A New Eigensolver for High Performance NVH Analysis: MCMS (Multi-level Component Mode Synthesis)

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1 Introduction

The Noise, Vibration, and Harshness (NVH) problem is one of the most important targets for comfort and quiet product design. Especially, in automobile industry, as the degrees of freedom of NVH automobile FE model increases, from millions of degrees of freedom to tens of millions of degrees freedom, the computational time for NVH analysis becomes serious bottle neck in automobile design and analysis process. Therefore, it is desired to reduce the computational time of eigenvalue problem analysis, which is the most important part of NVH analysis.

For several years LSDYNA has had a Block Shift and Invert Lanczos Eigensolver in both SMP and MPP implementations. But this solver is deemed too expensive for computing the number of modes required for NVH applications. In the early 2000s, the Automated Multi-Level Substructuring (AMLS) method was developed to reduce the computational costs of NVH analysis by the group at The University of Texas at Austin. As the AMLS method provides significant contributions on automobile NVH analysis, it becomes industry standard tool for NVH community. Therefore, many CAE software companies such as NASTRAN, ABAQUS, HyperWorks released similar feature which uses AMLS algorithm. At LSTC, for efficient NVH analysis, we have been adding MCMS (Multi-level Component Mode Synthesis) method as an implementation of AMLS (Automated Multilevel Substructuring Method) for NVH applications that require thousands of eigenmodes quickly.

MCMS generally produces approximate eigensolutions that are less accurate than those computed by Lanczos method, but the error can be tolerated in automobile NVH applications. The new MCMS method that significantly reduces the computation time for frequency response analyses on complex structures such as automobiles, submarines and airplanes.

2 Theoretical Background

In vibration analysis, the finite element (FE) method solves equations of the form

$$[\mathrm{M}]\{\ddot{u}\}+[\mathrm{C}]\{\dot{u}\}+[\mathrm{K}]\{u\}=[F] \tag{1}$$

where [K], [C], and [M] are mass, damping, and stiffness matrices, respectively. From eq.(1), the The eigenvalue problem of the FE model is represented by

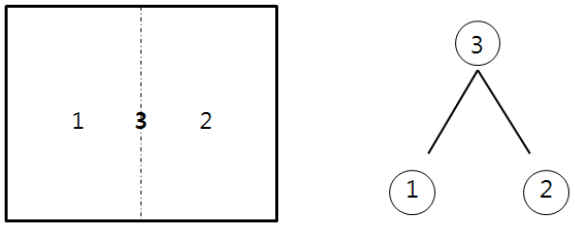
$$[K][\Phi]=[M][\Phi][\Lambda] \tag{2}$$

The matrix $[\Lambda]$ is the diagonal matrix of eigenvalues, $[\Phi]$ contains the corresponding eigenvectors and $[\Phi]^T[M][\Phi]=[I]$, where $[I]$ is an identity matrix. For many years LSDYNA has used the Block Shift and Invert Lanczos software, both in SMP and MPP, to solve the standard vibration analysis problem (2). Lanczos assumes that K and M is symmetric positive semi-definite.

In the late 1960s, Component Mode Synthesis (CMS) was introduced for reducing the size of a finite element model, particularly where many subsystems are connected. This method re-characterizes large finite element models into a set of relatively small matrices containing mass, stiffness and mode shape information that capture the fundamental low frequency modes of the structure. The Craig-Bampton method, which has been most

popularly used, combines the motion of boundary points with modes of the subsystem assuming the boundary points are held fixed.

In the Craig-Bampton method, substructure 1 consists of interior and boundary degrees of freedom. At the next level, substructure 3 consists of the interface degrees of freedom between substructure 1 and substructure 2. The partitioning can be described in a tree topology shown in Fig.1.



(a) (b)

Fig.1: A plate FE model partitioned with Craig-Bampton method: a single level tree topology

Based on the partitioning, the stiffness and mass matrices of each substructure is reordered as

$$K = \begin{bmatrix} K_{ii} & K_{ib} \\ K_{ib}^T & K_{bb} \end{bmatrix} = \begin{bmatrix} K_{ii} & K_{ib} \\ K_{bi} & K_{bb} \end{bmatrix} \tag{3}$$

$$M = \begin{bmatrix} M_{ii} & M_{ib} \\ M_{ib}^T & M_{bb} \end{bmatrix} = \begin{bmatrix} M_{ii} & M_{ib} \\ M_{bi} & M_{bb} \end{bmatrix} \tag{4}$$

The substructure response on the lowest level is represented by the Craig-Bampton method as

$$u = \begin{Bmatrix} u_i \\ u_b \end{Bmatrix} = \begin{bmatrix} \Phi_i & \Psi_i \\ 0 & I \end{bmatrix} \begin{Bmatrix} \eta \\ u_b \end{Bmatrix} = [T] \begin{Bmatrix} \eta \\ u_b \end{Bmatrix} \tag{5}$$

where Φ_i is the eigenvector matrix of the following algebraic eigenvalue problem and $\Psi_i = -K_{ii}^{-1} K_{ib}$.

$$K_{ii} \Phi_i = M_{ii} \Phi_i \Lambda_i \tag{6}$$

Note that the eigenpairs are truncated according to user defined cutoff value.

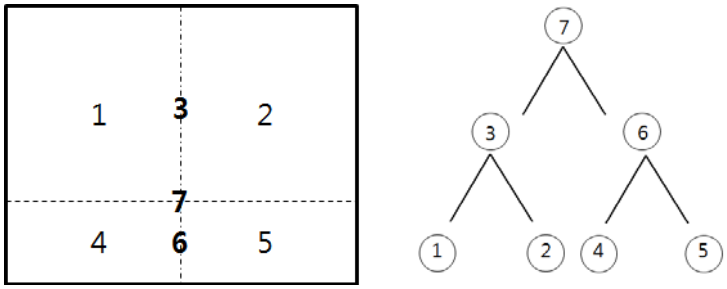
Once the transformation matrix $[T]$ is obtained, the substructure stiffness and mass matrices are transformed to the reduced matrices $[K_C]$ and $[M_C]$.

$$[K_C] = [T]^T \begin{bmatrix} K_{ii} & K_{ib} \\ K_{ib}^T & K_{bb} \end{bmatrix} [T] = \begin{bmatrix} \Lambda_i & 0 \\ 0 & K_{ib}^T \Psi_i + K_{bb} \end{bmatrix} \tag{7}$$

$$[M_C] = [T]^T \begin{bmatrix} M_{ii} & M_{ib} \\ M_{ib}^T & M_{bb} \end{bmatrix} [T] = \begin{bmatrix} I & \Phi_i^T (M_{ii} \Psi_i + M_{ib}) \\ 0 & \Psi_i^T (M_{ii} \Psi_i + M_{ib}) + M_{ib}^T \Psi_i + M_{bb} \end{bmatrix} \tag{8}$$

The key idea of Automatic Multi-Level Substructing (AMLS) method is a multi-level extension of Craig-Bampton method. It reduces the full scale model using a recursive application of the Craig-Bampton approach to a smaller model. The recursion is based on the elimination tree for the sparse factorization of the stiffness matrix. Since the eigensolution is computed for each smaller substructure model, it requires far less computing resources than Lanczos on the full scale model.

In the MCMS method, based on the transformation process for a single level explained above, multi-level transformation is processed as follows. A FE model of plate shown in Fig.2 illustrates a partitioning of the plate into substructures in a two level. substructure 1 consists of interior and boundary degrees of freedom. At the next level, substructure 3 consists of the interface degrees of freedom between substructure 1 and substructure 2. The substructure 7 consists of the interface degrees of freedom that separate substructure 1, 2, 3 and substructure 4, 5, 6. The partitioning can be shown in a tree topology in Fig.2.



(a) (b)

Fig.2: A plate FE model partitioned with two level and the substructure tree topology

The partitioning of the system matrices, K and M, becomes

$$[K]=\begin{bmatrix} K_{1,1} & 0 & K_{1,3} & 0 & 0 & 0 & K_{1,7} \\ & K_{2,2} & K_{2,3} & 0 & 0 & 0 & K_{2,7} \\ & & K_{3,3} & 0 & 0 & 0 & K_{3,7} \\ & & & K_{4,4} & 0 & K_{4,6} & K_{4,7} \\ & & & & K_{5,5} & K_{5,6} & K_{5,7} \\ & \textit{symm.} & & & & K_{6,6} & K_{6,7} \\ & & & & & & K_{7,7} \end{bmatrix} \tag{9}$$

$$[M]=\begin{bmatrix} M_{1,1} & 0 & M_{1,3} & 0 & 0 & 0 & M_{1,7} \\ & M_{2,2} & M_{2,3} & 0 & 0 & 0 & M_{2,7} \\ & & M_{3,3} & 0 & 0 & 0 & M_{3,7} \\ & & & M_{4,4} & 0 & M_{4,6} & M_{4,7} \\ & & & & M_{5,5} & M_{5,6} & M_{5,7} \\ & \textit{symm.} & & & & M_{6,6} & M_{6,7} \\ & & & & & & M_{7,7} \end{bmatrix} \tag{10}$$

For example, transformation matrix in Eq.(11) represents the transformation of the first substructure 1 and its ancestor substructure 3, 7.

$$[T_1]=\begin{bmatrix} \Phi_1 & 0 & \Psi_{1,3} & 0 & 0 & 0 & \Psi_{1,7} \\ 0 & I_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_7 \end{bmatrix} \tag{11}$$

Once all transformations for all substructures are finished, the transformed and reduced mass and stiffness matrices $[K_M]$ and $[M_M]$ can be represented as

$$[K_M] = [T]^T [K][T]$$
$$[M_M] = [T]^T [M][T] \tag{12}$$

where $[T] = [T_1][T_2]\cdots[T_7]$ is transformation matrix that contains substructure eigenvectors and the number of column is the total number of substructure eigenvectors. Once the MCMS transformation is finished, the global eigenvalue problem (2) can be transformed as

$$[K_M][\Phi_M] = [M_M][\Phi_M][\Lambda_M] \tag{13}$$

where $[\Lambda_M]$ is the diagonal matrix that contains eigenvalues for the reduced eigenvalue problem (13) and $[\Phi_M]$ is the matrix of its eigenvectors. The dimension of the reduced eigenvalue problem (13) is typically on the order of one hundred thousand, when the required eigenvector is over ten thousand for large scale FE model which has millions of degrees of freedom.

Finally, the approximate global eigensolution can be obtained from

$$[\Phi] \approx [T][\Phi_M], \quad [\Lambda] \approx [\Lambda_M]$$

3 Numerical Examples

3.1 Fuel Tank FE model

This model represents a fuel tank for an automobile. It is comprised of 61.488 shell elements.

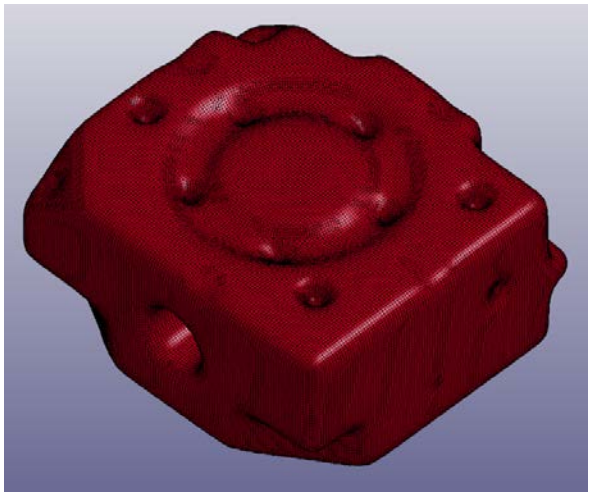


Fig.3: Fuel tank FE model

3.2 Bumper FE model

This model represents a bumper for a passenger car and has 168,860 shell elements.

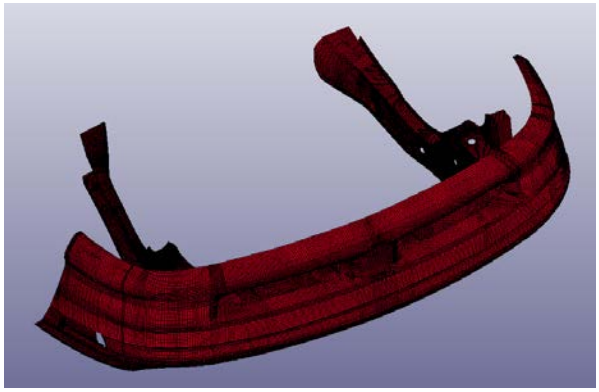


Fig.4: Passenger car bumper FE model

3.3 Door FE model

The door model represents a passenger car door using 484,588 shell elements. A second smaller version of the door model uses 121566 shell elements.

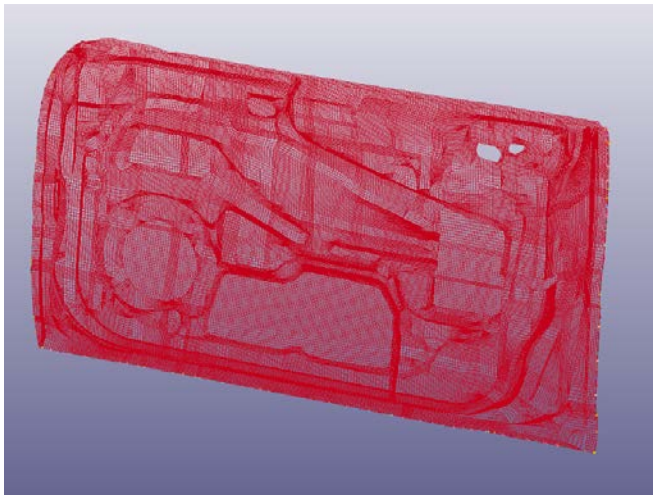


Fig.5: Passenger car door FE model

3.4 Results

For all of these test cases the new MCMS feature is computing the approximate eigenvalues correctly.

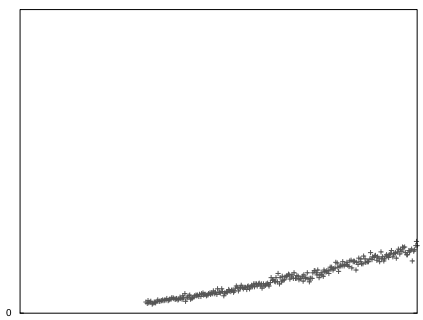


Fig.6: Relative error of Eigenvalues computed using MCMS compared to Lanczos

Figure 6 displays the relative difference for the first 400 eigenvalues computed using MCMS and Lanczos for the fuel tank model. For the larger eigenvalues the relative error is approaching 2%. The performance oft he MCMS implementation is still being enhanced in LSDYNA. We hope to show performance results comparing MCMS with Lanczos during the conference.

4 Summary

This paper provides an overview of LSC effort to implement the MCMS in LSDYNA for the NVH applications. We provide a motivation for the work and an overview of the theoretical background of the algorithm based on a recursive appllication of the Craig-Bampton Component Mode Synthesis algorithm. We briefly showed 3 test cases we are using in our development efforts. And demonstrated the results we are getting. We hope to present performance numbers comparing Lanczos and MCMS during the conference presentation.