tank is no longer fully filled (fig. 14 (a) and (b)).

As long as there is a space between the spheres and the lid, the spheres have too much movement freedom which completely skews the results. Consequently no mode could be measured.

Indeed, it has been established in the first part that the compactness and the spheres organization can have a huge influence on the media behaviour. Indeed, a first gravity step has to be performed on auto generated spheres. The compactness is then acceptable (around 57%) but is too small to prevent spheres reorganization during the loading.

Experimentally, some spheres are added in the tank and then compressed by the bottom compression plate to offset this problem. This compression step has been extremely complicated to fix because the spheres experimental configuration does not easily permit to compress spheres or to add spheres to eliminate the space appearing after vibrations.

The same problem has been numerically encountered (Fig. 14 (a) and (b)) because of the impulse applied to the tank, which is a really good news for the young Discrete Element Method in LS-DYNA. Indeed, the numerical method is able to reproduce this physical problem.

So, the numerical solution looks like the experimental one but is easier to set up: an intermediate step between the gravity one and the pulse one consists in compressing the media by the bottom compression plate to reduce the spheres degrees of freedom.



Fig.14: Illustration of the settling phenomenon occurring during the tank oscillations ((a) experimentally and (b) numerically)

The tank is completely filled (100% filling) and all the tests are performed with this configuration. Experimentally, there are about 465 000 spheres with an average diameter of 3.38mm in the cylinder. Numerically, there are about 475 000 spheres, which represents 2% more spheres than in the experimental tank.

The numerical compression is really low and corresponds to a compression plate of around 0.1mm, whereas the numerical compression plate displacement is imposed to around 5mm. Experimentally compressing the media to a displacement of the millimetre order is more complicated than numerically. In order to maximally avoid spheres movements in the tank, the numerical compression is exaggerated. This have a negligible influence on the spheres mechanical behaviour as long as the spheres material law is considered purely elastic.

In this configuration, the next part presents the comparison between experimental and numerical results, and the issues encountered during the tests.

5.2 Results: eigenmodes for the fully filled cylinder

Once the spheres movements have been reduced as much as possible, the frequency spectrum could be studied (Fig. 15). Those spectrums show only two or three identifiable natural frequencies, which is less than 50% of the previous empty tank identifiable frequency number on the same frequency range. However, this result is experimentally and numerically visible on the frequency spectrums: there is a great correlation between those results.

When performing the mode shapes identification, both experimental and numerical identification methods show the same recognizable natural frequencies which correspond to bending modes.

Moreover, the magnitude of the second peak is really low. Experimentally, no other peak can be detected whereas numerically some small ones can be observable while zooming on the spectrum. Those peaks show kinds of mode couplings, but do not correspond to perfect mode shapes. The provenance of those "modes" has not yet been established, and it is possible that they finally correspond to structural noise. This track will be studied in-depth.

Thus, it was complicated to identify transversal modes. This issue can come from various sources:

- Since spheres add mass in the tank, the bending mode magnitude is amplified so that the transversal modes cannot be observable (weak magnitude) (on the Fig. 15 (c), a factor of 200 is measured between the magnitude of the two first bending modes).
- The coupling with the predominant bending mode is high and so the transversal modes can not be identified (the displacements induced by the bending mode, considering the applied pulse magnitude of 200N, is of the order of 0.01mm).
- This damping seems to come from the "internal" spheres movements inside the tank. The spheres initially chosen for their availability have a too viscous behaviour to allow lobe modes to occure (topological properties).



Fig. 15: (a) Experimental frequency spectrum, (b) experimental accelerometers positioning, (c) numerical frequency spectrum and (d) numerical accelerometers positioning, on the fully filled tanks

Some solutions were found to verify those potential explanations, and will soon be set up:

- Experimentally, to use a shaker to apply a pulse on the structure and not a transversal loading;
- Experimentally, not to excite the bending mode by beginning to vibrate the structure at a frequency higher than the bending mode one.
- Numerically, to apply a pulse load which does not excite the bending mode.
- Numerically, to try to modify the spheres properties (topological ones) to test the feasibility of the transversal mode excitation.

Those tests will be performed as the next step of the project.

6 Summary

In the TANKYOU project, the first aim was to evaluate the possibilities of representing a continuous fluid dynamic behaviour with a discrete medium (spheres). The study is then divided in two steps:

- Evaluating the mechanical properties of a sphere and on the granular media made by those spheres;
- Evaluating the vibratory behaviour of this granular media in a vibrating tank.

As a first step, the numerical tests aimed at determining the spheres mechanical properties and the link between one sphere properties and the media one. To do so, some experimental tests coupled to numerical tests permitted to determine some of the spheres properties:

- i. Some compression tests on one sphere permitted to determine their Young's modulus. A first finite elements model reproducing the hollow sphere in compression was compared to a DES full sphere model.
- ii. Some experimental compression tests on a set of spheres allowed to determine and verify the numerical parameters previously identified during the sphere scale studies. The establishment of the relation between a unique sphere properties and the granular medium ones has been considered.

As a second step, the experimental and numerical tests focused on the tank and the granular media vibratory behaviour. In order to set up a modal identification method (experimentally and numerically), the first tests were conducted on a empty tank. The experimental tests consisted in vibrating the tank at increasing frequences in a given range. This permitted to build the tank frequency spectrum and the mode shapes associated to the identified natural frequencies. The same process has been conducted numerically, but in order to save computation time, the applied load corresponded to a white noise (located pulse on the tank). The obtained results showed great correlations between experimental and numerical tests considering both frequency spectrums and mode shapes. Those results permitted to validate both the experimental and numerical identification methods, thanks to an annex analytical study.

Those identification methods were then applied to a tank fully filled with spheres, which properties were evaluated previously. The results showed again a good correlation between experiments and numerical models. However, the spheres settling and reorganization in the oscillating cylinders caused some issues to the tank filling. After having offseted this, only the two first bending modes could have been identified by the experimental and numerical methods. Several reason of this behaviour have been pointed out, particularly the influence of the bending mode on the other ones and the spheres friction properties.

Another way for study consists in examining the bending frequencies evolution depending on the compression levels applied on spheres. The comparative approach combining experiments, the DEM and the analytical study shows multiple physical phenomenon understanding prospects (relative movements, non-linearities, ...). From the approach combining manufacturing, numerical models, analytical models and experimental measurements, a new efficient granular material may be defined.

7 Ackowledgement

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