

Development of a Tool for Automatic Calibration of Material Models in LS-DYNA

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1 Abstract

LS-DYNA offers a wide variety of material cards to cover different needs in diverse applications. However, choosing the most proper material model from among 250 keywords can be quite confusing. Therefore, we intend to develop a tool for a smart material database, which can search for the most proper material keyword with regard to the user's application. The algorithm of this tool is developed and implemented in a beta version of the program in order to examine its reliability.

The user will be guided step-by-step through some questions for the purpose of entering the desired material and simulation properties. The questions are general set and include some examples, so as to avoid the need for advanced material science. Due to a default value by each step, the user can skip questions that are not relevant to his or her application. Each material keyword is coupled with a profile, which includes the material properties covered by the corresponding keyword. The program searches beyond the tags regarding the user's inputs in order to find the appropriate material model for the desired application.

This algorithm is implemented in MATLAB and has been coupled with a database of material cards in Microsoft Excel. By answering some general questions (e.g. about the density or Young's modulus of the desired material), the program can find the most proper material card from among the existing ones in the database. The output of the program is a prioritized list of material cards that matches the needs of the user. This program is tested for some pre-defined inputs. Consequently, the suggested material cards were comprehensive and similar to the expectations contained in publications.

The distinction of this work is its encyclopedic knowledge about material cards for use in full vehicle crash simulations. However, it is possible to develop the algorithm for other applications, such as metal forming, etc.

2 Introduction

FE-Simulation is considered to be one of the main tools for research and development in the automotive industry. Among other solvers, LS-DYNA is an advanced general-purpose multiphysics simulation software which is widely used for implicit and explicit analysis. Although the basic principle of the software remains similar for a variety of applications, several modeling differences should be considered. This study is dedicated to investigate crash simulations and the focus is on determining the proper material model for this specific application.

LS-DYNA offers a wide range of material models (material keywords or *MATs) in order to ensure sufficient reliability of the simulation model. However, selecting the most proper material model in LS-DYNA may be confusing. Therefore, a tool is being developed that aims to suggest the proper material keyword for the user's desired application. The algorithm of this tool is based on the information and knowledge from the LS-DYNA keyword user's manual and some published papers. This tool is the primary version of a smart material database for LS-DYNA, which can search for the most proper material keyword with regard to the user's application. The Institute of Automotive Technology at TUM (Technische Universität München) intends to develop this database for a community of users and developers from universities and research institutes.

3 Approach

The algorithm of the program consists of two steps. The first step is determining the proper mechanical behavior model of the material, called the material keyword. The second step is calibrating the selected keyword with specific mechanical properties or creation of the material card.

3.1 Selection of the Material Keyword

The beta version of the tool is developed for four material types commonly used in crash applications: rubber, foam, plastic/metal and composite. First, the material keywords of LS-DYNA are processed in three steps:

1. A list of available material keywords is obtained for each material type. For this, some validated simulation models (e.g. full vehicle models from National Crash Analysis Center at the George Washington University, dummy and barrier models from Livermore Software Technology Corporation, etc.) and some publications [1] to [25] were studied to find the most common keywords for each material type in crash simulations.
2. To distinguish the material keywords, the mechanical properties of each material model are analyzed. The minimum mechanical properties required to distinguish the keywords of each material type have been categorized as a list of items with different values.
3. Finally, one table for each material keyword has been developed, which is called the “keyword’s profile” here. This profile contains all of the material properties and their relevant items that could be covered through that specific material keyword.

After the material type is selected, the user is asked several questions to detect the desired material properties. Each question addresses a property from the keyword’s profile. According to the user’s choice, a value will be assigned to the available keywords from the list of the selected material type. These values express a quantitative representation for the weight of each question. The keyword will become 0 if the desired property is not covered, 3 if the desired property will be covered, and 5 if the keyword is the most proper material model for that desired property.

The weights are chosen empirically and differences between the values are provided to avoid the compensation of not allowed property in final results. The values which are ascertained during the questioning phase will be added together for each material keyword. Finally, the keywords will be sorted from the highest to the lowest value, with the first represented keyword being the most proper material model for the user’s application.

3.2 Calibration of the Material Keyword

The second part of the algorithm is developed to calibrate the selected keyword from the previous section with material properties and creation of a material card. For this aim, a database of material cards is developed, which can be expanded with more cards. The database consists of blocks for each keyword, which contains different material cards. For example, the block of the ***MAT_PIECEWISE_LINEAR_PLASTICITY** keyword contains several material cards for different aluminum alloys, plastics, steels, etc.

After selecting the keyword from the user, the algorithm will ask a specific number of questions from the relevant block to find the most proper material cards for the selected keyword. The questions can be skipped if the user does not have enough information. Answers to the questions will be converted into a vector, which will be compared with the vector of stored material cards in the database in order to find the proper material card that covers the desired properties. Consequently, these vectors will be reported to the user as a list of available material cards.

The user’s uncertainty about the exact amount of the values is addressed in this algorithm as well. The user can specify a variation limit for the desired material properties, which will be applied to the vector values. The matching principle of these vectors is shown in Fig.1 .

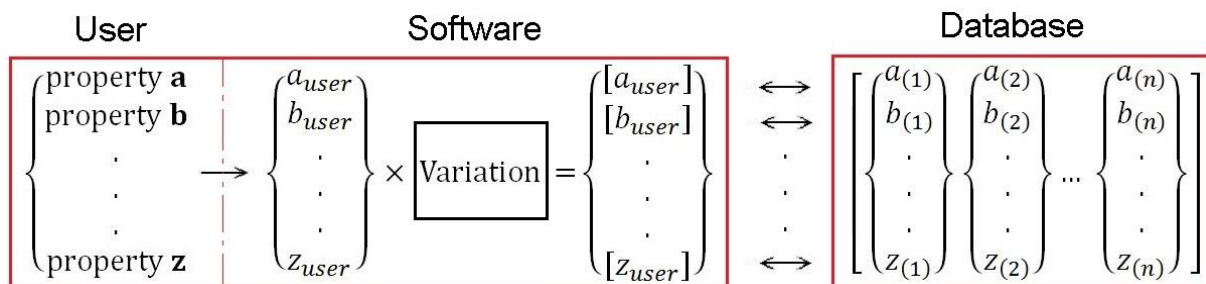


Fig.1: Material card vectors and their matching principle

Since LS-DYNA does not have a default unit system, users need to provide all the values in a consistent unit system. In the database for this study, all of the values are provided in SI units.

For the beta version of this program, the decision was made to use Microsoft EXCEL for the database, since it can be coupled with MATLAB and is a frequently used software program. Fig.2 shows the state of the algorithm when the user has selected ***MAT_VISCOELASTIC**.

4 Keywords’ Profiles

In this section, the three steps of keyword processing described in section 3.1 is presented for each material type. The first table of each part shows the common keywords that are considered for the

relevant material type here. The second table indicates the minimum mechanical properties required to distinguish the keywords of each material type and the possible values for each item. The third table presents the keywords' profiles, which contain the properties covered by the relevant material keyword.

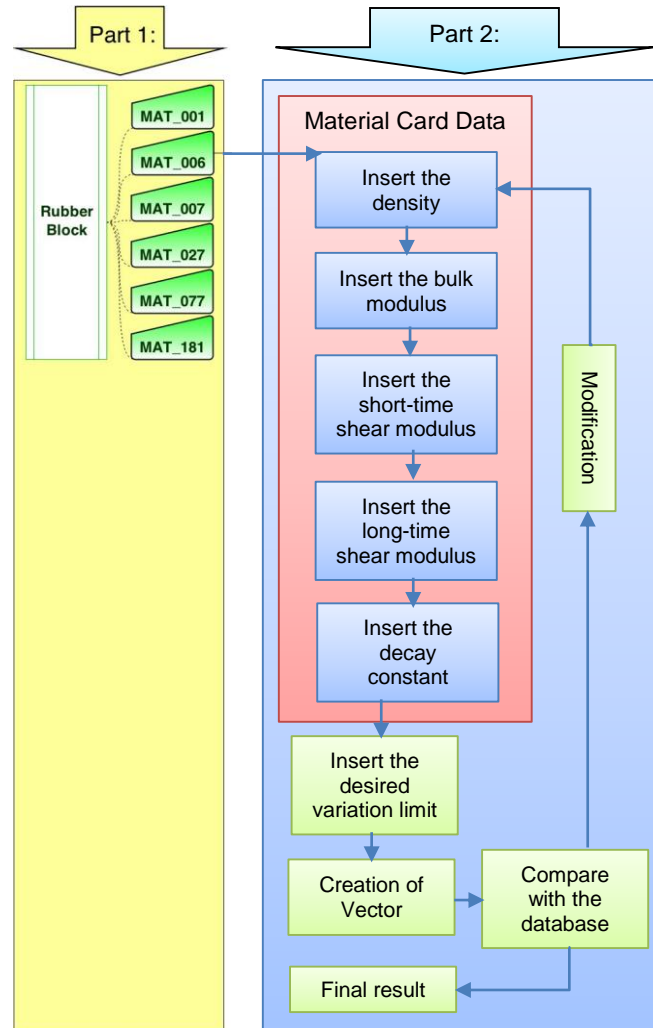


Fig.2: Algorithm for the calibration of material cards

4.1 Rubber

The keywords that are considered for modeling rubbers are listed in Table 1.

3 Digit numerical designation	Descriptive designation
*MAT_001	*MAT_ELASTIC
*MAT_006	*MAT_VISCOELASTIC
*MAT_007	*MAT_BLATZ-KO_RUBBER
*MAT_027	*MAT_MOONEY-RIVLIN_RUBBER
*MAT_077	*MAT_HYPERELASTIC_RUBBER *MAT_OGDEN_RUBBER
*MAT_181	*MAT_SIMPLIFIED_RUBBER/FOAM

Table 1: List of *MATs for rubbers

Table 2 indicates the minimum mechanical properties required to distinguish the keywords for rubber. They are categorized as six items, with each one having its own value.

Items	Values			
Mechanical model	Linear elastic	Viscoelastic	Hyperelastic	Combination of visco and hyperelastic
Stress-strain behavior	Only constants	Uniaxial	Multiple	
Strain range	small	medium	Large	Very large
Rate dependency	Yes	No		
Failure criteria	Yes	No		
Damage effect	Yes	No		

Table 2: List of the required mechanical properties for rubbers, based on information from [1] to [6]

The mechanical properties shown above can be explained as follows:

- **Stress-strain behavior:** This item specifies how the stress-strain behavior is modeled. “Only Constants” means that this behavior is only modeled by considering some constant properties such as Young's modulus and Poisson's ratio. Uniaxial means defining this behavior using a single uniaxial load curve, and multiple means using a family of uniaxial curves to model the stress-strain behavior.
- **Strain range:** There is no international standard for classifying the applicable range of strain. In this study, the four ranges of 0 to 100%, 100 to 300%, 300 to 500% and 500 to 700% [5] are set for the low, medium, large and very large range of applicable strains, respectively.
- **Rate dependency, failure criteria and damage effect** are not examined in detail. Material keywords can use different approaches in order to consider these properties. In this algorithm, just their ability to consider these properties is enough to distinguish them.

Table 3 demonstrates the keywords' profile for the rubber block.

Material model	Stress-strain behavior	Strain range	Strain-rate	Failure	Damage
*MAT_001 (or *MAT_ELASTIC)					
Elastic	Only Constants	0 ÷ 100 %	NO	NO	NO
*MAT_006 (or *MAT_VISCOELASTIC)					
Viscoelastic	Only Constants	0 ÷ 100 %	YES	NO	NO
*MAT_007 (or *MAT_BLATZ-KO_RUBBER)					
Hyperelastic	Only Constants	0 ÷ 100 %	NO	NO	NO
*MAT_027 (or *MAT_MOONEY-RIVLIN_RUBBER)					
Hyperelastic	Uniaxial	0 ÷ 200 %	NO	NO	NO
*MAT_077 (or *MAT_ODGEN_RUBBER)					
Combination	Uniaxial	0 ÷ 700 %	YES	NO	NO
*MAT_181 (or *MAT_SIMPLIFIED_RUBBER/FOAM)					
Combination	Multiple	0 ÷ 500 %	YES	YES	YES

Table 3: Keywords' profiles for the rubber block, based on information from [4], [7] to [9]

4.2 Plastic/Metal

The keywords that are considered for modeling plastics and metals are listed in Table 4.

3 Digit numerical designation	Descriptive designation
*MAT_001	*MAT_ELASTIC
*MAT_003	*MAT_VISCOELASTIC
*MAT_024	*MAT_PIECEWISE_LINEAR_PLASTICITY
*MAT_081	*MAT_PLASTICITY_WITH_DAMAGE
*MAT_082	*MAT_PLASTICITY_WITH_DAMAGE_ORTHO
*MAT_123	*MAT_MODIFIED_PIECEWISE_LINEAR_PLASTICITY

Table 4: List of material keywords for plastics and metals

Table 5 indicates the minimum mechanical properties required to distinguish the keywords for plastics and metals. They are categorized as six items, with each one having its own value.

Items	Values	
Stress-strain behavior	Elastic	Elasto-Plastic
Curve model	Bilinear	Piecewise
Rate dependency	Cowper- Symonds	LCSS LCSR
Damage effect	Yes	No
Lattice structure	Isotropic	Orthotropic
Failure criteria	Basic mode	Comprehensive mode

Table 5: List of the mechanical properties for plastics and metal, based on information from [6], [10] to [13]

The mechanical properties of Table 5 can be explained as follows:

- **Curve model:** In some keywords, such as ***MAT_PLASTIC_KINEMATIC**, the strain-stress curve is simply described by two inclined lines for elastic and the plastic region. In this model, which is called **bilinear**, the Young's modulus and tangent modulus are used in order to define the slope of these two lines. In some other keywords, such as ***MAT_PIECEWISE_LINEAR_PLASTICITY**, the elastic area is still modeled by an inclined line, while the plastic region would be described by a combination of lines. These lines are intended to provide the best fit of the experimental test curve. They can be demonstrated by a set of points. This model is called "**piecewise**" here.
- **Rate dependency:** In the **Cowper-Symonds** model, the strain rate affects only the yield stress, σ_y and the Young's modulus, E remains unchanged [11]. **LCSS** (or load curve ID of stress vs. strain) is a particular option of LS-DYNA in which the plasticity data for each stress-strain curve is input directly as a family of curves. These curves can be demonstrated as a set of stress values against effective strain [6]. **LCSR** (or load curve ID of stress vs. strain rate) is another particular option of LS-DYNA, defining strain rate scaling effect on yield stress. This option permits the direct input of normalized yield stress vs. strain rate data to represent the rate dependency. The resultant interpolated scale factor is used to scale the plasticity curve [6].
- **Lattice structure** and **damage effect** are not examined in detail in this study. Material keywords can use different approaches in order to consider these mechanical properties. In this algorithm, just their ability to consider these properties is enough to distinguish them.
- **Failure criteria:** There are several methods for defining the desired failure criteria. They are studied under "phenomenological failure models" theories in finite element modeling. These models describe failure in materials in terms of mechanical variables such as stress, strain, temperature, strain rate, etc. If these variables reach a critical value, failure is expected in the material. As a general rule in this study, failures based on a plastic strain or minimum time step size are classified as **basic failure**

criteria, while the term **enhanced** (or **comprehensive**) **failure criteria** comprises failure modes regarding stress, strain or plastic thinning [13], [14] and [15].
 Table 6 demonstrates the keywords' profiles for the plastic/metal block.

Stress-strain behavior	Curve model	Rate dependency	Damage	Lattice structure	Failure criteria
*MAT_001 (or *MAT_ELASTIC)					
Elastic	Linear	NO	NO	Isotropic	Basic mode
*MAT_003 (or *MAT_PLASTIC_KINEMATIC)					
Elastoplastic	Bilinear	CP	NO	Isotropic	Basic mode
*MAT_024 (or *MAT_PIECEWISE_LINEAR_PLASTICITY)					
Elastoplastic	Piecewise	CP+LCSS+LCSR	NO	Isotropic	Basic mode
*MAT_081 (or *MAT_PLASTICITY_WITH_DAMAGE)					
Elastoplastic	Piecewise	CP+LCSS+LCSR	YES	Isotropic	Basic mode
*MAT_082 (or *MAT_PLASTICITY_WITH_DAMAGE_ORTHO)					
Elastoplastic	Piecewise	CP+LCSS+LCSR	YES	Orthotropic	Basic mode
*MAT_123 (or *MAT_MODIFIED_PIECEWISE_LINEAR_PLASTICITY)					
Elastoplastic	Piecewise	CP+LCSS+LCSR	NO	Isotropic	Comprehensive

Table 6: Keywords' profiles for the plastic/metal block, based on information from [6], [12] and [14]

4.3 Composite

The keywords that are considered for modeling composites are listed in Table 7 [16].

3 Digit numerical designation	Descriptive designation
*MAT_054	*MAT_ENHANCED_COMPOSITE_CHANG
*MAT_055	*MAT_ENHANCED_COMPOSITE_TSAI-WU
*MAT_058	*MAT_LAMINATED_COMPOSITE_FABRIC
*MAT_059	*MAT_COMPOSITE_FAILURE

Table 7: List of material keywords for Composites

Table 8 indicates the minimum mechanical properties required in order to distinguish the keywords for composites. They are categorized as three items, with each one having its own value.

Items	Values	
Element type	Shell element	Solid element
Laminate thickness	Thin lamina	Thick lamina
Lattice structure	Moderately anisotropic	Highly anisotropic

Table 8: List of the required mechanical properties for composites, based on information from [15], [17] to [20]

The mechanical properties shown above can be explained as follows:

- **Laminate thickness:** The thickness of composite materials can be measured by a local span, a , to thickness, h , ratio as a/h . In a reasonable consideration, if this ratio is less than 20, the lamina can be called a **thick** lamina, otherwise it is called a **thin** lamina [21].

- **Lattice structure:** The anisotropy condition is important in terms of both stiffness and strength. It is possible to define the anisotropic moduli ratio as:

$$\frac{E_2}{E_1}$$

Where E_1 is the usual modulus in the fiber direction and E_2 is the transversal modulus. This anisotropic moduli ratio varies as:

$$0 \leq \frac{E_2}{E_1} \leq 1$$

The 0 limit is the plain strain case and the 1 limit is that of isotropy. It should be noted that even the value 0.9 would have the isotropic case, since it has close and reasonable representation. The conjugate value 0.1 would have the plain strain form as a close and reasonable representation as well [18]. Therefore, a value between them, for example 0.5, can be considered as the **moderate anisotropy** range, while the values near 0 can be expressed as **highly anisotropic**.

Table 9 demonstrates the keywords' profiles for the composite block.

Element type	Laminate thickness	Lattice structure
*MAT_054 (or *MAT_ENHANCED_COMPOSITE_CHANG)		
Shell element	Thin lamina	Highly anisotropic
*MAT_055 (or *MAT_ENHANCED_COMPOSITE_TSAI-WU)		
Shell element	Thin lamina	Moderately anisotropic
*MAT_058 (or *MAT_LAMINATED_COMPOSITE_FABRIC)		
Shell element	Thick lamina	Highly anisotropic
*MAT_059 (or *MAT_COMPOSITE_FAILURE)		
Solid element	Thin lamina	Highly anisotropic

Table 9: Keywords' profiles for the composite block, based on information from [16] to [20]

4.4 Foam

The keywords that are considered for modeling foams are listed in Table 10.

3 Digit numerical designation	Descriptive designation
*MAT_057	*MAT_LOW_DENSITY_FOAM
*MAT_062	*MAT_VISCOUS_FOAM
*MAT_063	*MAT_CRUSHABLE_FOAM
*MAT_083	*MAT_FU_CHANG_FOAM
*MAT_163	*MAT_MODIFIED_CRUSHABLE_FOAM

Table 10: List of material keywords for foams

The algorithm for foams has a different structure than other material types. Based on different behavior of foams as regards compression, tension and shear, foam materials can be classified into five classes in explicit codes [22]. It is possible to use these classes to determine the proper material card in LS-DYNA [23]. The mechanical properties and the relevant LS-DYNA keywords of these classes are shown in Table 11.

Classes of foam material laws	Mechanical properties			Relevant LS-DYNA keyword	
	Reversibility status	Mass concentration	Foam cell structure		
Soft polyurethane	Reversible	30 – 60 (g/l)	Open cell	*MAT_057	*MAT_083
Comfort foam	Reversible	60 – 70 (g/l)	Open cell	*MAT_062	*MAT_083
Reversible energy-absorbing polyurethane	Reversible	50 – 110 (g/l)	Closed cell	*MAT_057	
Irreversible energy-absorbing polyurethane	Irreversible	50 – 110 (g/l)	Closed cell	*MAT_063	*MAT_163
Expanded particle foam	Reversible	20 – 200 (g/l)	Closed cell	*MAT_083	

Table 11: Mechanical properties and relevant LS-DYNA keywords of the foam classes, based on information from [22] to [24]

In terms of material keywords, their profiles are evaluated using the same approach as for other material types. Table 12 demonstrates the keywords' profiles for the foam block.

Reversibility	Cell construction	Mass concentration	Damage	Failure	Strain rate
*MAT_057 (or *MAT_LOW_DENSITY_FOAM)					
Reversible	Open and closed cell	30 – 110 (g/l)	NO	YES	YES
*MAT_062 (or *MAT_VISCOUS_FOAM)					
Reversible	Open cell	60 – 70 (g/l)	NO	NO	YES
*MAT_063 (or *MAT_CRUSHABLE_FOAM)					
Irreversible	Closed cell	50 – 110 (g/l)	NO	YES	NO
*MAT_083 (or *MAT_FU_CHANG_FOAM)					
Reversible	Closed cell	20 – 200 (g/l)	YES	YES	YES
*MAT_163 (or *MAT_MODIFIED_CRUSHABLE_FOAM)					
Irreversible	Closed cell	50 – 110 (g/l)	NO	YES	YES

Table 12: Keywords' profiles for the foam block, based on information from [22] and [25]

The algorithm for foam consists of two parts. The first part again comprises questions, tags, evaluation and sorting for determining the most proper classes of foam. Then, the user should decide to either use one of these classes or simply continue on with the questions. In the second part, the data obtained from the previous part will be added to new data in order to report the most proper material keyword.

5 Results of the Program

An example is provided for the purpose of investigating the output of the developed tool. Let us consider a user who searches for an appropriate material card merely by knowing the characteristics provided in Table 13.

Material type	Stress-strain behavior	Damage effect	Lattice structure	Failure criteria	Mass density	Poisson ratio
Metal	Elastoplastic	YES	Isotropic	Basic mode	2730 $\frac{kg}{m^3}$	0.33

Table 13: An example about the desired mechanical characteristics

When the program runs, once the user answers the respective questions, the software reports a sorted list as the first output (Figure 3). The most proper keyword is the first one on the list. It also provides the accuracy percentage of the most proper and second most proper suggested material keyword.


```

Command Window
Following are the suggested results according to their priorities
'MAT081'
'MAT082'
'MAT024'
'MAT123'
'MAT003'

MAT081 is your most proper material card. The accuracy percentage of MAT081 is:
Percentage_MAT081 =
                100

MAT082 is the second proper material card. The accuracy percentage of MAT082 is:
Percentage_MAT082 =
                9.4444e+001
    
```

Fig.3: First result; suggestion list of *MATs

As can be seen in Fig.3, *MAT_081 is the most proper material keyword with 100% accuracy. The reason for this 100% accuracy is that the selected answers are exactly the same as those provided in the profile of this keyword.

Next, to calibrate the material card, the user can simply enter the numbers that are available, which are density and Poisson's ratio in this example. Finally, the software outputs the full list of available material cards from the database along with some extra information, such as their sources (Figure 4). Other values are blacked out to protect the right of the material card's producer.

```

The following Material Cards obtained from our DataBase are perfectly fitted with your answers:
Card Number #1
Columns 1 through 9
2.7300e+003 ██████████ 3.3000e-001 ██████████ 0 ██████████ 0 ██████████ 0
Columns 10 through 12
██████████ 0 0
'Title: block front cladding'
'Source: LSTC.214 SHELL BARRIER.120203 V3.0'
    
```

Fig.4: Second result; calibration of the material card

6 Summary and Discussion

A beta version of a smart material database for LS-DYNA has been developed, which can search for the most proper material keyword with regard to the user's application.

The user has to answer some questions in order to provide enough information for the algorithm. The program attempts to ask questions about minimum required properties in order to specify the material cards. There are several similar characteristics in each sub-unit which the user is not asked about. For example, almost all of the material keywords in the plastic and metal block are strain rate sensitive, therefore, the users are not asked to either use or not use this property. It should be noted that these questions are based on differences and similarities of the material keywords. Thus, they will not necessarily remain the same, if new material keywords are added to the algorithm. The capability for further development has been considered within all intermediate steps required to obtain the algorithm, so that the final structure can be updated with minimum required effort.

The developed algorithm is implemented in MATLAB and coupled with a database in Microsoft EXCEL in order to examine its reliability. For the next generations of this tool, other programming languages will be used, which are much more powerful for IT applications.

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