Numerical Modeling of 'Concrete Response' to High Strain Rate Loadings

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Abstract

The dynamic characterization of concrete is fundamental to understand the material behavior in case of earthquakes and extreme dynamic events like Blast and impact. Extensive research is available on the study of quasi-static or nearly static behavior of concrete, but limited investigations/research exists on the prediction of dynamic response, especially under high strain loadings. Numerous material models are available for modeling the dynamic behavior of concrete, this research focusses mainly on numerical simulations of the quasi static and dynamic behavior of the concrete including the strain rate effects. For this research, popular material models MAT_072R3 (KCC), MAT_084 (WINFRITH), MAT_ 272 (RHT) and MAT_159 (CSCM) were implemented, that are available in the explicit dynamic software LS-DYNA. Single element tests verification subjected to varying strain rates in tension as well as in compression were the starting point of validation/comparison of different material models. The single element tests on different strain rates confirms the experimental behavior. Followed by, to study the behavior of concrete in varying strain rates, the numerical simulations of three-point loading test is carried out. The strain rate behavior of concrete for the different material models were analyzed and the suitability of the material models for the scenario is discussed. Backed up with the above numerical analysis results, unreinforced concrete slab subjected to blast loadings is studied, which usually involves high strain rates.The damage behavior was studied for all the material models. Followed by, the high strain loading in the form of penetration of the missile is studied for all the different concrete models and the material model behaviors for this loading is explained. Finally, the analysis results were summarized and concluded.

1 Introduction

Concrete is the most common building material used in critical infrastructural facilities such as Nuclear Reactors, Bridges, Dams, Protective structures etc. In recent years due to the occurrence of various unforeseen manmade activities, ensuring the safety of these structures subjected to high strain loadings such as blast, impact etc. has been one of the primary concerns of structural designers. The safety of these structures can be ensured by two means, i.e. one by the full scale model testing of the structure, and another by the method of Numerical Analysis. However full scale model testing is difficult to conduct and will cost heavy for these types of high strain loads, which leads the structural designers to rely on the responses predicted by numerical Analysis. For Reinforced Concrete structures subjected to high strain loadings such as blast or impact loads, the practitioners are looking forward for refined material models to obtain a consistent sets of numerical analysis results, enabling them to provide innovative and improved resistance to these kind of loads.

To predict the behaviors of concrete structures subjected to various types of high strain loads, the concrete material models needs to be confirmed first to simulate the behavior at the smaller material specimen levels. Even though it is very difficult to predict the exact response of concrete structures subjected to high strain rate loads, the concrete material models should be able to capture the basic properties of concrete. Advances in material modeling and FE analysis have made it easy to support engineer's requirements. Amongst the available FE software, LS DYNA is widely applied in analyzing structural responses to shock and impact loads which is having numerous concrete constitutive models. Each of these concrete constitutive models have its own advantages and disadvantages, so appropriate selection of material models suitable to the concerned application is of prime importance.

Thus the primary purpose of this paper is to examine the strain rate behaviors of four popular concrete material models such as MAT_RHT, MAT_CSCM_CONCRETE, MAT_WINFRITH_CONCRETE and MAT_72_R3, which might enable users to select appropriate concrete material model suitable for their analysis and design purposes. This is achieved by explaining each concrete material model briefly at first and then examining them with various strain rates. The prime reason behind selecting these particular concrete models for the study is the relatively simple inputs required for defining the behavior. The material models are first applied in a single element simulations to understand their capability in capturing the strain rate effects. The models are then applied in solving structural problems under various high strain loading conditions, including blast and impact loads.

2 Theoretical Background

In this section, each material model is explored in detail, and the possible applications of each material model is explained.

2.1 MAT072R3 (KCC) - MAT_72_R3

This Karagozian & Case (K&C) Concrete model is a three- invariant model. It uses three shear failure surfaces, includes damage and stain-rate effects, and has origins based on the Pseudo-TENSOR Model. The inputs needed to work with this model are simple and it also has a parameter generation capability, based solely on the unconfined compression strength of the concrete. An Equation of state is also required for the pressure- Volume strain response. This material model is intended for analyzing RC Structural responses to blast and impact loadings. It has been applied in analyzing many RC structures subjected to quasi- static, blast and impact loads [2,8,5]

2.2 Winfrith Concrete Model – MAT_084

The Winfrith concrete model (MAT084) was developed to solve RC Structures subjected to impact loadings, and was implemented to LS-DYNA in 1991. Although the input is not as simple as the KCC model, still the keywords are relatively simple. Another distinguishing feature of this material model is that it allows up to three orthogonal crack planes per element and the cracks can be visualized as fringe plot [d3crack]. This model has been used mainly to obtain responses of RC Structures subjected to impact loadings [3,4,1]

2.3 Continuous Surface Cap (CSC) Model – MAT_159

The Continuous Surface Cap Model, developed in 1990’s was aimed at roadside safety analyses was made available in LS-DYNA around 2005. Similar to the KCC Model (Mat 072_R3), the CSCM Model also supports the automatic generation of all the parameters required for the analysis. The application of this material model along with its validation can be found in some References [10, 11].

2.4 RHT Model – MAT_272

The RHT concrete model is an advanced plasticity model for brittle materials developed by Riedal et al [9, 10]. It is particularly useful for modeling the dynamic loading of concrete as well for other brittle materials such as rock and ceramics. The RHT concrete model implements a strain rate law, which uses a Dynamic Increase Factor (DIF) for tension at varying strain rates. The DIF is represented by a ratio of dynamic and static tensile strength. To predict the correct behavior of the concrete subjected to penetration, Spalling and Scabbing, DIF data for tension and compression are required.

3 Strain rates in loading and its effects

Strain rate is the change in strain (deformation) of a material with respect to time. The strain rate at some point within the material measures the rate at which the distances of adjacent particles of the material change with time in the neighborhood of that point. When structures made of concrete is subjected to high strain loadings, concrete is known to show an increase in its strength. This increase in its strength is typically reported in terms of the ratio of dynamic to static strength, called the Dynamic Increase Factor (DIF). The Strain rate of the loadings can be simply expressed in terms of velocity as shown in equation below.

$$\text{Strain rate of Loading} = \frac{v(t)}{L_0}$$

Therefore as we can see, strain rate is nothing but the ‘velocity of the body’ divided by its length. It is expressed in terms of “Per Second” It is very important to understand the behavior of concrete structures subjected to high strain loads, as the behavior of concrete structures vary with respect to strain rates. For example, in the case of projectile and fragment impacts, cracking, spalling and scabbing are mainly influenced by strain rate in tension, whereas the behavior of concrete subjected to Blast (Near-range/close-in effects), penetration of missile is influenced by the strain rates in compression.

Blast loadings produces very high strain rates, which alters the dynamic mechanical properties and failure mechanisms of target structures and its corresponding structural elements. Due to the effect of these high strain rate loads, the strengths of steel reinforcements and concrete in a typical Reinforced Concrete Structure increases significantly. The Figure below shows a typical strain rate variation for different types of loadings. It can be seen that the high strain rate loads such as blast has a strain rate of 10^2 - 10^4 s^{-1} whereas ordinary Quasi- Static loads come in the strain rate range of 10^{-6} to 10^{-4} s^{-1}.

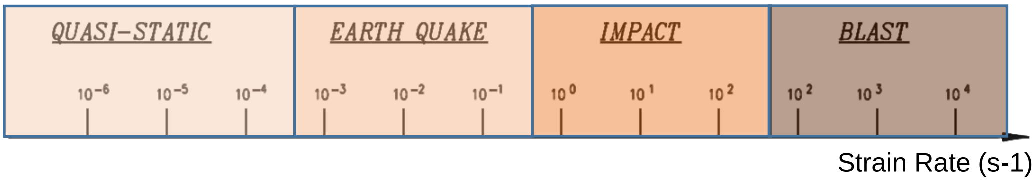


Figure 1: Strain Rates associated with different types of loading [13]

Dynamic Strength of the Concrete:

Structural elements subjected to high strain loads such as blast exhibits a higher strength compared to a similar structural element subjected to a static loading. This increase in strength for both the concrete and reinforcement is because of the rapid rates of strain that occurs in dynamically loaded members. These increased stresses or dynamic strengths are used to calculate the element's dynamic resistance to the applied blast load. Thus, the dynamic ultimate resistance of an element subjected to a blast load is always greater than its static ultimate Strength.

In a typical Reinforced Concrete Structure, both the concrete and the reinforcements are found to show an increase in strength when subjected to high strain loads. Higher the strain rate of the loadings, higher the compressive strength of the concrete. This parameter is employed while designing protective reinforced concrete structures for high strain rate loadings. Hence this paper focusses mainly on the study of increase in strength observed in concrete when subjected to high strain loads. The increase in strength of the concrete changes with respect to type of Blast whether it is a ‘Close in Range’ Blast or an ‘Far Range Blast’. According to UFC 3-340-02, Close-in Range of blast produces shock loads of high intensity which are non-uniform in nature and acting for a smaller periods of time. These extreme high pressure concentrations produces a punching kind of failure of an element. Whereas in the case of Far Range of Blast, the shock wave pressures produced are fairly uniform which acts for relatively larger periods of time, and the deflections required to absorb the loadings are also relatively small.

4 Single Elements Test

The basic performance of concrete in compression and tension is first studied by means of single element test simulations. The tests conducted were Unconfined Uniaxial Compression (UUC) and Unconfined Uniaxial Tension (UUT). The Unconfined Compressive Strength of the concrete used for the test is 30 MPa and the maximum aggregate size is 20 mm.

The stress strain curve for the four concrete models is generated when subjected to UUC as shown in figure below. The straining velocity for the analysis is 0.0254 mm/msec, where the strain rate is 0.001/ msec. It can be seen that MAT_72_R3 and MAT_RHT is showing more strength and the MAT_CSCM and MAT_WINFRITH_CONCRETE are relatively close in capturing the peak strength of the concrete which is 30 MPa.

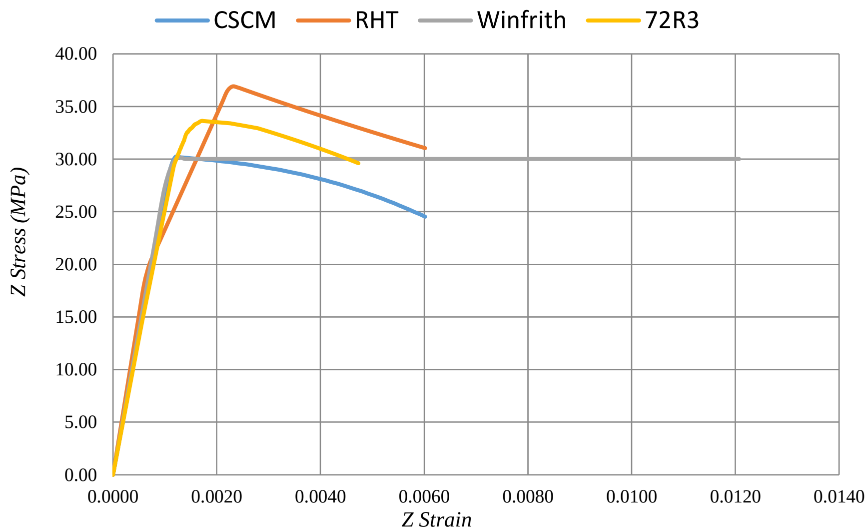


Figure 2: Stress- Strain curves - UUC

Followed by the uniaxial compression test, each of the material models were analysed with varying strain rates of 0.001, 0.01 and 0.1 per msec in single element simulations subjected to compression as well as tension.

MAT_RHT

The variation in the strength with different strain rates are well captured in Mat_RHT and is showed in the figures below.

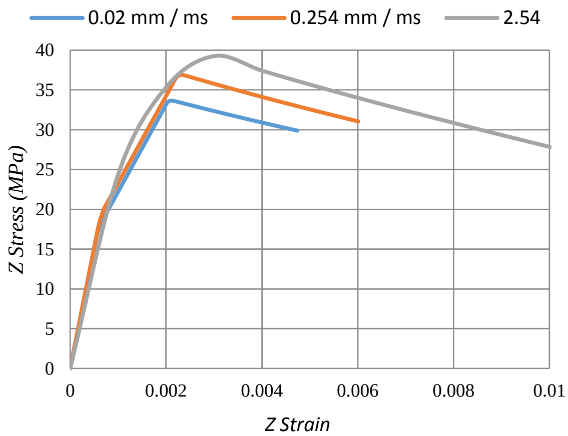


Figure 3: Showing stress strain curve in compression for MAT_RHT

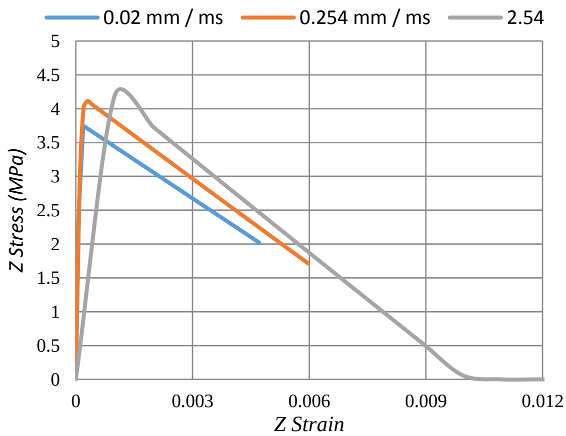


Figure 4: Showing stress strain curve in tension for MAT_RHT

MAT_CONCRETE_DAMAGE_REL3 (72_R3)

For MAT_72_R3, the dynamic increase factors were fed into LCRATE keyword. The following figures are the results obtained for tension and in compression.

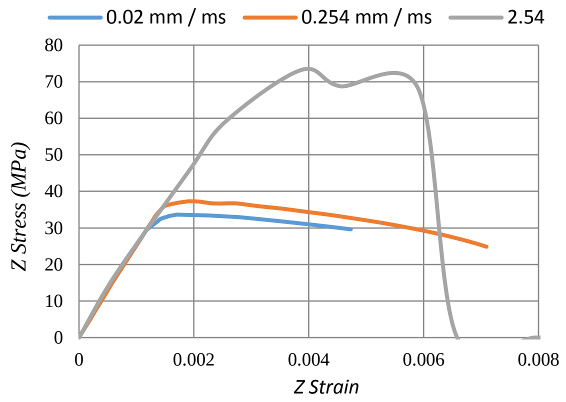


Figure 5: Stress strain curve in compression for MAT_72_R3

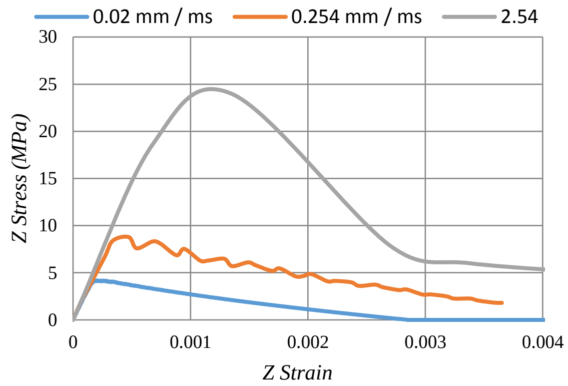


Figure 6: Stress strain curve in tension for MAT_72_R3

MAT_CSCM_CONCRETE

SRATE parameter in the material model is used for the strain rate effects.

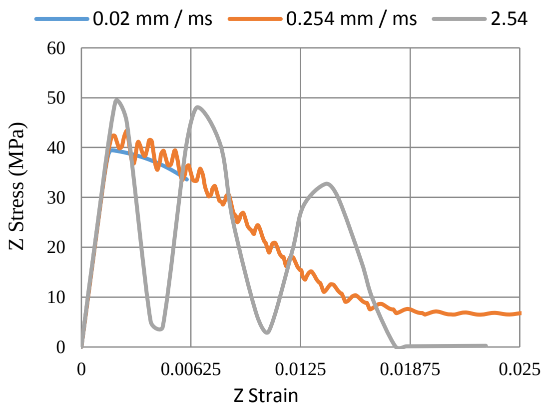


Figure 7: Stress strain curve in compression for MAT_CSCM_CONCRETE

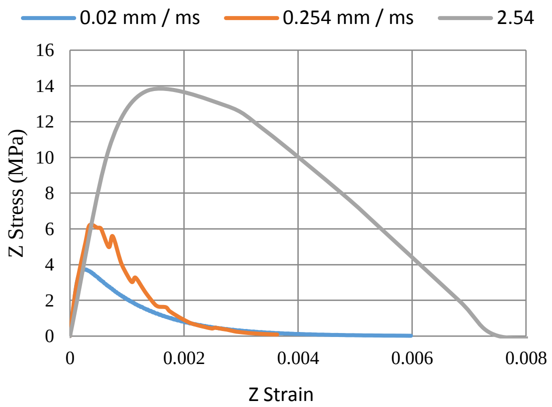


Figure 8: Stress strain curve in tension for MAT_CSCM_CONCRETE

MAT_WINFRITH_CONCRETE

RATE parameter in the material model is used for the strain rate effects.

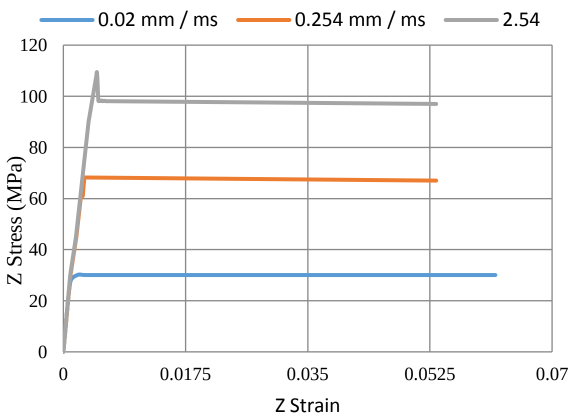


Figure 9: Stress strain curve in tension for MAT_WINFRITH_CONCRETE

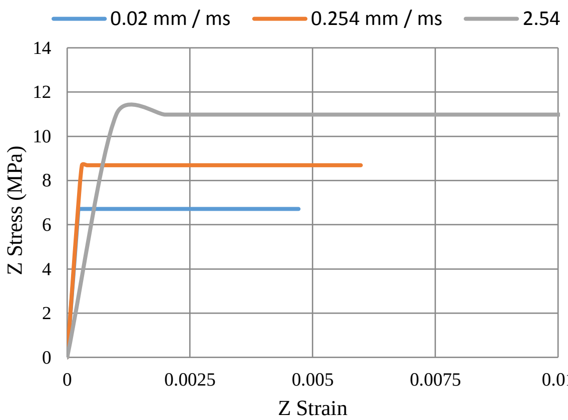


Figure 10: Stress strain curve in tension for MAT_WINFRITH_CONCRETE

The target strength under compression is 30 MPa and for tension it is around 3.83 MPa from the empirical relations. From the above graphs, we can infer that MAT_RHT and MAT_72_R3 performed better in responding to high strain rates, whereas the other concrete material models such as

MAT_CSCM_CONCRETE and MAT_WINFRITH_CONCRETE showed an abrupt increase in strengths when subjected to high strain rate for both compressive and tensile loadings. Based on UUT and UUC test observations MAT_RHT and MAT_72_R3 are capturing the strength increase in a better way.

5 Three Point Bending Test

In this section, the classical ‘three point bending test is modified to test the behavior of concrete material models in bending subjected to different strain rates. For this purpose, a beam is modeled (using solid elements) having a span of 4 meters with cross section of 0.25 X 0.3 meters and compressive strength of concrete as M40.

The Loading on the beam is achieved to have three different strain rates by using the option of ‘BOUNDARY_PRESCRIBED_MOTION_RIGID_BODY’ in LS-DYNA. A rigid box is modeled on the top of the concrete beam for the load application purpose, which helps to idealize the simulation as three point bending test as shown in the figure below. Two different velocities used for the comparative study are 3 m/sec and 0.003 m/sec which corresponds to the different strain rates in Impact and Earthquake ranges respectively. The results for all these concrete models are summarized in this section.

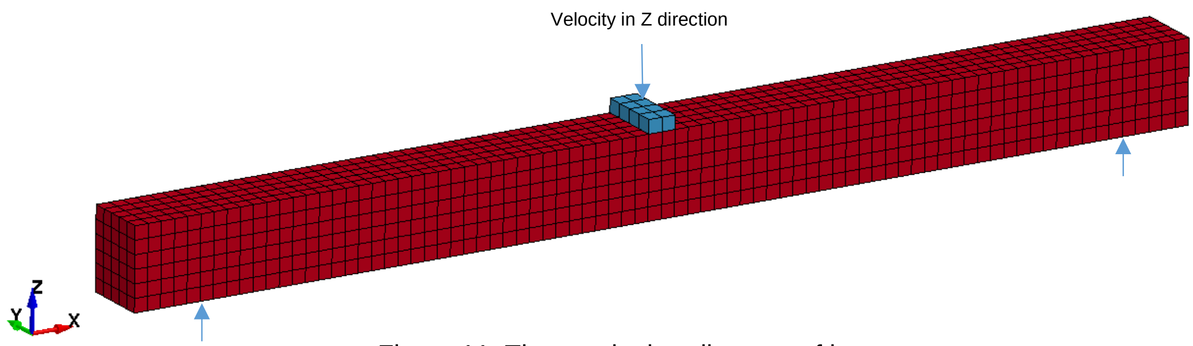


Figure 11: Three point bending test of beam

5.1 Mesh Study

To select the appropriate size for the mesh, a preliminary mesh study is conducted using MAT_RHT concrete model. The chosen mesh sizes are 75mm, 50 mm and 25 mm and analyzed for the strain velocity of 3 m/sec. First the failure pattern is observed. Since the beam was failing due to the plastic hinge forming in the bottom face as shown in the figure below, tensile stress is taken as the comparison parameter.

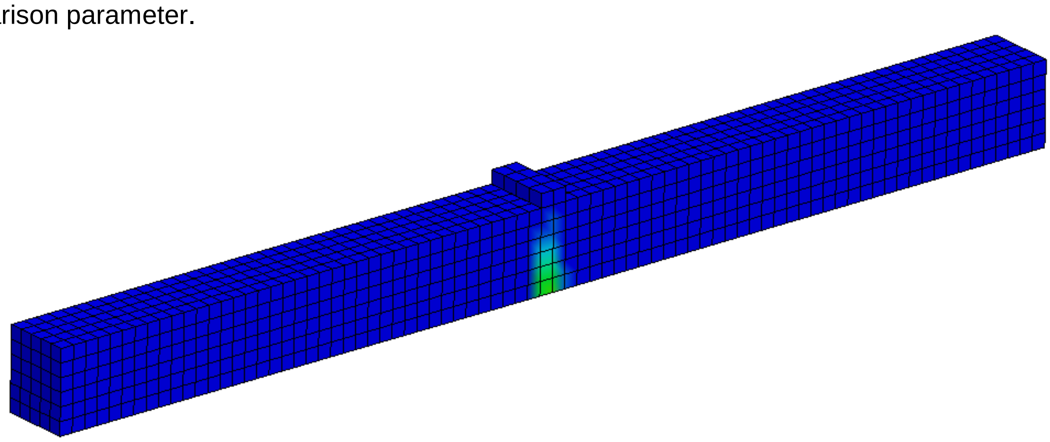


Figure 12: Plastic Hinge Formation at the bottom face

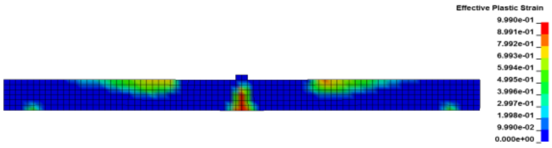
Mesh Size (mm)	Failure tensile stress (MPa)
75	6.16
50	6.25
25	6.35

The percentage difference in the tensile stress in the models with mesh 75 mm and 50 mm is around 1.46 percent and the percentage difference in the tensile stress between 50mm and 25 mm models is around 1.6 percent. Since the percentage difference in the tensile stress is within 2 percent, 50 mm mesh size is adopted for the future study.

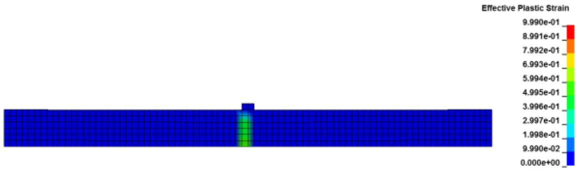
5.2 Material Model Behaviors in Varying Strain Rates

The failure mode in the concrete beam subjected to the varying strain rates is explained here. The results were tabulated in the subsequent table for all the material models.

MAT_CSCM_CONCRETE

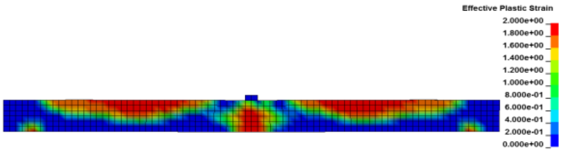


Velocity=3 m/s, t= 0.00129 s

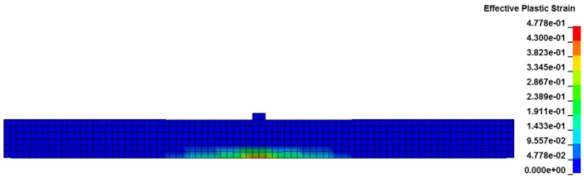


Velocity=0.003 m/s, t= 0.4 s

MAT_72_R3

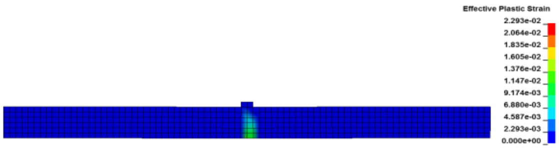


Velocity=3 m/s, t= 0.00129 s

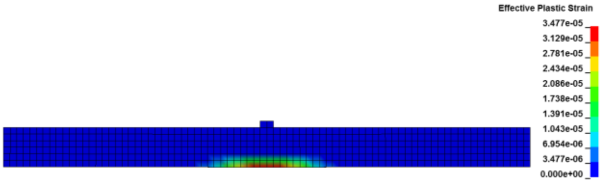


Velocity=0.003 m/s, t= 0.4 s

MAT_RHT

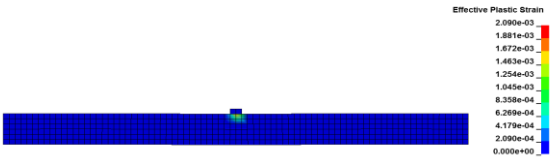


Velocity=3 m/s, t= 0.00129 s

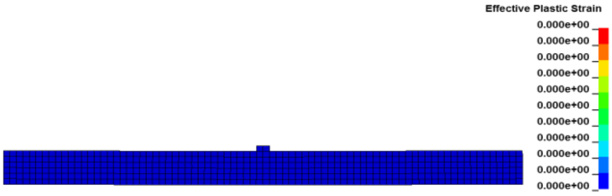


Velocity=0.003 m/s, t= 0.4 s

MAT_WINFRITH_CONCRETE



Velocity=3 m/s, t= 0.00129 s



Velocity=0.003 m/s, t= 0.4 s

Based on the formation of the plastic hinge, the failure stresses are extracted, and each material response for varying strain rates are compared in the table below.

Material Model	Straining Velocity = 3 m/sec	Straining Velocity = 0.003 m/sec
RHT	6.25 MPa at 0.2 ms (T)	4.77 MPa, at 0.6 s (T)
CSCM	5.58 MPa, at 0.3 ms (T)	2.9 MPa, at 0.3 s (T)
WINFRITH	41.5 MPa, at 0.7 ms(C)	4.51 MPa, at 0.4 s
72_R3	8.04 MPa, at 0.2 ms (T)	4.06 MPa, at 0.5 s (T)

As we can see from the results that all the material models shows an increase in strength when subjected to high strain rate loadings. The main difference observed being is the formation of Plastic Hinge and the Failure stress. In all the material models except MAT_WINFRITH_CONCRETE, the failure was found to happen in the tension zone with the formation of the tension cracks. Whereas MAT_WINFRITH_CONCRETE captured an inverse behavior where the plastic hinge forms on the compression face with loading velocity of 3 m/sec, and no plastic hinge formation under a loading velocity of 0.003 m/sec. The above behavior of MAT_WINFRITH_CONCRETE is very much deviated with the results observed with the other material models, which showed tension zone to be failed first.

6 Blast Load on Concrete Slab

Effect of Blast on an Unreinforced Concrete Slab is studied in this section. Blast is simulated for different test scenarios to consider different strain rates of blast. The FE model of the slab with dimensions 5m (L) x 5m (B) x 0.2 m (t) and grade of the concrete as M40 is adopted for the simulation. The blast load with varying strain rates is achieved by keeping the TNT charge at 10 meter, 2.2 meter and 1.7 meter from the surface of the slab with the charge weight being 100 kgs. LOAD_BLAST_ENHANCED is used to simulate the blast effect rather than actual simulation through ALE elements.

The results obtained from the analysis are tabulated below, which helps us to understand the behavior of the concrete material models as a slab subjected to varying strain rates. The strain rates of blast is selected such that it replicates the 'Far Range' and 'Near Range' blast Effects. Structural elements subjected to 'Far Range' pressures responds to it conventionally, with the plastic hinge occurring at the tensile face. The above behavior is same as observed in bending action due to quasi static loading. But when the same slab is subjected to Impulsive loads such as 'Near Range of Blast', the structural elements responds in such a way that the plastic hinge formation may be altered as shown in the figure 17 below. This behavior is subjected to the examination here through the Numerical simulation.

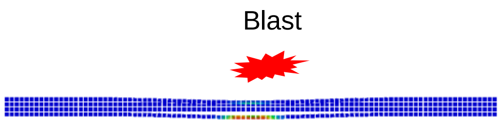


Figure 16: Formation of Plastic Strains at the Bottom face

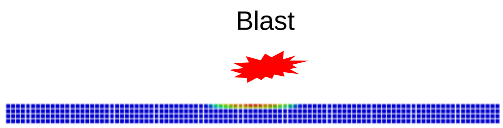


Figure 17: Formation of Plastic Strains at the Top Face

The pressures calculated as free air blast according to UFC 3-340-02 [14] for 100 kgs of TNT charge kept at 10 m, 2.2 m, and 1.7 meters above the slab are 0.523 MPa with t_o=9.29 ms, 4.283 MPa with to=1.34 ms and 6.639 MPa with to=0.966 ms respectively(to = Time Period of the Blast). The responses of the slab with four different concrete models for these pressure pulses are tabulated in the table below.

Pressures	0.523 MPa 9.29 ms Case 1:100 kgs at 10m	4.283 MPa 1.34 ms Case 2:100kgs at 2.2 m	6.639 MPa 0.966 ms Case 3: 100 kgs at 1.7 m
MAT_RHT	F_t = 5 MPa at 14.5 ms	F_c = 48.5 MPa, at 2.2 ms F_t = 6.2 MPa, at 5 ms	F_c = 55.5 MPa, at 2.7 ms F_t = 6.3 MPa, at 5 ms
MAT_CSCM_CONCRETE	F_t = 4.39 MPa at 12.8 ms	F_c = 54.4 MPa, at 3.0 ms F_t = 3.59 MPa, at 0.7 ms	F_c = 64.8 MPa, at 3.0 ms F_t = 3.8 MPa, at 0.49 ms
MAT_WINFRITH_CONCRETE	F_c = 29 MPa at 12.8 ms	F_c = 54.4 MPa, at 3.0 ms F_t = 3.59 MPa, at 0.69 ms	F_c = 67.7 MPa, at 0.2 ms F_t = 2.77 MPa, at 0.49 ms
MAT_72_R3	F_t = 3.18 MPa at 16 ms	F_c = 49.6 MPa, at 3.0 ms F_t = 9.32 MPa, at 0.692 ms	F_c = 59.2 MPa, at 3 ms F_t = 10.6 MPa, at 0.49 ms

For a particular time period, the figures below shows the plastic strains formed when the slab is subjected to three different High strain loadings.

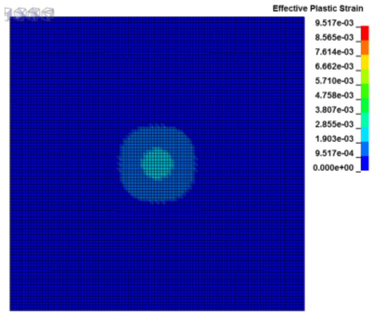


Figure 18: Bands of Plastic strain occurring at bottom face in MAT_RHT for Case 1

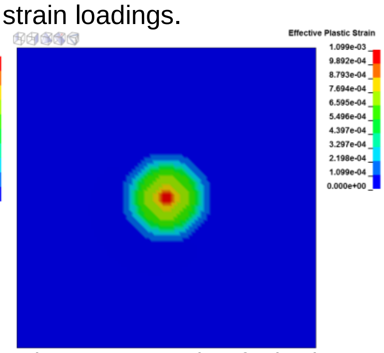


Figure 19: Bands of Plastic strain occurring at bottom face in MAT_RHT for case 2

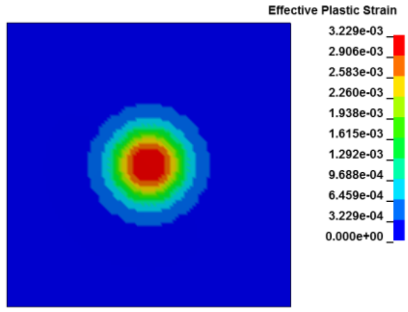


Figure 20: Bands of Plastic strain occurring at bottom face in MAT_RHT for case 3

The above table is summarized with reference to the formation of plastic hinge and the time of formation of plastic hinge is also tabulated, which gives us an idea of type of failure the material might undergo when the different range of blast load acts on a slab element. Firstly it can be seen that all of the material models shows an increase in strength when acted by increasing strain rate loadings.

In Case 1 scenario, wherein tensile kind of failure is expected, MAT_RHT, MAT_CSCM_CONCRETE, and MAT_72_R3 shows only tension plastic strains, whereas MAT_WINFRITH_CONCRETE shows compression plastic strains also. In Case 2, which is also more like far range blast, MAT_RHT and MAT_WINFRITH_CONCRETE models shows the plastic strains to be generated in the tension zone first and gradually extending to the compression zone which leads to the failure of the material. In Case 3, near range of blast where punching kind of failure is expected MAT_RHT and MAT_WINFRITH_CONCRETE capture the same. So from the results it can be inferred that MAT_RHT captures the entire range of strain rate loading whereas MAT_WINFRITH_CONCRETE doesn't exactly capture the far range blasts.

7 Penetration of a Missile in an Unreinforced Concrete Slab

To study the behavior of concrete slab subjected to varying strain rate loadings due to the penetrating missile, a slab is modeled having a dimension of 5 m x 5 m with a thickness of 0.2 m. The initial velocity of the missile is applied at 50 m/sec. Lagrangian type of element formulation is used to model solid section, since the Eulerian element formulation is supported only by MAT_72_R3 [6], and not by any other concrete material models chosen for this study. Due to the large computational time required for the analysis, the material models here were only compared with each other for its numerical responses obtained for 50 m/sec projectile speed and not for varying velocities of projectile speeds. The projectile mass being 245 kgs, and the restraints for the model was applied at the faces for all three translations.

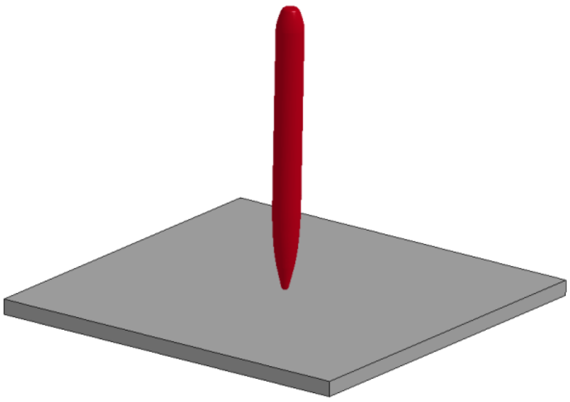


Figure 21: Undeformed shape of the missile penetrating the slab

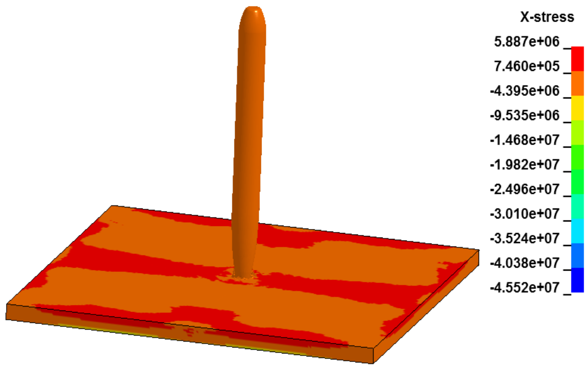


Figure 22: Showing the x-direction stress developed in the slab

The following are the results obtained for the simulations of 245 kgs of missile penetrating the slab at 50 m/sec initial velocity. The Plastic Strains are observed to happen in the following way at the time of 4 ms, for the four material models

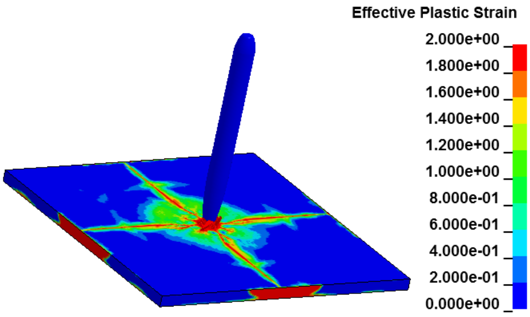


Figure 23: Plastic Hinge formation in MAT_72_R3 at 4 ms

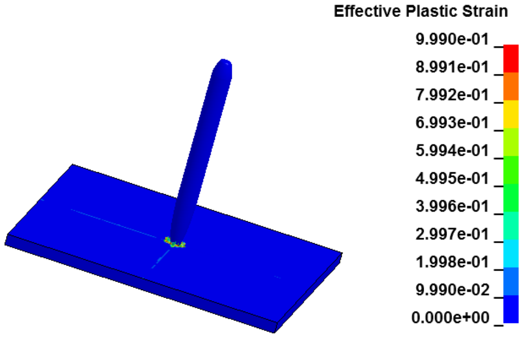


Figure 24: Plastic Hinge formation in MAT_CSCM_CONCRETE at 4 ms

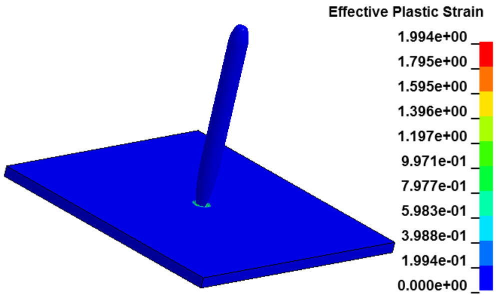


Figure 25: Plastic Hinge formation in MAT_RHT at 4 ms

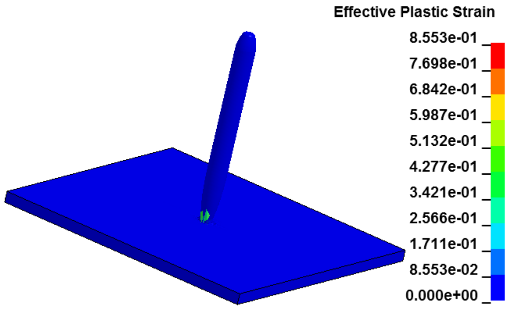


Figure 26: Plastic Hinge formation in MAT_WINFRITH_CONCRETE at 4 ms

The final Velocities of the projectile was found to be 47.6 m/s in MAT_72_R3, 48.08 m/s in MAT_CSCM_CONCRETE , 47.16 m/s in RHT and 44.4 m/s in MAT_WINFRITH_CONCRETE. The failure bands are captured accurately only in the case of MAT_72_R3 where radial crack pattern is observed.

CONCLUSIONS

Based on the numerous analyses results it can be concluded that selecting the appropriate material model is an important step in a numerical simulation process, as the structural responses and the failure patterns changes with respect to the selected material models and loading rates.

For Quasi- Static loadings, from the single element test simulations, it can be inferred that MAT_WINFRITH_CONCRETE, MAT_RHT and MAT_72_R3 may be employed, which showed an accurate capture of the compression strength of the concrete. And it can be seen be seen that MAT_RHT and MAT_72_R3 showed better results with an agreeable increase in strength when subjected to high strain loadings. From the beam bending tests which is replica of impact test, it is evident that MAT_RHT and Mat_72_R3 are the best suitable material models since it capture the failure behavior and strain rate effects reasonably.

Blast study on the slab has shown that all the material models capture the strain rate effects. In near range blast (Case 3) MAT_RHT and MAT_WINFRITH_CONCRETE are able to predict the compression failure. In near range blast (Case 2) only MAT_RHT is able to predict the compression failure.

In the penetration analysis, all the material models were able to predict just the concentrated failure and corresponding plastic strains, whereas only MAT_72_R3 showed an agreeable plastic strains, with the diagonal bands. Also MAT_72_R3 is the only material model which supported ALE formulations, which is most preferred in the case of large deformation simulations.

Hence from all these observations, it can be concluded that even though all the material models were fairly successful in evaluating the increase in strength due to increase in strain rate of the loads. From different applications study it can be cleared inferred that selection of material models is crucial for an analysis, where thorough investigation is required prior to each analysis in selecting a suitable material model. In such case, this study can be taken as starting point for various applications/simulations of concrete.

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