

# Sound Radiation Analysis of a Tire with LS-DYNA<sup>®</sup>

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## Abstract

*In vibro-acoustic problems, which are assumed to be weak acoustic-structure interactions, the vibration of structural response is computed first. The obtained result is taken as boundary condition for the acoustic part of the vibro-acoustic problem. Consequently, the radiated noise at any point into space can be calculated. This paper presents a case study of applying the steady state dynamics (SSD) coupling with boundary element method (BEM) in LS-DYNA for calculating sound radiation of a tire.*

## Introduction

LS-DYNA is a widely used finite element code, intended to solve complex mechanical problems. One of the recent developments of the code is the addition of a vibro-acoustic solver [1], which enables users to perform a variety of vibro-acoustic simulations in the frequency domain.

## Steady State Dynamics

The steady state dynamics (SSD) in LS-DYNA[2] calculates the steady state response of a structure subjected to known harmonic excitations. The excitation spectrum can be given as nodal force, pressure or base accelerations. The excitation spectrum takes complex variable input. In other words, both the amplitude and the phase angle of the excitation are considered. The frequency domain dynamic features: SSD are based on the results of modal analysis of the structures, e.g. the natural frequencies and modal shapes. They use either mode superposition method, or mode acceleration method or some other modal combination methods. Thus one needs to run implicit eigenvalue analysis preceding the frequency domain dynamic analysis.

The SSD feature can be activated by the keyword \*FREQUENCY\_DOMAIN\_SSD. The results are also complex and have either real and imaginary parts, or amplitude / phase angle pairs. The amplitudes of the response are given in a binary plot file d3ssd, which is accessible by LS-PrePost<sup>®</sup>. A complete results including the amplitude and phase angle can be found in ASCII

database NODOUT\_SSD for nodes specified in keyword \*DATABASE\_HISTORY\_NODE, and database ELOUT\_SSD for elements specified in the following keywords

\*DATABASE\_HISTORY\_SOLID  
\*DATABASE\_HISTORY\_BEAM  
\*DATABASE\_HISTORY\_SHELL  
\*DATABASE\_HISTORY\_TSHELL.

### **BEM Model for Acoustic System**

In frequency domain, the acoustic wave propagation in an ideal fluid in absence of any volume acoustic source is governed by Helmholtz equation [3] given as follows:

$$\Delta p + k^2 p = 0 \quad (1)$$

Where  $k = \omega/c$  denotes the wave number,  $c$  is the sound velocity,  $\omega = 2\pi f$  is the pulsation frequency and  $p$  is the pressure at any field point.

Equation (1) can be transformed into an integral equation by using Green's theorem. In this case, the pressure at any point in the fluid medium can be expressed as an integral of both pressure and velocity over a surface as given by the following equation:

$$Cp(P) = \int_{\Gamma} (G \frac{\partial p}{\partial n} - p \frac{\partial G}{\partial n}) d\Gamma \quad (2)$$

where  $G = \frac{e^{-ikr}}{4\pi r}$  is the Green's function,  $n$  is the normal on the surface  $\Gamma$ ,  $C$  is the jump term resulting from the treatment of the singular integral involving Green's function, and  $r$  is the distance between the field point  $P$  and surface integration point. The normal derivative of the pressure is related to the normal velocity by  $\frac{\partial p}{\partial n} = -i\omega p v_n$ .

The knowledge of pressure and velocity on the surface allows calculating the pressure of any field points. This constitutes the main idea of the integral equation theory. In practical cases, the problems are Neumann, Dirichlet or Robin ones. In Neumann problem, the velocity is prescribed on the surface while in Dirichlet case the pressure is imposed on the surface. Finally, for robin problems the acoustic impedance, which is a combination of velocity and pressure, is given on the surface. Hence, generally only half of the variables are known on the surface. By using the variational indirect method or the collocation method, a linear equation system can be established, which provides solution for the other half of the variables on the surface. Then the integral equation (2) can be used to calculate the acoustic pressure at any field points.

The BEM acoustic solver has been coupled with the FEM dynamic analysis capability of LS-DYNA to provide an integrated solution for vibro-acoustic problems. Two options are available to couple the BEM acoustic solver with the FEM dynamic analysis. For the first one, the traditional time domain FEM is performed and the time domain dynamic response of the structure is converted to frequency domain by using Fast Fourier Transform (FFT); for the second one, frequency domain steady state dynamics is performed (using \*FREQUENCY\_DOMAIN\_SSD) and it gives response in frequency domain directly. The obtained boundary velocities (accelerations) provide boundary condition for the subsequent

BEM acoustic computation. Both the variational indirect BEM and collocation direct BEM are available in LS-DYNA.

### Sound radiation prediction of a tire

Tire noise is one of main sources in automotive noise, especially pass-by noise. In this paper, we consider only radiation noise caused by structural vibration. The tire model is shown in figure 1.

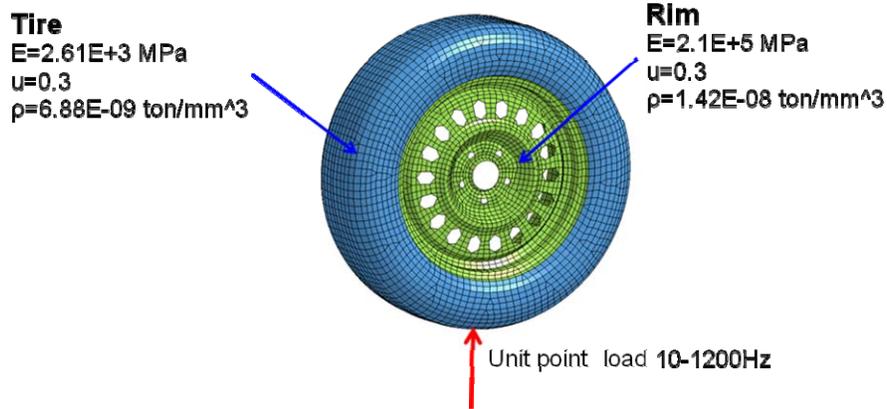


Figure 1: Tire Model

The harmonic point force is applied to the nodes which are in contact with ground. Tire surface velocity can be obtained from SSD analysis in figure 2.

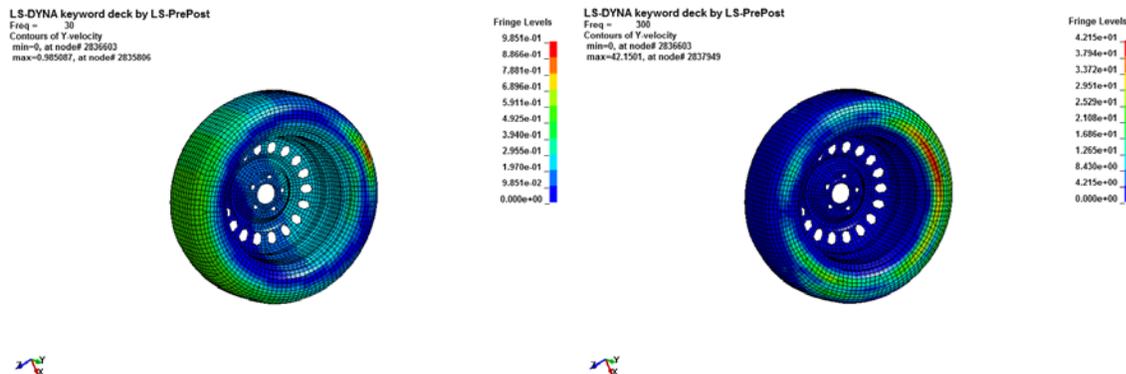


Figure 2: Surface velocities at different frequency

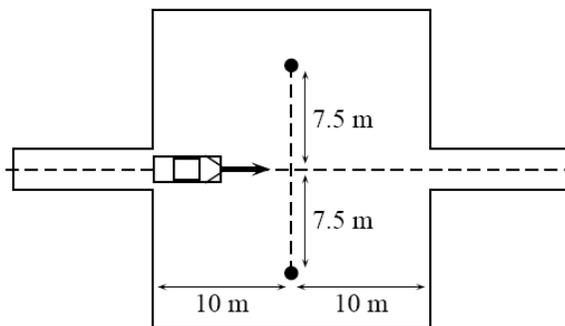


Figure 3: the setup for a pass-by noise test form the ISO 362 Standard.

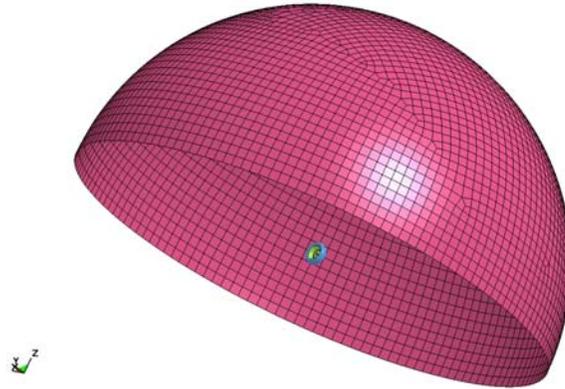


Figure 4: R7.5 hemi sphere field point used for pass-by noise test

To calculate the sound radiation, a hemisphere field points (radius= 7.5m) are used for pass-by noise test [4], which is shown in figures 3 and 4. Considering tire on road are reflecting surface radiation case, reflecting surface was modeled as rigid shown in figure 5. The keyword \*FREQUENCY\_DOMAIN\_ACOUSTIC\_BEM\_HALF\_SPACE can be used for reflecting surface case. Figure 6 shows the sound pressure level of different frequency at field points.

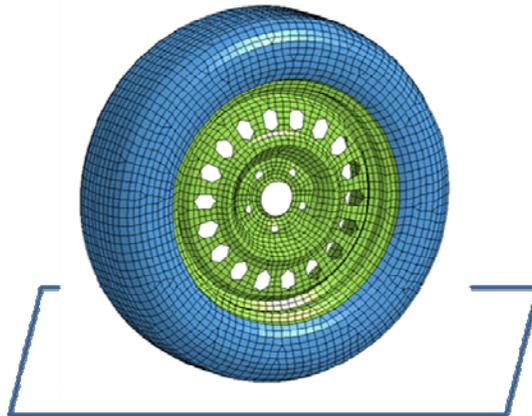
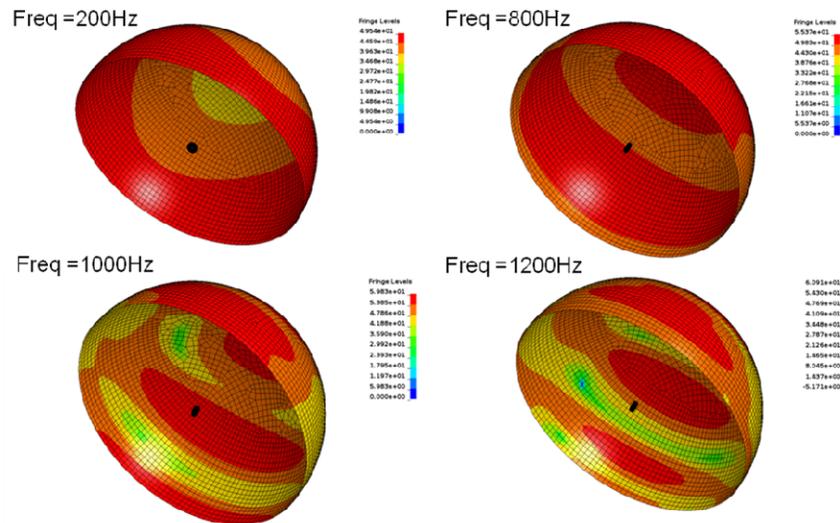


Figure 5: Reflecting surface was modeled as rigid



**Figure 6: Sound pressure level contours of field points**

In the classical approach, the acoustic response is calculated by solving the system of equations for each loading conditions. For a multi-load case solution such as tire vibration due to different road condition, Modal acoustic transfer vectors (MATVs) approach can be used. The modal acoustic transfer vectors (MATVs) are the modal counter part of the ATVs: they express the acoustic transfer function in modal coordinates from a radiating structure to a field point, and, therefore, list the acoustic contribution from each individual structural mode. The acoustic response in the field point is obtained by recombination of the MATV with the corresponding structural modal responses. Working in modal coordinates results in an important data reduction. However, MATVs are no longer independent from the structural model as they are linked to the structural modal basis. Whenever the structural modal basis changes, e.g. due to structural modifications, the set of MATVs need to be re-evaluated.

In general, the ATVs are known as [5], the acoustic pressure  $p(\omega)$  at a field point is computed by a rapid vector-vector product of ATV and normal vibration shape on the surface  $\{v_n(\omega)\}$ , which requires very little calculation time:

$$p(\omega) = \{ATV(\omega)\}^T \{v_n(\omega)\} \quad (3)$$

The process for computing the structural response frequently relies on modal superposition, so that:

$$\{v_n(\omega)\} = j\omega[\Phi_n]\{mrsp(\omega)\} \quad (4)$$

Where  $[\Phi_n]$  is the matrix of modal vectors, projected on the local normal direction of the radiating surface, and  $\{mrsp(\omega)\}$  is the modal response (vector of modal participation factors). The structural analysis in LS-DYNA can provide the participation factors of the modes, at each frequency, for each load case.

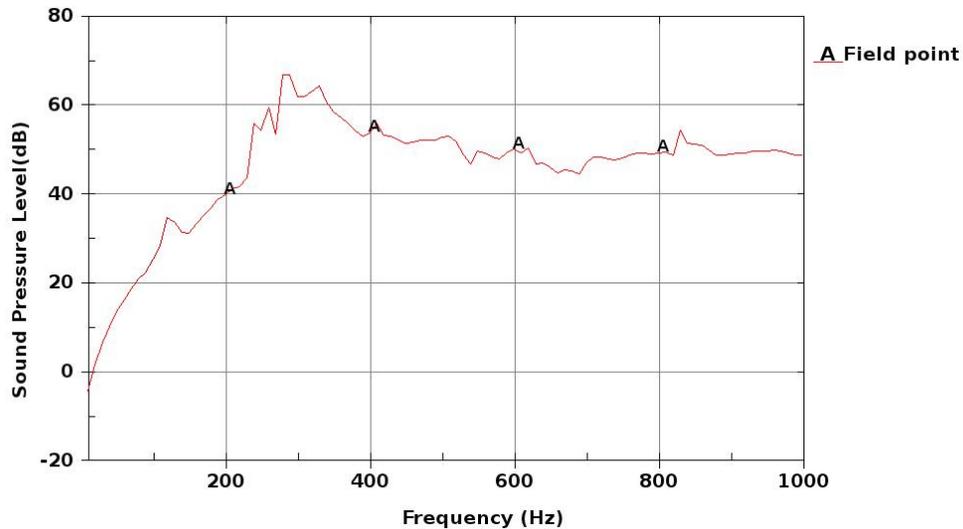
Considering again Equation (3), we can use the mode shape vectors (for the radiating surface only) as surface vibration shapes, thus deriving Modal Acoustic Transfer Vectors  $MATV_i$  as:

$$MATV_i = j\omega \{ATV(\omega)\}^T \{\phi_{ni}\} \quad (5)$$

Where  $\{\phi_{ni}\}$  is the (projected) shape vector of the  $i^{th}$  mode shape. Then, the final acoustic result is determined by multiplying the MATVs by the modal participations:

$$p(w) = \{MATV(\omega)\}^T \{mrsp(\omega)\} \quad (6)$$

To run MATV, we use the keyword \* FREQUENCY\_DOMAIN\_ACOUSTIC\_BEM\_MATV. Figure 7 shows sound pressure level of node 2839929 from hemi sphere field point (Figure 4)



**Figure 7: Sound pressure level at field point**

### Conclusions

This paper introduces briefly steady state dynamics and acoustics boundary element method of LS-DYNA for solving frequency domain vibration and acoustic problems. The BEM coupled with SSD analysis in LS-DYNA are used to predict the sound radiation of tire. The objective of implementing these features is to provide users the capabilities to deal with frequency domain vibration and acoustic problems, which are very common in auto industries.

### References

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