

Recent developments for frequency domain analysis in LS-DYNA[®]

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1 Introduction

Since Is971 R6 version, a series of frequency domain features have been implemented to LS-DYNA. They include FRF (Frequency Response Functions), SSD (Steady State Dynamics), random vibration, acoustics (by finite element method and boundary element method), response spectrum analysis and fatigue analysis. Those features were developed to meet the need from users in various industry, e.g. NVH analysis of automobiles, durability analysis of machines and metal parts. They extend the application of LS-DYNA to the new area of frequency domain analysis.

Some new developments and updates have been made for the frequency domain features, since the last European LS-DYNA Users' Conference in 2015. The main developments and updates include

- ERP (equivalent radiated power) computation based on SSD
- Direct SSD solver
- Torque and rotational dof excitation for FRF and SSD
- New database for acoustic simulation
- Fatigue analysis with mean stress correction

This paper provides a brief review of these updates and new developments. Some examples are included to demonstrate these updated features or capabilities.

2 ERP computation based on SSD

2.1 Background

In auto NVH analysis, computing radiated noise from vehicles is an important task. The traditional numerical methods, like FEM and BEM, are generally slow and memory intensive. As an alternative, ERP (equivalent radiated power) method has been well established and widely used. It is a reduced calculation method to characterize the structure borne noise, especially in an early phase of the product development. With ERP, engineers can have a quick look at the possible maximum radiated power for specific excitations in the frequency domain. ERP can also provide the information about panel contribution towards radiated noise so that engineers can get some suggestions when they need to do structural optimization towards noise.

2.2 Brief theory

In LS-DYNA, ERP calculation has been implemented as an option for the feature Steady State Dynamic analysis (keyword ***FREQUENCY_DOMAIN_SSD**). For the results, they are given in 1) binary plot database, D3ERP; and 2) ASCII xyplot files, ERP_abs and ERP_dB. They are all accessible to LS-PrePost.

The ERP calculation is based on plane wave assumption for the radiated acoustic waves. First we calculate the ERP density, which is defined as

$$ERP_{\rho} = \frac{1}{2} \rho c \operatorname{Re}[v_n \cdot \bar{v}_n] \quad (1)$$

Where ρ is the density of acoustic medium, c is the sound speed and v_n is the normal velocity on the surface of the structure.

The ERP absolute value radiated from the vibrating panels is the integral of the ERP density over the whole surface and is given by

$$ERP_{abs} = \int_S ERP_{\rho} dS = \frac{1}{2} \rho c \int_S \operatorname{Re}[v_n \cdot \bar{v}_n] dS \quad (2)$$

It can be converted to a dB value if a reference value ERP_{ref} is provided.

$$ERP_{dB} = 10 \log_{10} (ERP_{abs} / ERP_{ref}) \quad (3)$$

2.3 Keyword

We use the keyword ***FREQUENCY_DOMAIN_SSD_ERP** to activate ERP calculation. A typical keyword input looks like this:

```

$#   mdmin   mdmax   fnmin   fnmax   restmd   restdp   lcflag   relatv
      1       100       0.       2000.
$#   dampf   lcdam   lctyp   dmpmas   dmpstf   dmpflg
      0.01
$#                               memory   nerp     strtyp   nout     notyp   nova
                                       1
$#   r0      c      erprlf   erpref
      1.21   340.0   1.       5.e-13
$#   pid     ptyp
      1
$#   nid     ntyp     dof     vad     lc1     lc2     lc3     vid
      131    0       3       0       100    200

```

nerp defines the number of ERP panels. With pid and ptyp each of the ERP panels can be defined. The ERP panel can be defined as part ID, part set ID, or segment set ID, depending on the value of ptyp. For more details about the keyword, please refer to LS-DYNA Keyword User's Manual [1].

2.4 Post-processing

The ERP calculation results are saved in binary plot database D3ERP, which is activated by the keyword ***DATABASE_FREQUENCY_BINARY_D3ERP**. D3ERP is accessible to LS-PrePost and the following results are included in the database for each excitation frequency:

- Normal velocity on the surface (real part, imaginary part and magnitude)
- acoustic intensity
- ERP density

The frequency variation of the ERP absolute value and ERP dB value are saved in the ASCII files: ERP_abs and ERP_dB. They can be plotted as xy curves with LS-PrePost.

2.5 Example

In this example we consider a simplified engine model, which has 16041 nodes and 13484 solid elements. The whole surface of the engine is defined as sound radiating panel, by a set of segments (***SET_SEGMENT**). To model the excitation from the ground, we assume that the model is fixed on a shaker table and a constant horizontal acceleration 0.02g is provided for the range of frequency 10-1000 Hz.

The ERP density results at frequency 100 Hz are shown in Fig. 1. It helps to visualize the contribution towards radiated noise from the whole surface, which may vary with frequencies.

As mentioned before, the ERP computation can help to estimate the radiated noise from a structure without going through the time consuming FEM or BEM computation. It (with D3ERP) can also suggest the hot spot area which may need special attention of designers to optimize the acoustic behavior of the structures.

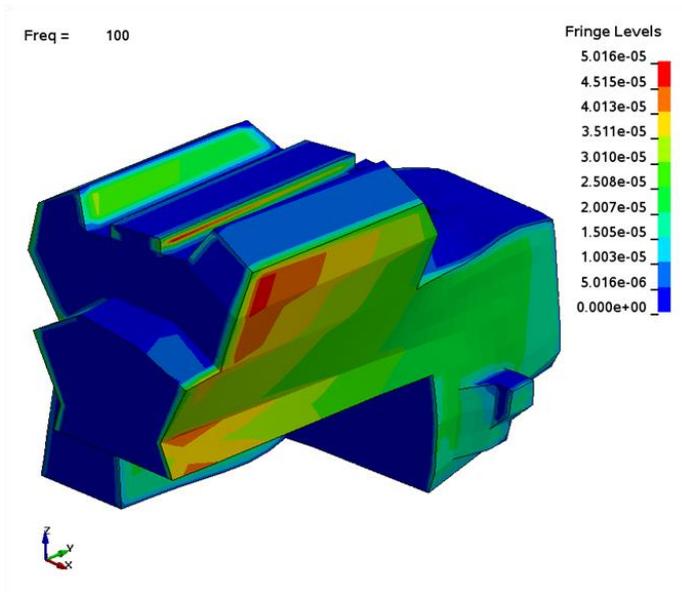


Fig.1: ERP density plot at frequency 100 Hz

3 A direct SSD solver

3.1 Brief theory

The current SSD (Keyword `*FREQUENCY_DOMAIN_SSD`) is based on modal superposition. One has to run modal analysis first (keyword `*CONTROL_IMPLICIT_EIGENVALUE`) in order to run SSD. An option to run SSD in a direct way was implemented recently. It is based on the global stiffness, mass and damping matrices of the system and no modal analysis is needed. To run the direct SSD solver, we use the keyword `*FREQUENCY_DOMAIN_SSD_DIRECT`.

The governing equation system for structural dynamics is

$$Ma(\omega) + Cv(\omega) + Ku(\omega) = F(\omega) \quad (4)$$

Where, M , C , K , and F are mass, damping and stiffness matrices and loading vectors. They can all be dependent on excitation frequencies. a , v and u are nodal acceleration, velocity and displacement vectors.

Considering the following equations for harmonic vibration:

$$\begin{aligned} a &= -\omega^2 u \\ v &= i\omega u \end{aligned} \quad (5)$$

Equation (4) can be rewritten as

$$\left[-\omega^2 M + i\omega C + K \right] u(\omega) = F(\omega) \quad (6)$$

Equation (6) can be solved by a direct solver or an iterative solver like GMRES.

The results of the direct SSD analysis are still written to the binary database "D3SSD", and ASCII files "NODOUT_SSD" and "ELOUT_SSD". "D3SSD" shows the fringe plot of the SSD results; "NODOUT_SSD" and "ELOUT_SSD" save the nodal and element results for selected nodes and elements. They are saved in binout file and can be extracted using LS-PrePost 4.2 and above versions.

Comparing the direct SSD and the mode-based SSD, the main difference is the unknown variables. For direct SSD, the physical nodal displacements u are used as the primary variables. As the consequence it may result in a large system of equations. For the mode-based SSD, the primary variables are the model coefficients (or generalized coordinates). So the number of unknown variables is equal to the number of eigenmodes adopted in the simulation and it is usually moderate

(considering that for many structures, hundreds of eigenmodes can be sufficient for most applications). Besides, with mode-based SSD, the system of equations is decoupled using the orthogonality property of the eigenvectors. So, the mode-based SSD can be cheaper than direct SSD, if the system has a large number of dof.

3.2 Example

A rim model is considered in this example. It is constrained along the edge of the hole in the center. A y-directional base acceleration is applied to the model for the frequency range 1-2001 Hz. Dynamic response of the rim is obtained using direct SSD and mode-based SSD. Fig. 2 and Fig. 3 show y-directional velocity and acceleration response at 2001 Hz and 1971 Hz, which are provided by D3SSD.

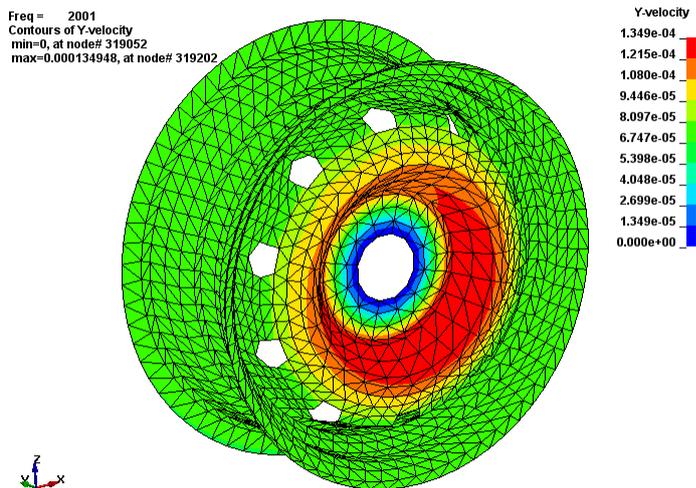


Fig.2: Y-directional velocity fringe plot at frequency 2001 Hz

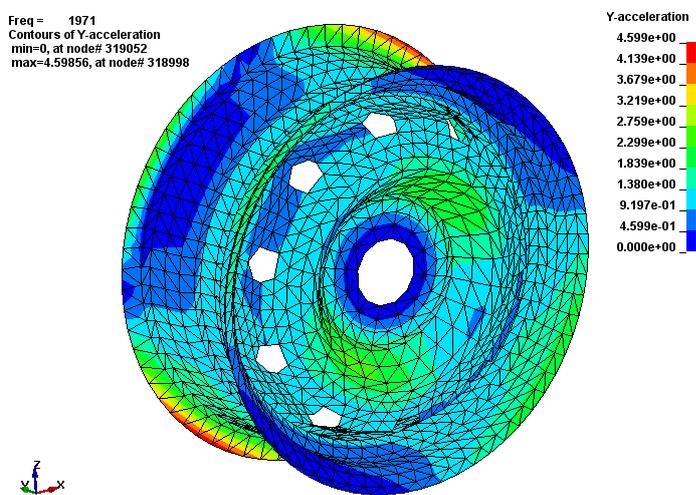


Fig.3: Y-directional acceleration fringe plot at frequency 1971 Hz.

Fig. 4 compares the y-directional acceleration at one node given by direct SSD and mode-based SSD. A reasonable match can be found for the results by the two methods.

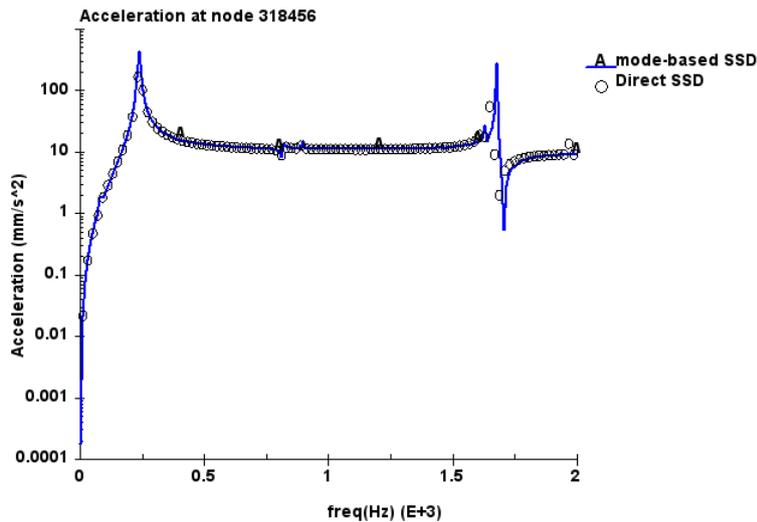


Fig.4: Y-directional acceleration given by mode-based SSD and direct SSD

One has to note that the direct SSD is usually slower than the mode-based SSD, as the number of unknowns is larger. The advantage of the direct SSD is that one can easily incorporate the frequency-dependent properties like stiffness and damping in the system, which would be very hard when the mode-based SSD is used.

4 Torque and rotational dof excitation for FRF and SSD

4.1 Keyword

In FRF and SSD, torque and rotational dof excitation have been implemented. They are used to model the moment input and specified rotational dof excitation, which can be found in many industry applications. They are defined by variables vad1 and vad for FRF and SSD, as shown below.

```
*FREQUENCY_DOMAIN_FRF (CARD 1)
$#      n1      n1typ      dof1      vad1      vid1      fnmax      mdmin      mdmax
      875001      0          3          8

*FREQUENCY_DOMAIN_SSD (CARD 4)
$#      nid      ntyp      dof      vad      lc1      lc2      lc3      vid
      875001      0          3          8
```

The new excitation types can be defined by VAD (or VAD1) as follows.

<u>VARIABLE</u>	<u>DESCRIPTION</u>
VAD	Excitation input type
...	...
EQ. 8:	torque
EQ. 9:	base angular velocity
EQ.10:	base angular acceleration
EQ.11:	base angular displacement

4.2 Example

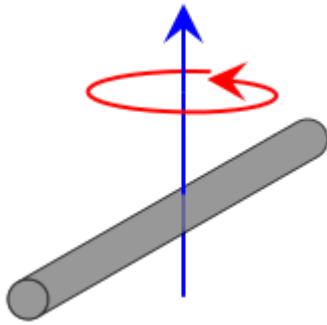


Fig.5: A torque excitation

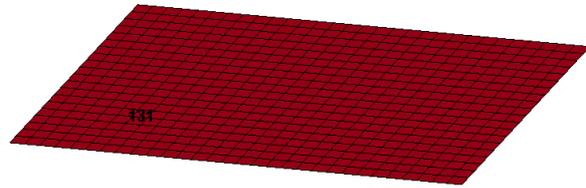


Fig.6: A rectangular plate with torque input (at node 131)

A rectangular plate (Fig. 6) is subjected to torque excitation shown in Fig. 5. The excitation is applied at node 131 on the plate shown in Fig. 6.

The FRF response (z-directional displacement due to unit torque loading) for the model given by LS-DYNA is compared with the results given by Nastran in Fig. 7. One can see that the two sets of results match well.

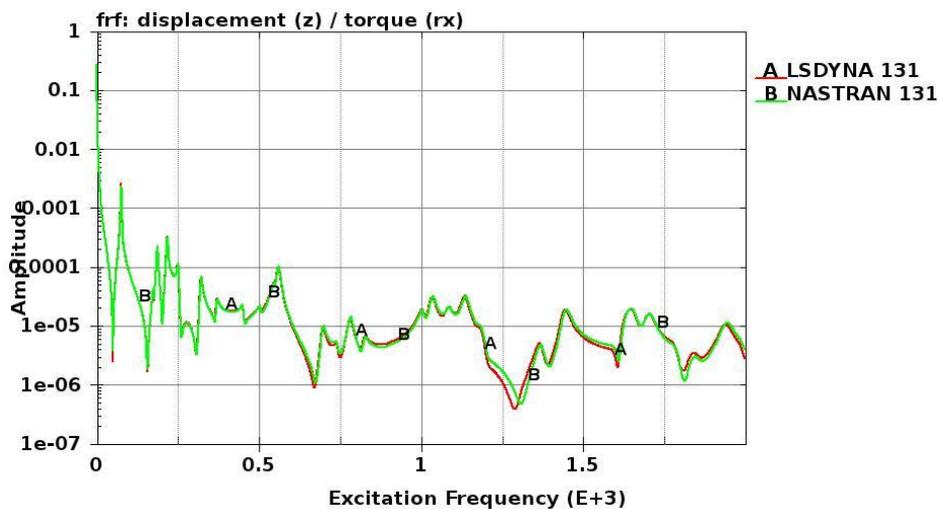


Fig.7: FRF results by LS-DYNA and Nastran

5 New database for acoustic simulation

5.1 Introduction

In BEM Acoustic computation (keyword `*FREQUENCY_DOMAIN_ACOUSTIC_BEM`), a new binary database D3ACC is added to show the element contribution to the acoustic pressure at field points, as well as the contribution percentage of the elements. To activate the output of this database, the keyword `*DATABASE_FREQUENCY_BINARY_D3ACC` is needed. Here, "ACC" in "D3ACC" means ACoustic Contribution.

The idea of this contribution analysis is that according to equation (7), the acoustic pressure p at a field point P is given as a combination of the pressure p_j contributed by each of the boundary elements.

$$p(P) = \sum_{j=1}^N \int_{\Gamma_j} \left(G \frac{\partial p}{\partial n} - p \frac{\partial G}{\partial n} \right) d\Gamma_j = \sum_{j=1}^N p_j(P) \quad (7)$$

In equation (7), G is the acoustic fundamental solution in free space and n is the normal direction of the boundary elements.

To compute the contribution percentage of an element to the acoustic pressure at a field point, vector projection like the one shown in Fig. 8 is used. This is because the acoustic pressure, in frequency domain, is given as complex variables.

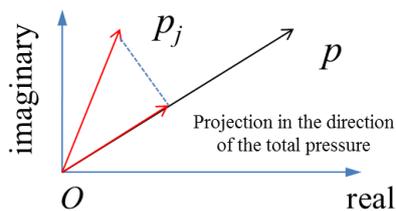


Fig.8: Computation of contribution percentage using vector projection

5.2 Example

A simplified compartment model is adopted to illustrate the D3ACC database. For simplicity, the compartment is assumed to be subjected to uniform normal velocity 10 mm/s for the whole range of frequency.

Fig. 9-10, show the real and imaginary parts of the element acoustic pressure contribution towards field point 100002 at frequency 200 Hz. Please note, the element acoustic pressure contribution (and contribution percentage) not only vary with frequency, but also vary with field points.

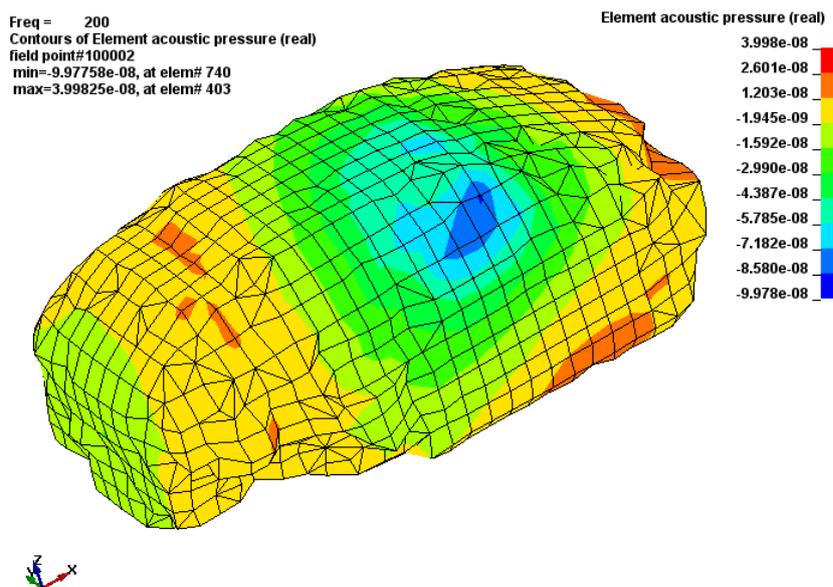


Fig.9: Real part of element acoustic pressure contribution for field point 100002 at 200 Hz.

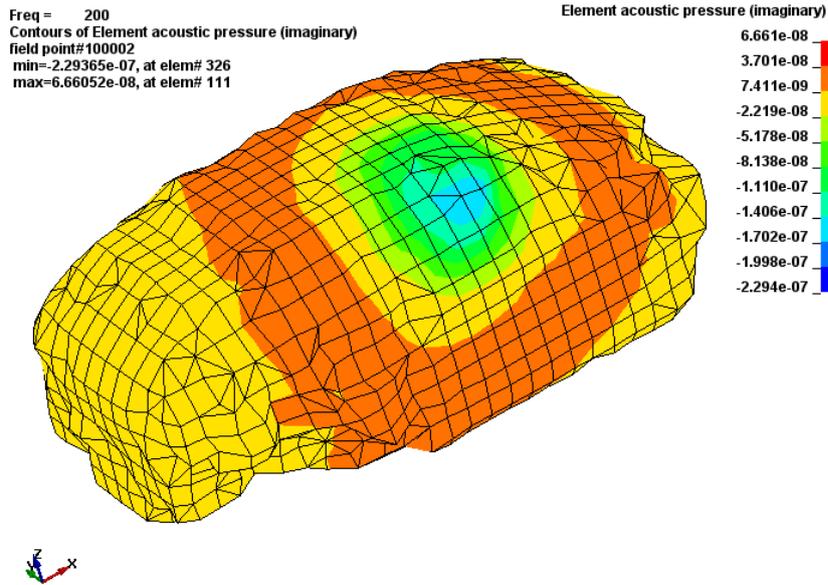


Fig.10: Imaginary part of element acoustic pressure contribution for field point 100002 at 200 Hz.

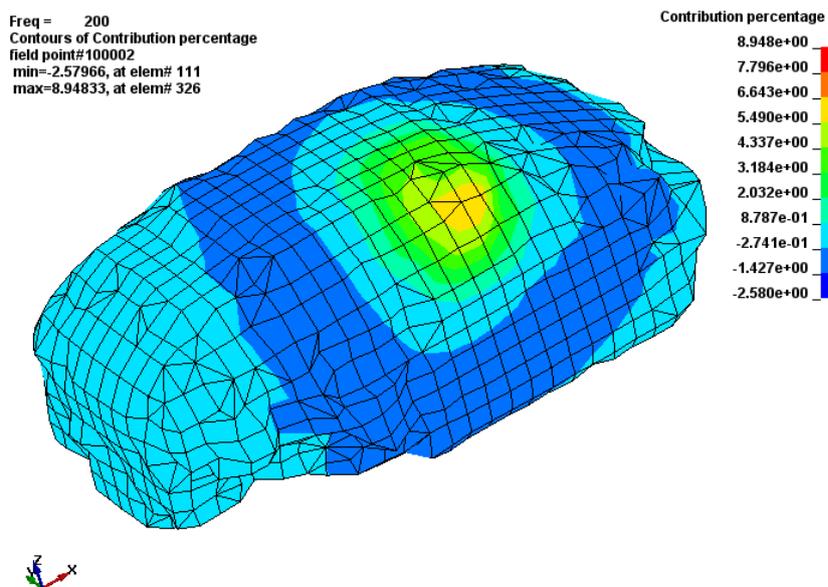


Fig.11: Element contribution percentage for acoustic pressure at field point 100002 at 200 Hz.

6 Fatigue analysis with mean stress correction

6.1 Introduction and brief theory

A series of frequency domain fatigue analysis methods have been implemented to LS-DYNA, including random vibration fatigue (`*FREQUENCY_DOMAIN_RANDOM_VIBRATION_FATIGUE`), and harmonic vibration fatigue (`*FREQUENCY_DOMAIN_SSD_FATIGUE`) (e.g. sine sweep). The fatigue analysis is based on material's SN fatigue curve.

$$N S^m = a \tag{8}$$

Where, S is the alternating stress range, N is the number of cycles for failure under stress S . m and a are material parameters depending on the type of material.

The SN fatigue curve testing is usually performed under 0 mean stress condition (fully reversed testing) in the lab. However in practice, many metal structures are subjected to vibration with nonzero mean stress due to residual stress, or pre-stress due to thermal loading or other mechanical loading. The existence of mean stress changes the SN curve of materials, as illustrated by Fig. 12.

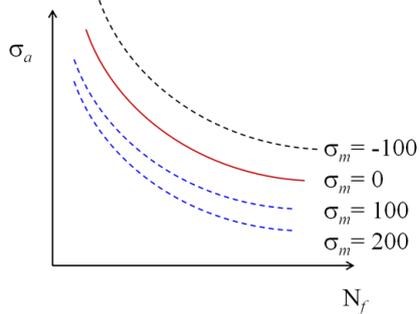


Fig. 12: SN fatigue curve under different mean stress

As shown in Fig. 12, a positive mean stress (tension) makes the material weaker and a negative mean stress (compression) makes the material stronger (more resistant to the fatigue failure under same alternating stress level).

A lot of empirical equations have been introduced to correct the SN curve with the consideration of mean stress [2].

The Goodman's equation:

$$S = \frac{S_a}{1 - \sigma_m / \sigma_u} \quad (9)$$

The Soderberg equation:

$$S = \frac{S_a}{1 - \sigma_m / \sigma_y} \quad (10)$$

Gerber equation

$$S = \frac{S_a}{1 - (\sigma_m / \sigma_u)^2} \quad (11)$$

Here, S_a is the alternating stress amplitude, σ_a is the mean stress, σ_y is the yield stress and σ_u is the ultimate tensile strength.

In LS-DYNA fatigue analysis, the mean stress is introduced by keywords `*INITIAL_STRESS_SHELL`, `*INITIAL_STRESS_SOLID`, ...

The mean stress correction method is set by keyword `*FATIGUE_MEAN_STRESS_CORRECTION`.

6.2 Example

The figures 14-15 show the cumulative damage ratio plot for a metal bracket model, with the consideration of mean stress correction using Goodman's equation (equation 9) and Soderberg's equation (equation 10).

The mean stress state of the structure is shown in Fig. 13.

The model is subjected to random vibration under PSD base acceleration. Dirlik method is used for the fatigue analysis [3].

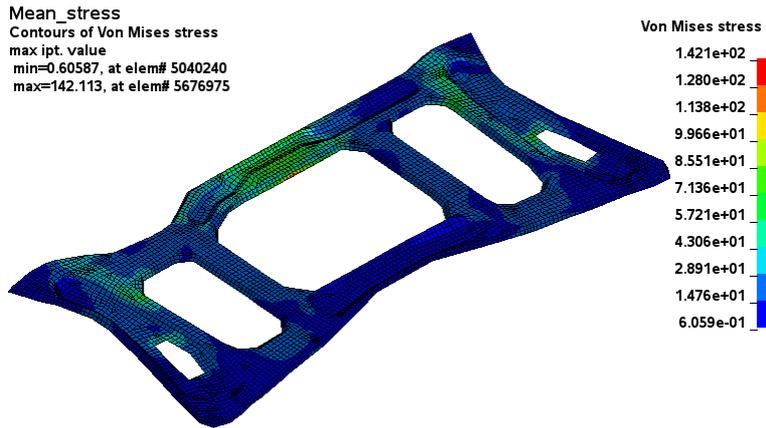


Fig.13: Von mises stress state of the mean stress

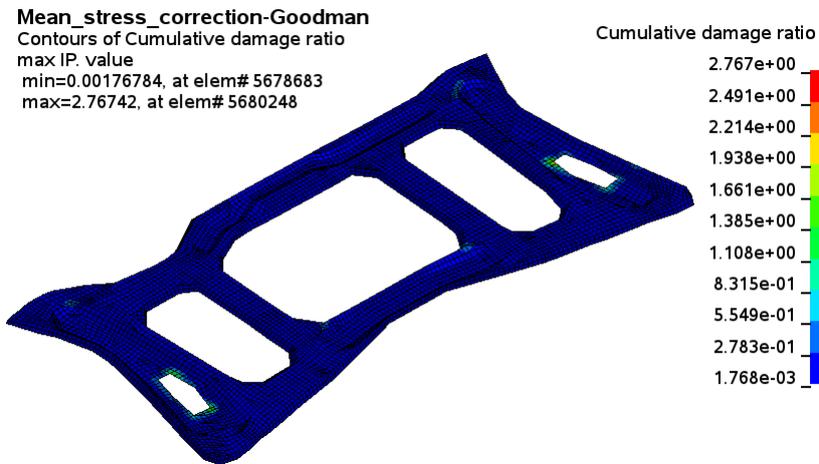


Fig.14: Cumulative damage ratio of the bracket (use Goodman's equation for mean stress correction)

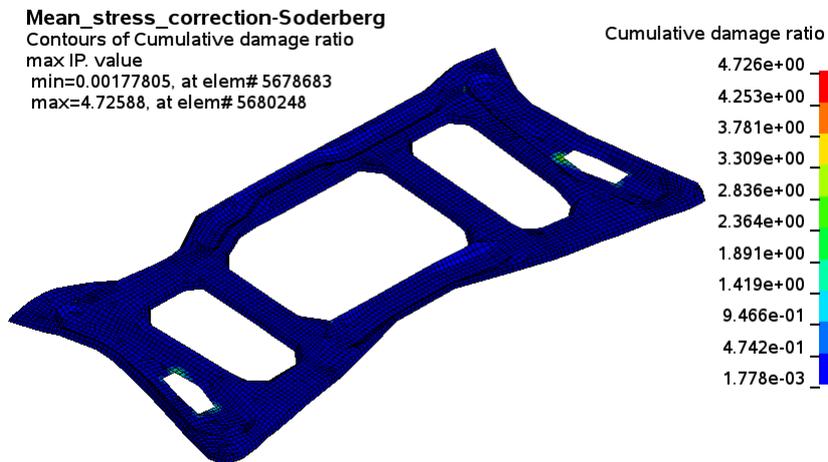


Fig.15: Cumulative damage ratio of the bracket (use Soderberg's equation for mean stress correction)

The following table shows the maximum cumulative damage ratios with and without mean stress correction.

Mean stress correction method	Maximum damage ratio
No mean stress correction	2.480
Goodman's equation	2.767

Soderberg's equation	4.726
Gerber's equation	2.510

Table 1: Maximum damage ratio with and without mean stress correction

One can see, with the mean stress correction, the damage ratio of the structure can be bigger, which means the fatigue life of the structure becomes shorter due to the existence of the mean stress, under the same vibration condition.

7 Summary

The paper gives a brief review of the recent updates and development for the frequency domain features of LS-DYNA.

Some examples are given for ERP (equivalent radiated power) computation, direct SSD (Steady State Dynamics), FRF and SSD computation with torque and rotational degree-of-freedom excitation, new database in acoustic computation, and fatigue analysis with mean stress correction.

Some of the developments were performed to answer the request from users. The authors are grateful to the users for their comments and feedback.

8 Literature

- [1] Livermore Software Technology Corporation: "LS-DYNA Keyword User's Manual", 2016.
- [2] Bader Q., Kadum E.: "Mean Stress Correction Effects On the Fatigue Life Behavior of Steel Alloys by Using Stress Life Approach Theories", International Journal of Engineering & Technology, Vol :14, No:4, 2014, pages 50-58.
- [3] Ringeval, A., and Huang, Y., "Random Vibration Fatigue Analysis with LS-DYNA", 12th International LS-DYNA Users Conference, Dearborn, Michigan, 2012.