Topology optimization methods based on nonlinear and dynamic crash simulations

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Abstract

Topology optimization for crashworthiness has been investigated during the last years, starting from methods based on linear elastic and static simulations [1] or so-called equivalent static loads (ESL) obtained by a single nonlinear crash simulation with a subsequent optimization loop based on the linear stiffness matrix and the corresponding sensitivities [2, 3]. Both methods do not consider material nonlinearities in their optimization process, which are essential for structural components designed for energy absorption, although it is well-known that plasticity and failure play an important role.

As alternative, optimization methods have been proposed, which use fully nonlinear and dynamic crash simulations. The first method, presented for example in Patel's PhD thesis, is based on a hybrid cellular automata approach (HCA) and derives optimal structures using a homogenized energy density approach where each finite element (cell) is modified until the highest degree of homogeneity is achieved [4, 5]. Because this is not fully appropriate for thin-walled structures, Hunkeler modified the approach (HCA-TWS – Hybrid Cellular Automata for Thin-walled Structures) such that deformation energy is only homogenized between larger structural entities (i.e. thin ribs / walls) [6, 7]. The most recent method, the EA-LSM, a combined level set method (LSM) and evolutionary approach, was then proposed by Bujny et al. where more appropriate objectives and constraints can be used with the drawback of higher computational costs [8, 9].

In this paper, the latest results for HCA-TWS and EA-LSM will be presented, see also [10]. Special focus is here the investigation of the influence of different material models for plasticity. Examples are inspired by recent material model developments for magnesium alloys with a characteristic anisotropy in the plasticity model [11]. As a result, it is shown that the optimal topologies depend on the material model and that it is necessary to use nonlinear and dynamic finite elements for crash topology optimization.

Keywords Crashworthiness, Nonlinear Topology Optimization, Hybrid Cellular Automata, Level Set Methods, Material Models, Plastic Anisotropy.

1 Introduction and State of the Art

Topology optimization is well-established in most areas of structural engineering. For crashworthiness, due to its high inherent nonlinearity, this is still not the case. Here, most published and probably also not published work have been based on linear elastic, static finite element methods (FEM) neglecting (i) all dynamic effects like rate dependency of the material behavior or inertia effects, (ii) material nonlinearities like plasticity and failure, and (iii) nonlinearities due to contact. Efforts to include dynamic behavior by considering a set of static loads to represent the different phases of the crash have helped slightly [1], but the drawback of linear material modeling remained. Neither plasticity with or without hardening nor failure have been integrated into these optimizations although it is well-known that optimal topologies differ when geometrical, contact, or material nonlinearities are important. This was already shown for non-crash cases and is even more important for crashworthiness. Fig. 1, for example, shows that the result of a topology optimization depends on material nonlinearities [12].

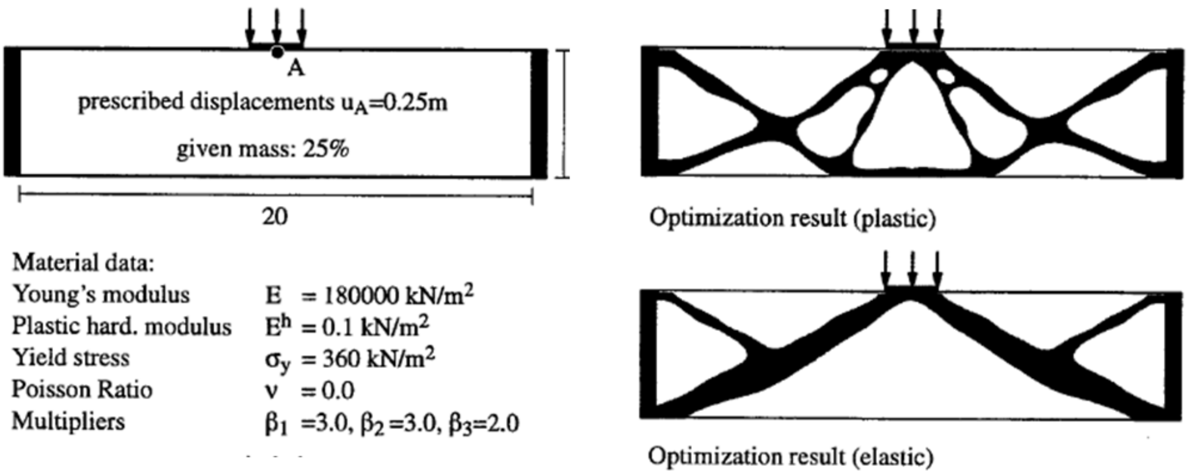


Figure 1: *Topology optimization results for a static case with linear and nonlinear material models, [12]*

In this example, structural ductility, defined as the integral of strain energy over a given range of a prescribed displacement, was taken as objective together with a mass constraint. At the beginning of the optimization, the design space was completely filled and the densities, ρ_i, of the finite elements (voxels) were used as design variables. Three functions, b_j; j = 1…3, were defined, which depend on this density, to scale the elastic material tensor, the hardening modulus and the yield strength. Taking the idea of penalization established in the standard SIMP method (Solid Isotropic Microstructure with Penalty for intermediate densities), three parameters, β_i, were defined as exponents such that $b_j = (\rho_i/\rho_0)^{\beta i}$ with ρ_0 as initial density. This example is one of the first studies on the usage of plasticity for (static) topology optimization. It shows clearly the difficulty to consider advanced material models as required for crash analysis. Such a scaling of material parameters driven by only one parameter (here density) becomes challenging for more realistic material models than the linear hardening model used here; see also the more detailed discussion in [12]. These approaches use a high number of design variables, i.e. voxel elements, and a gradient-based optimization method, which is only feasible via sensitivities computed by adjoint methods. They cannot, at least currently, be applied to nonlinear and time-dependent crash analysis based on explicit FEM where gradients are difficult to calculate and adjoint methods not available.

In addition, it is important here that a standard elasto-plastic material model was used (von Mises with linear, isotropic hardening), which assumes plastic incompressibility. This means that enclosed areas where the densities reach near-zero values still contribute strongly to the mechanical behavior not allowing plastic deformation related to volume change. To avoid this effect, it is necessary to eliminate these elements as shown in the left image of Fig. 2, see also [13].

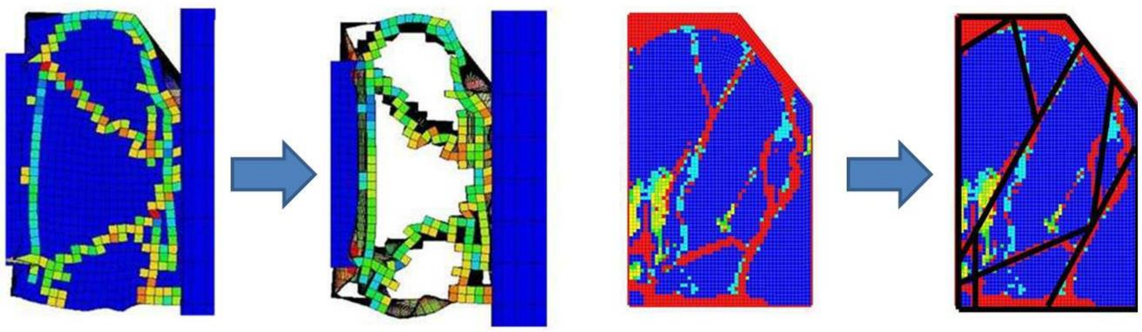


Figure 2: *Element elimination to reduce the effect of plastic incompressibility (left) and illustration of the necessity to interpret volume element results as thin-walled shell structures (right), [13]*

Stress concentrations due to the resulting non-smooth (jagged) boundaries influence strongly the results, especially for crash, because the nonlinear buckling and post-buckling behavior is triggered by these mesh-dependent quantities. The work published in [12] already addresses this problem; in the static examples discussed there, the influence of the jagged boundary can be neglected for elastic problems but is changing results drastically when plasticity is considered. A very fine mesh may be required to reduce these artificial effects. In [12], an adaptive strategy is proposed to overcome this problem based on isolines of one or more density levels (Fig. 3). This resembles ideas used in level set topology optimizations, which will be discussed more in detail below. Another issue is the required interpretation of the obtained optimal topologies based on three-dimensional voxels as a shell structure, see Fig. 3 (right).

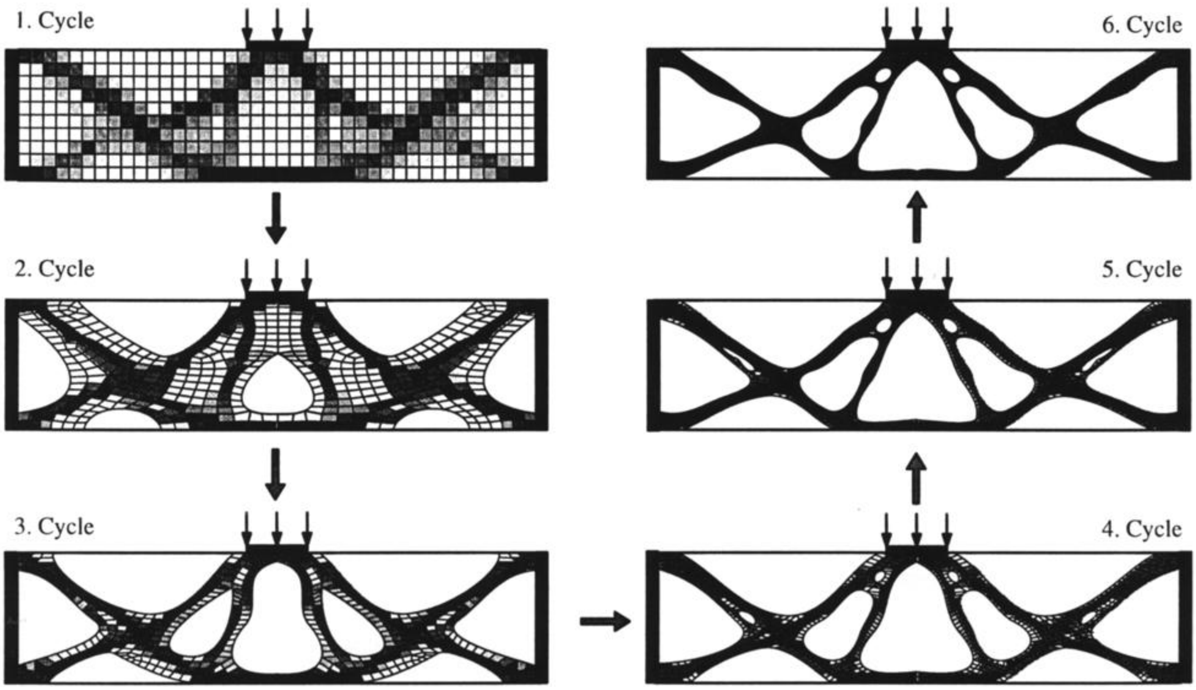


Figure 3: *Adaptive strategy to avoid stress concentrations due to jagged geometry, [12]*

This chapter is meant to illustrate the motivation and the difficulties of crash topology optimization. It is not the purpose of the authors to give a complete overview of relevant literature. Hence, more information with focus on topology optimization considering plasticity can be found, e.g., in [14-19]. An overview of topology optimization for crashworthiness was already published by the authors [1, 20]. In the following part of this paper, results from two methods, the hybrid cellular automata method for thin-walled structures (HCA-TWS) and the evolutionary level set method (EA-LSM) will be presented, both using truly nonlinear crash simulations with full nonlinearity. They illustrate the high influence of the material model for isotropic and anisotropic plasticity, this time for dynamic cases encountered in crashworthiness problems. The background theory of the two methods will be summarized first.

2 Topology Optimization Methods using Nonlinear FEM (Explicit)

2.1 Hybrid Cellular Automata for Thin-walled Structures (HCA-TWS)

To the authors' knowledge, the use of cellular automata (CA) for topological design of structures was first published in 1994 [21] addressing local homogeneity of physical quantities. Later applications were extended to hybrid cellular automata (HCA) considering also global aspects from FEM [22]. Based on this, Patel introduced HCA for crash topology optimization of structures modeled with volume elements (voxels) using nonlinear FEM [4]. Hunkeler transferred this to thin-walled structures modeled with shell elements [6, 20]. The main difference of the latter to the other approaches is that cells are now defined by sets of multiple finite elements (shells), i.e. that the cells are not identical to the finite elements. This allows folding and buckling of thin-walled structures having inside of the cells a non-homogeneous energy density distribution, which is essential for crashworthiness design. The approach was improved (higher stability and efficiency) and published in [10].

The general work flow is illustrated in Fig. 4 and discussed step-wise in the following.

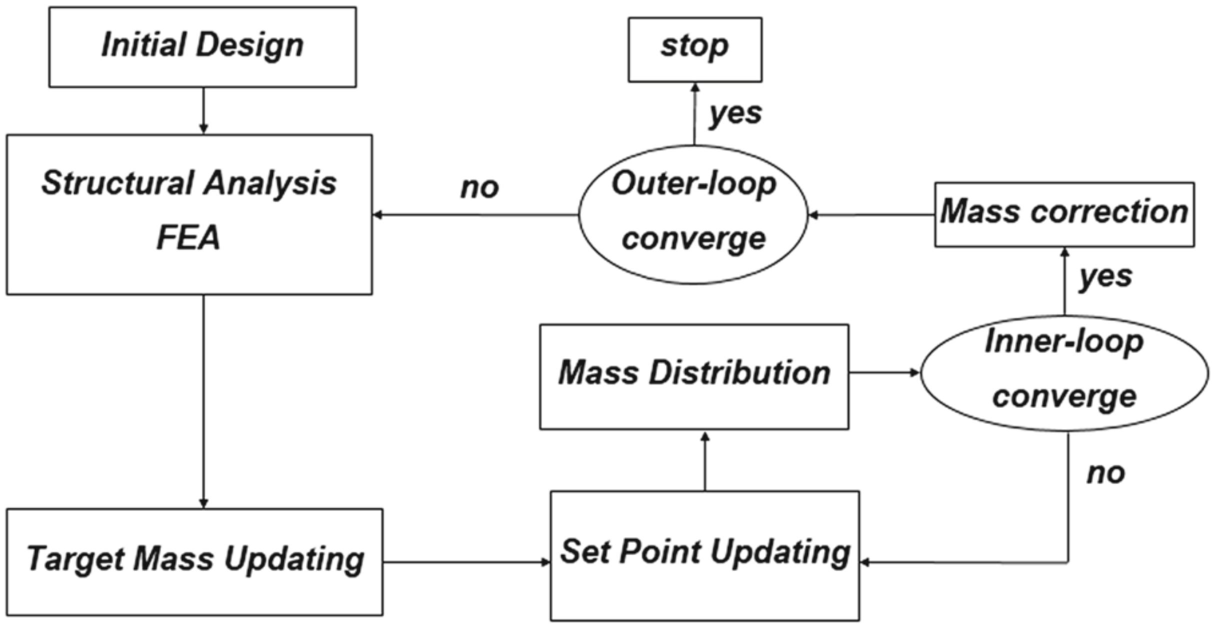


Figure 4: *Workflow of the HCA for thin-walled structures with nonlinear, dynamic FEM [10]*

Initial design. The first step, the initialization, includes the definition of the initial values of the n design variables, x_{0i}, $i = 1 \ldots n$, and their lower and upper bounds, x_{Li} and x_{Ui}. As variables, the thicknesses of the cell walls, t_i, are taken. In addition, the mechanical model and the load case must be defined together with the walls and their neighborhoods. This resembles the definition of ground structures in other approaches, e.g. [23]. The walls are later optimized such that they have the same total deformation energy as the neighboring walls (homogenization via CA). This definition is crucial for the proposed method; it means that the derived optimal design depends on this initialization. The overall mass fraction objective, M^*, and the constraint(s), C, have to be defined as well. It should be noted here that current HCA algorithms are not fully flexible concerning the definition of arbitrary constraints. Here, further improvements are required. In the work presented here, a monotonic relationship between mass and constraint is assumed.

Structural analysis (crash). In the outer loop, the model is evaluated via transient nonlinear FEM with explicit time integration (LS-DYNA). The current total mass, $m^{(j)}$, the states to be homogenized, $U_i^{(k)}$, and the value of the constraint, C, are formulated and passed on. In the example, the total deformation energy of the cells (i.e. entire walls) is chosen for $U_i^{(k)}$ and an intrusion constraint is taken as C.

Target mass updating. The target mass fraction, $M_f^{(k)}$, for the iteration step k is calculated via an a priori rule, which specifies if more or less mass reduces the constrained response. In case of constraining intrusion, more mass generally results in a reduction of intrusion and vice versa. Details on these rules are discussed in [6, 10, 20]. After this, the inner loop is started to derive the changes of the design variables in this step.

Set point updating. As initial set point (first outer iteration), the average internal energy of the cells is taken. Otherwise, the value of the last iteration is considered together with the previous mass fraction and target mass fraction; details and especially a new algorithm based on bi-sectional search are presented in [10], which improves the algorithm strongly in case of a higher number of design variables.

Mass distribution. The inner loop changes the design variables, here the wall thicknesses, according to the set point and the target mass fraction. It does not require any FEM simulations and is therefore computationally very efficient. The update of the design variables is done iteratively controlled by a sequential change of the set point. The special update rules are also published in [10].

Inner loop convergence and mass correction. The convergence of the inner loop is checked, i.e. it is controlled if the target mass fraction is reached. In case that elimination and re-introduction of cells is considered[1], the mass has to be corrected [10].

Outer loop convergence. The convergence (fulfillment of overall mass target) of the outer loop is checked and the optimization is either continued or terminated. The approach presented here drives the design normally to the constraint limit.

2.2 Evolutionary Level Set Method (EA-LSM)

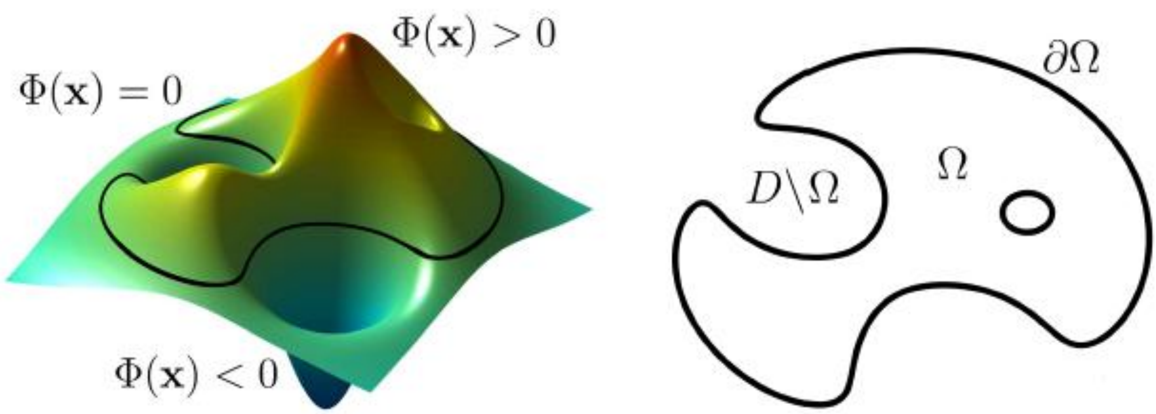


Figure 5: *Example of a level set function Φ (left) and the corresponding material domain $\partial\Omega$ and boundary $\partial\Omega$ for a two-dimensional design optimization problem [25]*

Level set methods (LSM) are based on an implicit parameterization of geometry, i.e. the material boundaries, with a level set function and an additional computational domain (here FEM), e.g. [24]. Interfaces between different phases of material are defined by iso-contours of a level set function, Φ. As shown in Fig. 5, positive values of Φ define regions, Ω, occupied by material, negative values define void. The boundary $\partial\Omega$ is described by the 0-th iso-contour. The optimization (shape and/or topology) varies this level set function and therefore the geometry and maps this information to a finite element mesh where it is evaluated. Normally, the changes are driven by gradient information originating from adjoint solvers or other advanced approaches; but this is not available in crash analyses using nonlinear, dynamic FEM (e.g. LS-DYNA). Adjoint solvers for explicit FEM do not exist and correct gradients are difficult to compute due to the intrinsic noise of the crash physics and/or numerical procedures. Hence, a non-gradient approach based on evolutionary strategies is proposed here, which was published for the first time in [25]. The principles of the approach are described below and details will be published in the near future [26].

Evolutionary strategies were developed independently of genetic algorithms in the 1970s by the research group of Rechenberg & Schwefel [27, 28]. They are mainly driven by mutation and are today very flexible and efficient because of their adaptation of strategy parameters (self-learning via CMA-ES, Covariance Matrix Adaptation Evolution Strategy), e.g. [25]. Their potential for crash shape and size optimization was, for example, explored in the work on multi-disciplinary optimization for car bodies by the first author [29] using methods developed by DIVIS[2].

[1] For example via SFE CONCEPT, www.homepage.sfe-group.org/en/products/sfe-concept
[2] www.divis-gmbh.de

Concerning the geometry representation, the work presented here is based on a level set function, which is a union of a set of local functions with a low number of free parameters. This is necessary to reduce the computational effort. Standard LSM do not need this because the gradient- and adjoint-based approach can handle a high number of design variables; to the opinion of the authors, this cannot be applied directly to crashworthiness problems. One of the local level set functions is shown in Fig. 6; parameters are length, position, angle, and width (for a 2D design problem).

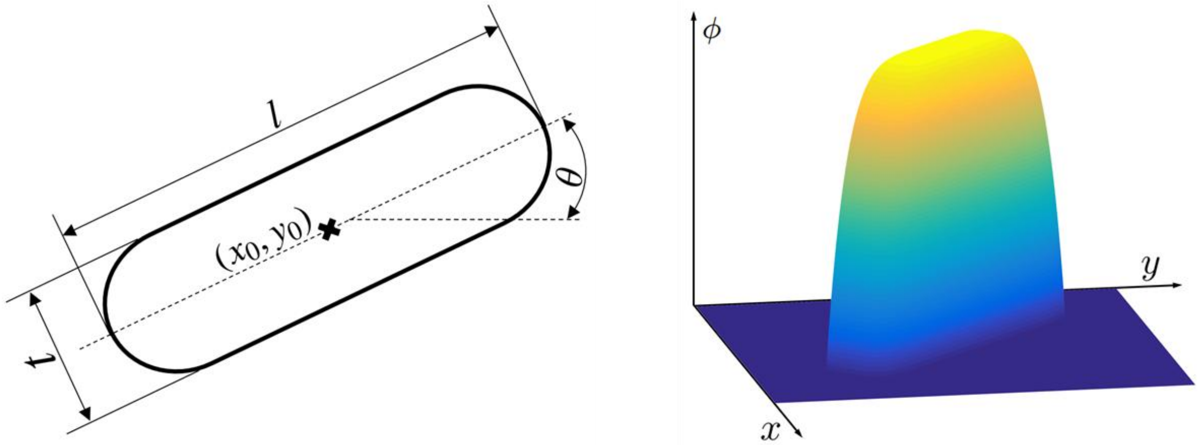


Figure 6: *Parameterization of an elementary structural component (left) and the corresponding level set function where negative values are set to zero (right), proposed in [30]*

3 Test Cases and Results

3.1 Material models

In this paper, first results for crash topology optimization using nonlinear and dynamic FEM are presented with a focus on the role of the plasticity model for the derived topologies. As discussed in the state-of-the-art part of this paper, there are only very few publications on nonlinear material topology optimization. Zhang et al. [19] give a good introduction into the topic based on SIMP. They used an anisotropic elasticity combined with an anisotropic plasticity model (Hoffmann yield function as generalized Hill yield function) with isotropic hardening and associated flow rule. Different yield strengths in tension and compression can be defined. This enables, in contrast to Hill, the consideration of not only pressure independent but also pressure dependent plasticity. Lower bounds for the interpolation of the plastic properties are higher than for elasticity to assure stability of the approach. As objective, the absorbed maximum plastic energy in the design domain is taken with a prescribed amount of material and displacement constraint. Sensitivities are obtained via an adjoint solver. The results motivate the study at hand and illustrate clearly the strong influence of the material model on the optimal geometries (in [19] only for static cases). For crash, advanced material models should be considered with elastic and plastic anisotropy as well as different yield for tension and compression. In fact, this should be extended such that failure is included, which is work currently under investigation by the authors.

In the first case presented here, which explores the potential of HCA for crash topology optimization, material models offered by LS-DYNA and an advanced model by MATFEM[3] are chosen based on [11]. Anisotropy is proposed, e.g., for magnesium wrought alloys with their hcp[4] crystal structure showing plastic anisotropy at room temperature and strong asymmetry of yield locus and hardening under tension and compression. This is due to twinning depending on the actual stress state. The results are compared with those obtained by a standard von Mises model and a modified von Mises model (LS-DYNA, MAT 024, MAT 124). Results for the MATFEM model will be published elsewhere.

[3] www.matfem.de

[4] hcp: hexagonal close packed

For the LSM examples shown in this paper, a simple material model (piecewise linear elastic-plastic material model) was chosen to establish the method. More advanced material models are currently investigated and will be published in the near future.

3.2 Test cases: Beam under transversal impact

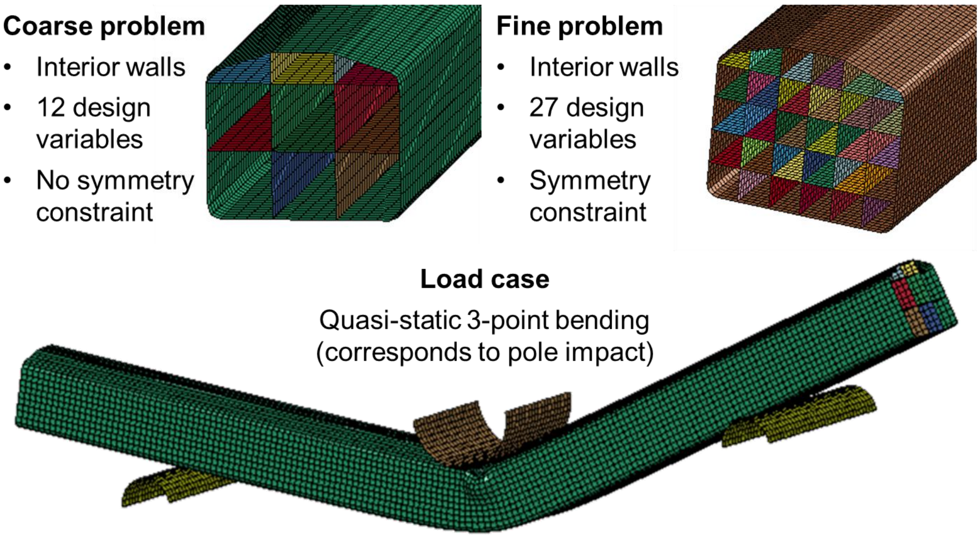


Figure 7: *Two sample problems for HCA method illustration [31]*

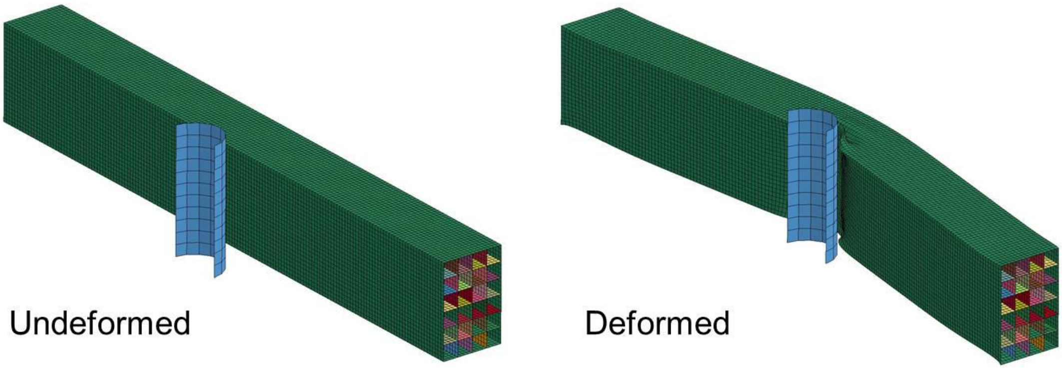


Figure 8: *Validation problem for high speed impacts comparable to the case shown in Fig 7.*

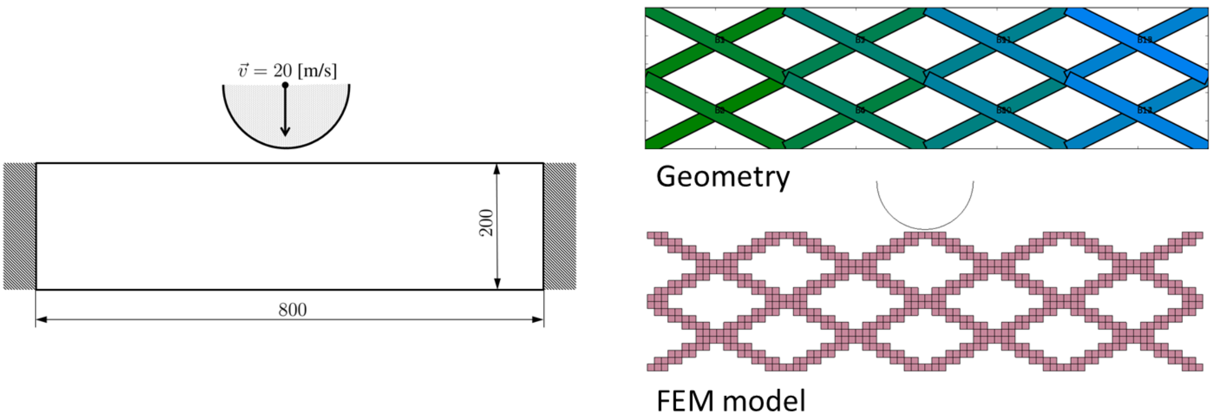


Figure 9: *Test case for LSM (left) with union of initial level set functions (right, top) and corresponding FEM mesh (left, bottom) [25]*

As example for the HCA, an extrusion beam is considered where the interior reinforcements modeled as ribs are optimized, see Fig. 7. Because these are the first results on this topic, an initial optimization was realized for a quasi-static case (3-point bending) corresponding to a pole impact. Two different initial geometries are defined as shown in Fig.7. As high speed impact case with simpler material models, see also [6, 7, 10, 20], and without addressing explicitly the role of the material model, a beam under transverse impact is shown here, which is fixed at both ends, see Fig. 8.

For the LSM, a beam under transverse impact is regarded without extrusion constraints in axial direction. Instead, the geometry was kept constant transverse to the beam axis, which makes the problem two-dimensional. More advanced parameterizations and optimizations will be published in the frame of ongoing PhD theses. The initial geometry is presented in Fig. 9.

4 Results

4.1 Results from HCA-TWS for Crash Topology Optimization

As results for the HCA, the quasi-static case with HCA leads to different topologies depending on the chosen material model. Fig. 10 shows the results for the coarse problem with a relative low number of cell walls. The top row with the isotropic von Mises model (red) led to an optimum after ca. 16 iterations (= inner loops); this means after only 16 FEM simulations. The bottom row shows a slightly different inner topology achieved after 14 iterations with slightly higher mass.

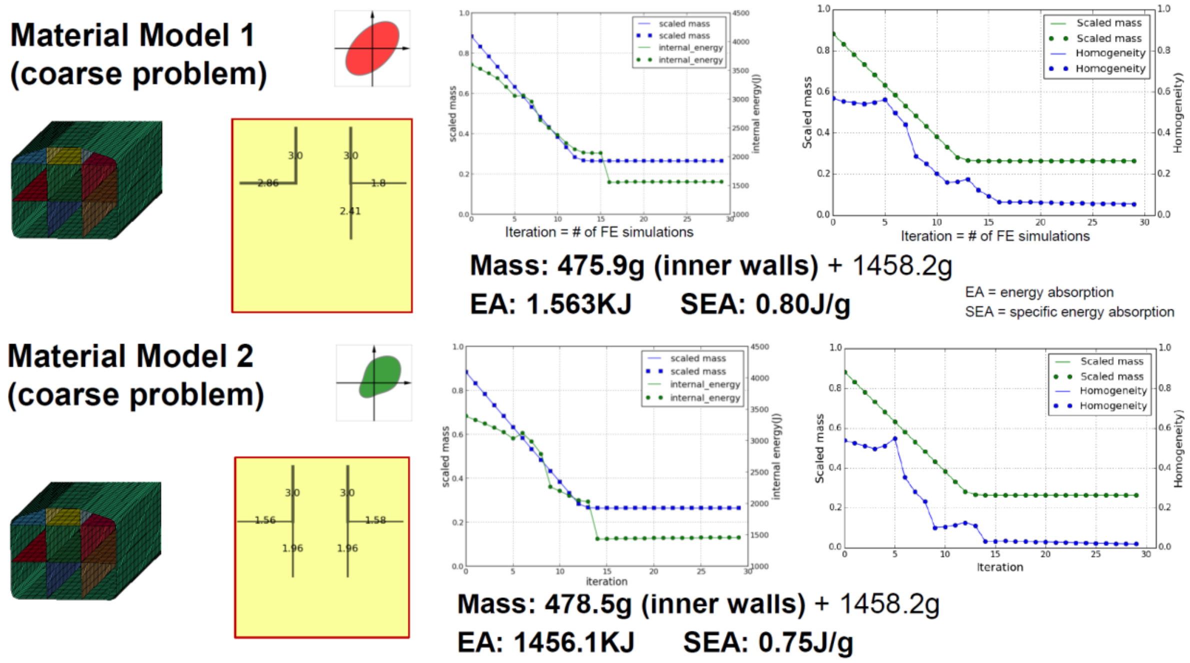


Figure 8: *HCA-TWS results for the coarse problem with two different material models [31]*

When a more refined inner ground structure is chosen, the results depicted in Fig. 11 are obtained showing stronger differences in the topology and the realized specific energy absorption (SEA, energy absorption to mass ratio). Again, the optimal results are obtained with a very low number of nonlinear crash simulations.

The figures also show the degree of homogeneity, i.e. the similarity of the internal energy of neighboring walls taken as driving quantity for the HCA. This is not always appropriate, especially if the extrusion constraints are not included in the optimization definition. Then, it is questionable that all walls should have identical internal energy; the energy will be concentrated at impact location and support regions. This should be considered when HCA or HCA-TWS are used for crash topology optimization. To overcome this, a less global definition of homogeneity requirements may be helpful. This needs investigations in the future. The LSM approach does not have this issue; it is more flexible with the definition of objectives and constraints and can treat all types of problems. Main drawback is the higher numerical effort for the optimization. Several hundreds of nonlinear crash simulations are normally required and not only a very low number (often less than 50) as for HCA.

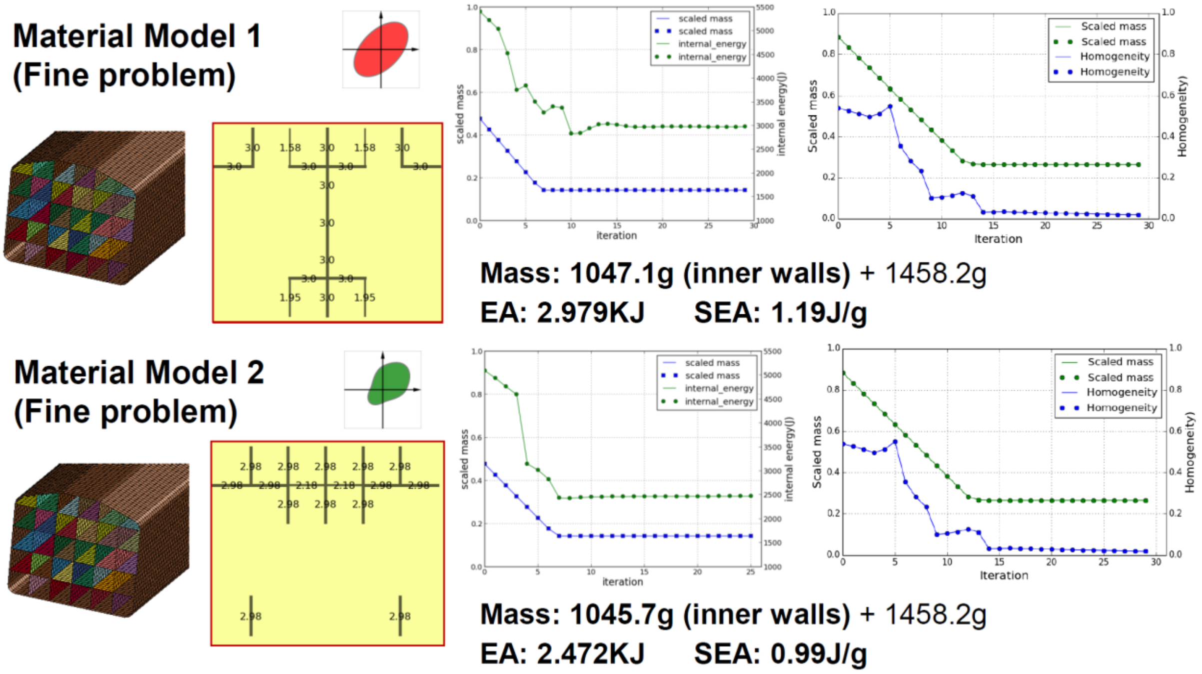
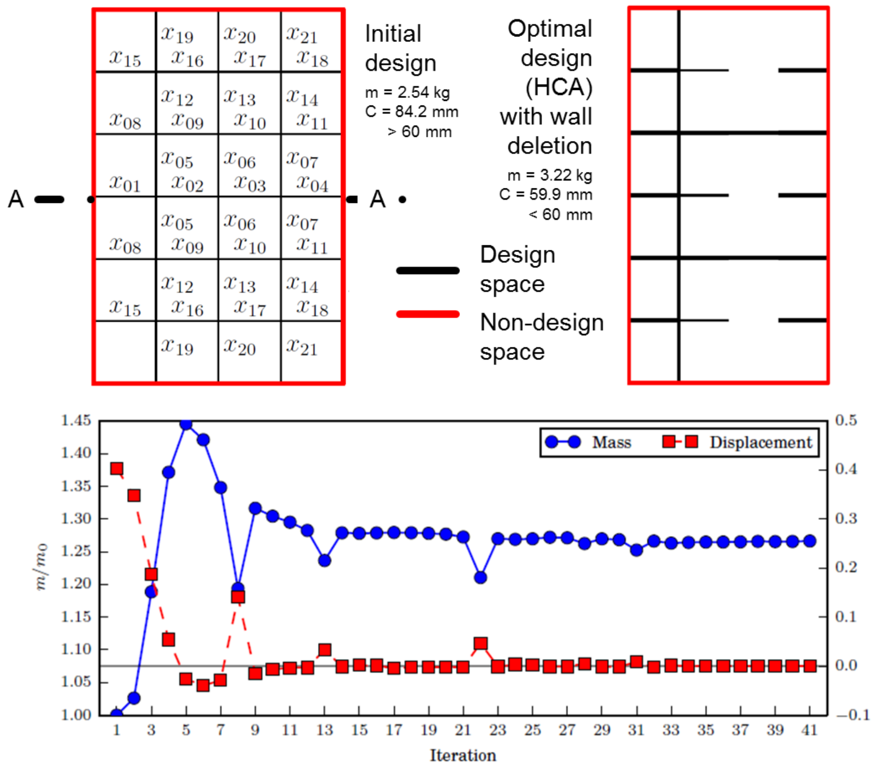


Figure 9: *HCA-TWS results for the refined problem with two different material models [31]*

For the high speed case optimized by HCA (Fig. 8), the results are comparable to those of the quasi-static case. Fig. 12 shows an exemplary optimal design obtained after 41 iterations (= nonlinear crash simulations). The mass is increased from initially 2.54 kg to 3.22 kg to drive the design into the feasible space (initial displacement constraint violation). Nevertheless, some walls are deleted with a threshold of 0.5 mm.



Design variable	Initial value	Optimal value
x01	0.6 mm	1.80 mm
x02	0.6 mm	1.13 mm
x03	0.6 mm	-
x04	0.6 mm	1.80 mm
x05	0.6 mm	1.80 mm
x06	0.6 mm	-
x07	0.6 mm	-
x08	0.6 mm	1.80 mm
x09	0.6 mm	1.78 mm
x10	0.6 mm	1.66 mm
x11	0.6 mm	1.80 mm
x12	0.6 mm	1.78 mm
x13	0.6 mm	-
x14	0.6 mm	-
x15	0.6 mm	1.80 mm
x16	0.6 mm	0.87 mm
x17	0.6 mm	-
x18	0.6 mm	1.80 mm
x19	0.6 mm	1.76 mm
x20	0.6 mm	-
x21	0.6 mm	-

Figure 10: *Results for the HCA-TWS (high speed impact) with optimal design after 23 simulations*

4.2 Results from EA-LSM for Crash Topology Optimization

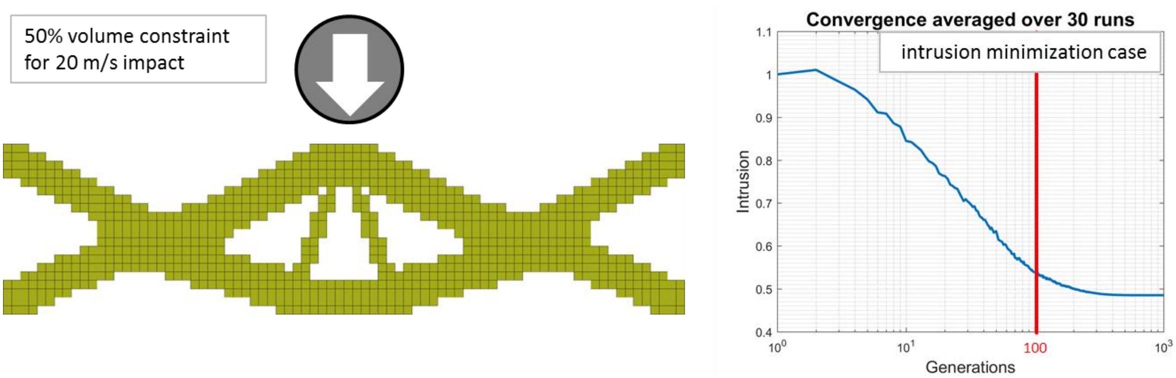


Figure 11: *Result of EA-LSM optimization, FE geometry and von Mises stress (left) and convergence plot (intrusion minimization case with mass and symmetry constraints) [25]*

The LSM results are obtained using state-of-the-art Covariance Matrix Adaptation Strategy (CMA-ES) with a stopping criterion after 1000 generations. An offspring population of the size of 17 was generated from 8 parents. A comma strategy was used, i.e. selection was performed among offspring only, neglecting parents to handle noise in crash computations. The convergence plot (here averaged over 30 repeated optimizations to enable comparison of random-based algorithms) shows the main improvements are realized before the 100th generation. In practical applications, the convergence is not essential and the quality of an algorithm is determined by fast improvements at the beginning of the optimization, see also [29]. Accepting the termination after 100 generations means that 1,700 simulations are required here. This is high compared to the HCA, but the definition of objectives and constraints is more flexible and appropriate for the LSM, which may therefore in some cases be recommended.

5 Summary

In this paper, two new methods for crash topology optimization were presented, the Hybrid Cellular Automata approach for thin-walled structures (HCA-TWS) and the level set method using evolutionary algorithms (EA-LSM). Both have the advantage that they use nonlinear and dynamic crash simulations such that all aspects of explicit FEM modeling for crash can be included. First results show that the optimal topologies depend on the material model (here shown for plasticity). This means that it cannot be recommended to use methods based on linear FEM for optimizations of structures developed for energy absorption. The latter may be used for structures with low deformation requirements and not energy absorption objectives as used for the safety cell of vehicles. The HCA-TWS has the advantage to deliver results after a very low number of simulations, but it is restricted by the choice of constraint(s) and especially by the definition of the objective functions. More general, but more computationally expensive is the EA-LSM, which can treat all types or crash problems.

Future work is necessary here to improve the methods, to explore further applications, and to study optimal topologies and their dependencies on more realistic and therefore complex material models. This is currently investigated by the authors and will be published in the future.

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