

Implementation of Peridynamic Theory to LS-DYNA for Prediction of Crack Propagation in a Composite Lamina

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ABSTRACT

Composite materials are commonly used in aerospace and automotive industry due to their high specific strengths. However, damage tolerance assessment of composite parts is done experimentally since there is no single criterion that is capable of predicting all of the failure modes. Thus numerical estimation of crack propagation in composite structures has been considered as an important research topic in academia and especially in aerospace industry. Composite materials are sensitive to stress raisers such as fastener holes and defects. Peridynamics is a nonlocal theory that overcomes the limitations of classical continuum mechanics caused by discontinuities in the structure like cracks and voids. By this aspect, Peridynamic theory differs from conventional finite element analysis. This paper focuses on crack propagation in fiber reinforced composite structures by implementing the bond-based Peridynamic Theory to LS-DYNA software. Peridynamic models of a composite plate with circular cutout are generated by implementing Peridynamic theory into LS-DYNA. Crack propagation under tensile load is investigated. To prepare the Peridynamic model, truss elements and mass elements are used in LS-DYNA. Numerical results are verified by comparing the experimental and numerical results from literature.

Keywords: Peridynamics, crack propagation, LS-DYNA, composite structures, truss elements, mass elements

1 INTRODUCTION

Composite structures are highly demanded in automotive and aerospace industries. Mechanical properties of composites can be tailored so that lightweight designs can be achieved. Generally, mechanical properties of a composite laminate depend on fiber orientations of individual plies. Because of this inhomogeneity, the failure mechanisms of composites are more complicated than metallic structures. Main failure modes can be divided into two subcategories as interlaminar failures (delamination) and intralaminar failures (matrix and fiber breakage). Final failure of a composite part occurs as a result of interactions of those failure modes. Thus failure prediction of composite structures are rather complicated than the metallic structures. This topic has been studied by researchers for many years and there have been various numerical methods to estimate failure load and failure type of composites. However, none of these techniques are confident enough. Thus in design phase, substantiation of load carrying capacity of the structures are done with experiments. Peridynamic theory which is a nonlocal theory of continuum mechanics is first proposed by Silling [1]. In classical continuum mechanics, a material point interacts only with its immediate neighbors. In Peridynamic Theory, a material point interacts with other material points within a horizon with a finite radius. This theory is a strong candidate for prediction of both crack initiation and propagation in composite materials [2-3]. In classical continuum mechanics, spatial derivatives of stress field are needed to evaluate the equation of motion. Eq. (1) and Eq. (2) gives the equation of motion for Classical continuum mechanics and Peridynamic Theory respectively [3].

$$-\rho(\mathbf{x})\ddot{\mathbf{u}} + \nabla \cdot \boldsymbol{\sigma} + \mathbf{b}(\mathbf{x}, t) = 0 \quad (1)$$

$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x}, t) = \int_H \mathbf{f}(\mathbf{u}' - \mathbf{u}, \mathbf{x}' - \mathbf{x}) dH + \mathbf{b}(\mathbf{x}, t) \quad (2)$$

In classical continuum mechanics Eq. (1) is not valid at the points of discontinuity such as cracks and voids and results in infinite stresses. In Peridynamic equation of motion, instead of spatial derivatives, integrals of interaction force densities between material points are needed. Since, Peridynamic equation of motion is an integro-differential equation; it can be evaluated at singularities. Peridynamic theory also does not need external failure criteria. Failure parameter (μ) is already implemented in its constitutive relation as given in Eq. (3) and Eq. (4).

$$\mathbf{f}(\boldsymbol{\eta}, \boldsymbol{\xi}) = \mu(\mathbf{x}' - \mathbf{x}, t) \frac{\boldsymbol{\xi} + \boldsymbol{\eta}}{|\boldsymbol{\xi} + \boldsymbol{\eta}|} c s \quad (3)$$

$$\mu(\mathbf{x}' - \mathbf{x}, t) = \begin{cases} 1 & , s(\mathbf{x}' - \mathbf{x}, t) < s_0 \\ 0 & , s(\mathbf{x}' - \mathbf{x}, t) > s_0 \end{cases} \quad (4)$$

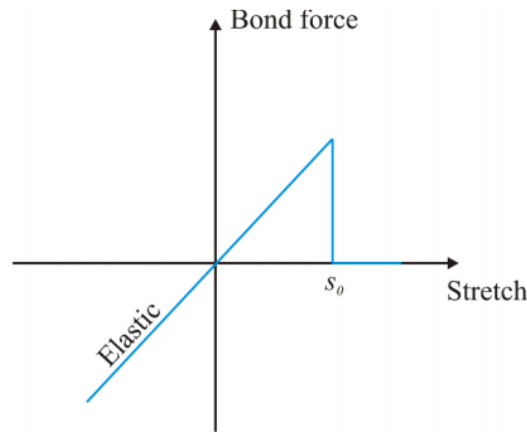


Fig.1: Constitutive relation for Peridynamic bonds [4]

As shown in Fig.1 when stretch between two material points reach a critical value, the pair-wise interaction force between material points reduce to zero.

In this study Peridynamic Theory is implemented to LS DYNA software using mass and truss elements. A Peridynamic model of a single ply with a circular cutout is generated under velocity boundary condition. Peridynamic model is validated by comparing elastic displacements in the middle of the plate with LS-DYNA finite element solution. Then failure modeling is implemented and failure initiation and propagation in 0° , 45° , and 90° plies.

2 PERIDYNAMIC MODELING OF A COMPOSITE LAMINA

In Peridynamic, pairwise interaction forces between material points are represented with Peridynamic bonds. Peridynamic model of lamina consist two types of bonds as illustrated in Fig. 2. Matrix bonds are oriented in all directions and they represent mechanical properties of the resin material. Fiber bonds which represent mechanical properties of fibers are oriented only in the fiber direction.

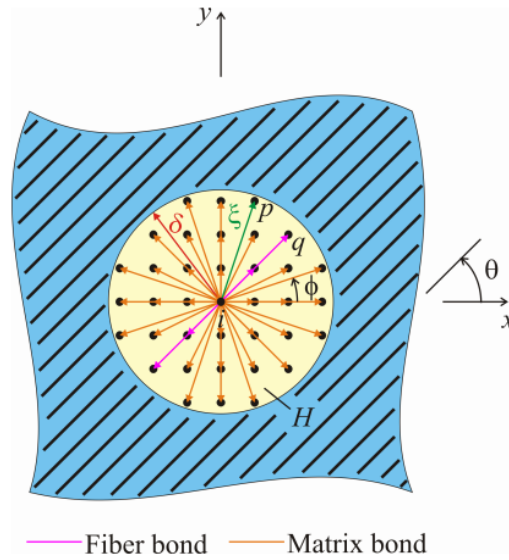


Fig.2: Fiber and matrix bonds [4]

Peridynamic bond constants for fiber (c_f) and matrix bonds (c_m) can be derived by equating strain energy density in continuum mechanics and Peridynamic Theory under isotropic expansion loading by using in Eq. (5) and Eq. (6) [4].

$$c_f = \frac{Q_{11} - Q_{22}}{\frac{1}{2} \sum_{q=1}^Q \xi_{qi} V_q} \quad (5)$$

$$c_m = \frac{24Q_{12}}{\pi t \delta^3} = \frac{8Q_{22}}{\pi t \delta^3} \quad (6)$$

where

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}, \quad Q_{66} = G_{12}, \quad \nu_{21} = \nu_{21} \frac{E_{11}}{E_{22}}$$

In Eqs. (5) and (6), ξ_{qi} stands for the initial length of the bond between material points q and i , V_q is used for defining the volume of cubic element that includes the material point q , δ defines the horizon of the material point, and t is the thickness of the plate.

Similar to the derivation of bond constants, the critical stretch values can be obtained in terms of the elastic constants and the critical energy release rate (G) [5]. Critical stretch value is taken as 0.04472 [3].

3 IMPLEMENTATION OF THE PERIDYNAMIC THEORY TO LS-DYNA

Peridynamic model of a lamina can be generated with mass elements and truss elements. Mass elements represent material points whereas truss elements represent interaction forces between material points [6]. These material points interact with each other within the spherical zone. For a lamina, horizon can be defined by a circular area with a specified radius called horizon, δ . According to Macek and Silling [6], the length of the horizon can be taken as $\delta = 3.015\Delta x$ which is an efficient horizon length for macro scale problems. Implementation of Peridynamic Theory to LS-DYNA is achieved using the following steps. First of all, Peridynamic bonds with six different lengths are defined as shown in Fig. 3.

Peridynamic model of composite lamina with a circular cutout consist of two groups of bonds which are categorized as matrix and fiber bonds. Bond constant calculations are performed for both of the bond categories using Eq. (5) and Eq. (6). Mechanical properties of truss elements between material points can be derived by equating force in Peridynamic bonds to force in truss elements. As stated by Macek and Silling, elastic modulus and cross sectional area for truss elements are defined by using Eq.(7) and Eq. (8) respectively [6].

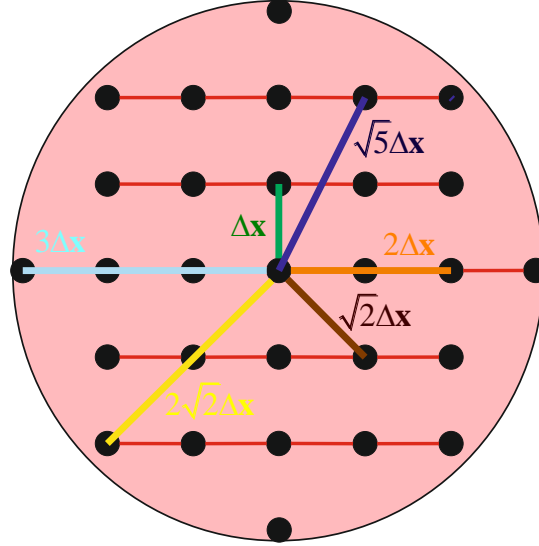


Fig.3: Six different bond length inside horizon for a material point included in Fiber reinforced lamina

$$E = c(V_i V_j)^{2/3} \quad (7)$$

$$A = (V_j V_i)^{1/3} \quad (8)$$

Here, V_i and V_j stands for the volumes of the material points which are connected through their center of gravity. In this paper, volume of material points is calculated using $\Delta x = \Delta y = \Delta z$. Then, volumes can be written as $V_i = V_j = \Delta x^3$. Also, c stands for bond constant for truss elements. Substituting volumes into Eqs. (7) and (8) the mechanical properties of truss elements are obtained as follows:

$$E = c \Delta x^4 \quad (9)$$

$$A = \Delta x^2 \quad (10)$$

4 GENERATING FINITE ELEMENT MODEL OF A COMPOSITE LAMINA USING PERIDYNAMIC THEORY

HYPERMESH and MATLAB are used to generate Peridynamic model of a lamina. While HYPERMESH is used for the purpose of node creating material points and LS-DYNA cards, MATLAB is used for defining truss elements between the material points. Peridynamic bonds and material points for a lamina are shown in Fig. 4. Here, the incremental distance in x and y directions assumed to be same. ($\Delta x = \Delta y$)

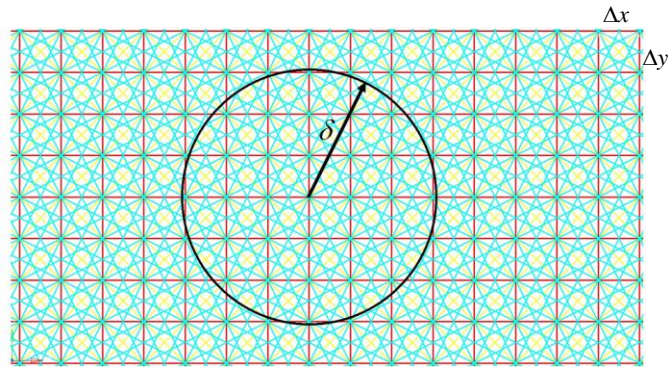


Fig.4: Truss elements(Peridynamic bonds) and material points of lamina

Within the scope of this paper, fiber reinforced lamina with a circular cutout under in-plane loading is considered as test subject. To understand how the fiber orientation angle affect deformation distribution and crack propagation under certain loading conditions, lamina is modeled with several fiber orientation angles as shown in Fig. 5. An initial velocity loading condition with ($V_0 = 2.75 \text{ mm/s}$) in $\pm y$ direction is used as illustrated in Fig. 5, several composite lamina models are created. Both conventional FEA and Peridynamic Theory implemented FEA (PTIFEA) is conducted. It can be seen that Peridynamic Theory requires much more computational cost than the conventional FEA. Dimensions of the plate in Fig. 5. are $50 \times 50 \text{ mm}$. When the grid size is set to $\Delta x = 2 \text{ mm}$, PTIFEA has nearly ten times greater number of elements than conventional FEA.

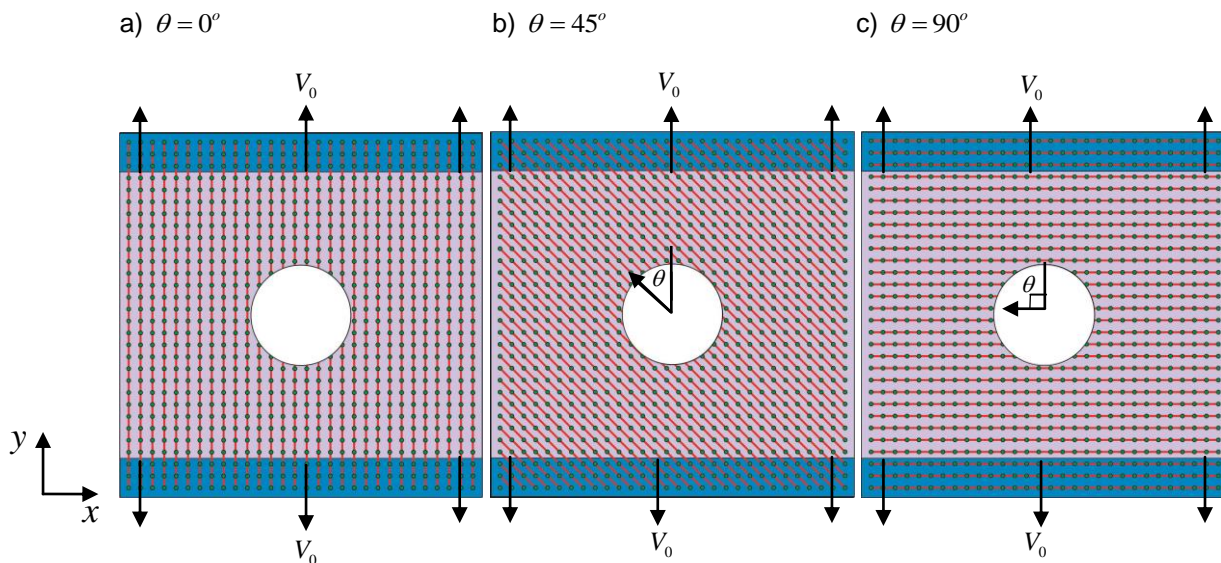


Fig.5: Fiber orientation with the angle of a) 0° , b) 45° , c) 90° for a composite lamina with a circular cutout

Mechanical properties of truss elements are defined using ***MAT_ELASTIC** card. This card does not include failure criteria. ***MAT_PLASTIC_KINEMATIC** is used for defining the failure criteria for truss elements. Kinematic hardening effects are ignored by setting the parameter $\beta=1$ [7]. Within this material card, suitable failure criteria can be defined by setting the parameter ϵ_s (Failure strain for eroding elements) which is taken as 0.04472 [3]. When failure strain is reached, truss elements called Peridynamic bonds are deleted.

5 ANALYSIS

Implementation of the Peridynamic Theory to LS-DYNA is successfully completed using an explicit solution scheme. Material properties of the ply are given in Table 1 using the reference [3]. Boundary conditions are shown in Fig.6. Peridynamic simulations of the lamina with 0° , 45° , and 90° orientations with the loading conditions as shown in Fig.5 are performed using explicit solver. In order to validate PTIFEA into LS-DYNA, each case is also solved using FEA.

Table 1: Mechanical properties and Boundary conditions [3]

Description	Value
Velocity boundary Condition	2.75 mm/s
Elastic modulus in Fiber Direction (E_{11})	159.96 GPa
Elastic modulus in transverse direction (E_{22})	8.96 GPa
In Plane Poisson's ratio (ν_{12})	1/3
In Plane shear modulus (G_{12})	3.0054 GPa

5.1 Verification of Peridynamic Theory with FEA

Using `*DEFINE_CURVE` and `*BOUNDARY_PRESCRIBED_MOTION_SET_ID` card the velocity boundary condition is applied. The results for the displacement components are obtained in x and y directions along the pre-defined paths as shown in Fig. 6a. and Fig. 6b. respectively. Additionally, diameter of the hole is set at $D = 10 \text{ mm}$. Note that all the results presented herein are taken at 10 ms.

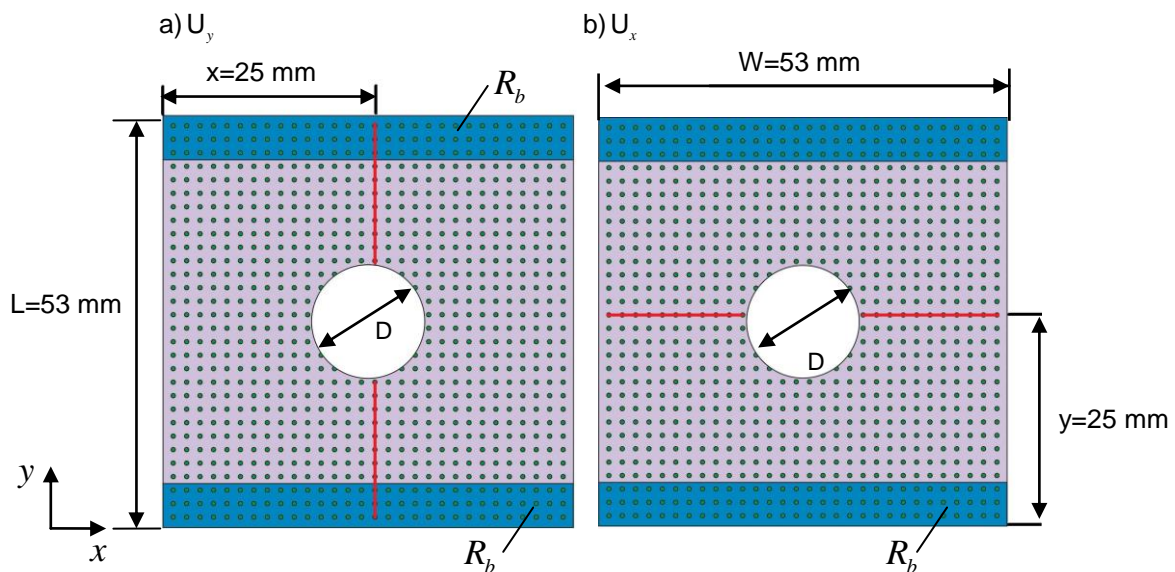


Fig.6: Paths that results are taken and boundary regions for loading (R_b)

Fig. 7- 9. show the comparison between FEA and PTIFEA for U_x and U_y displacements for different fiber orientation angles. The results are in good agreement.

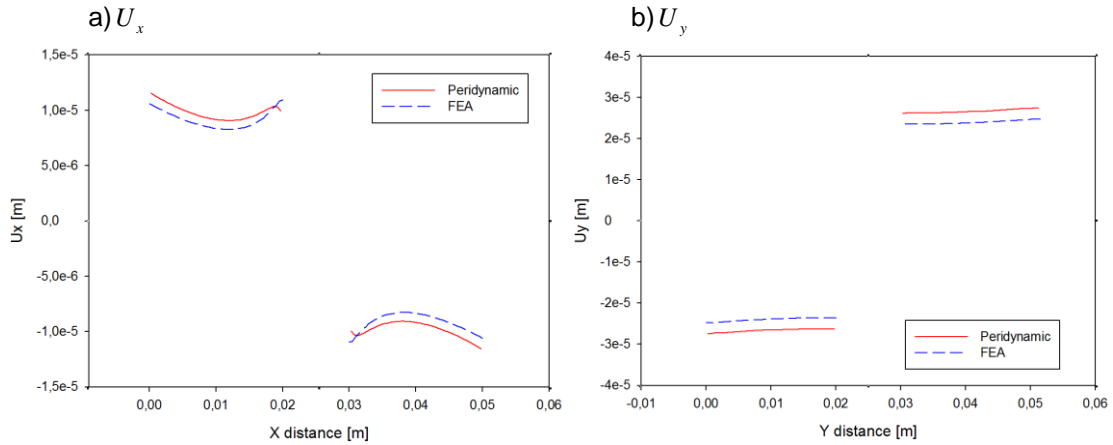


Fig.7: a) U_x and b) U_y displacement graphs for lamina with 0° fiber orientation(for paths see Fig.(6))

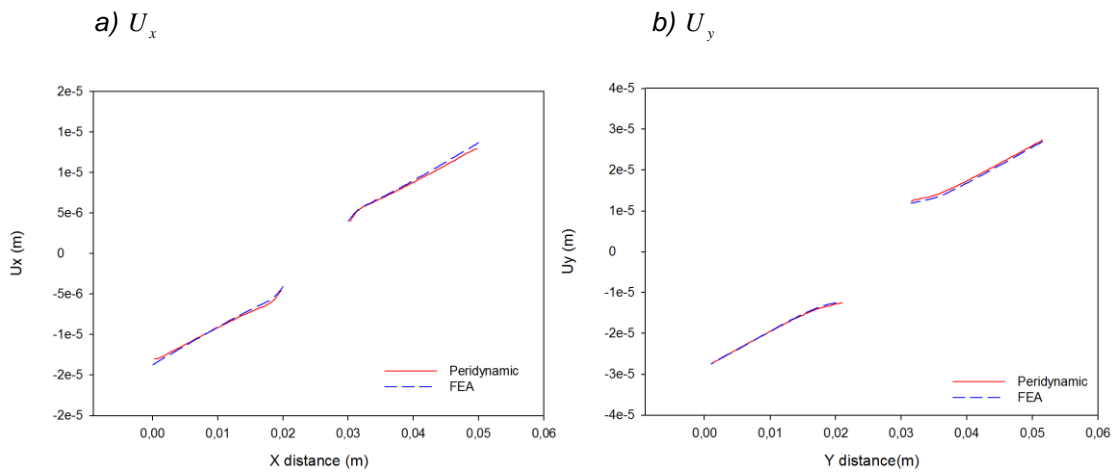


Fig.8: a) U_x and b) U_y displacement graphs for lamina with 45° fiber orientation(for paths see Fig.(6))

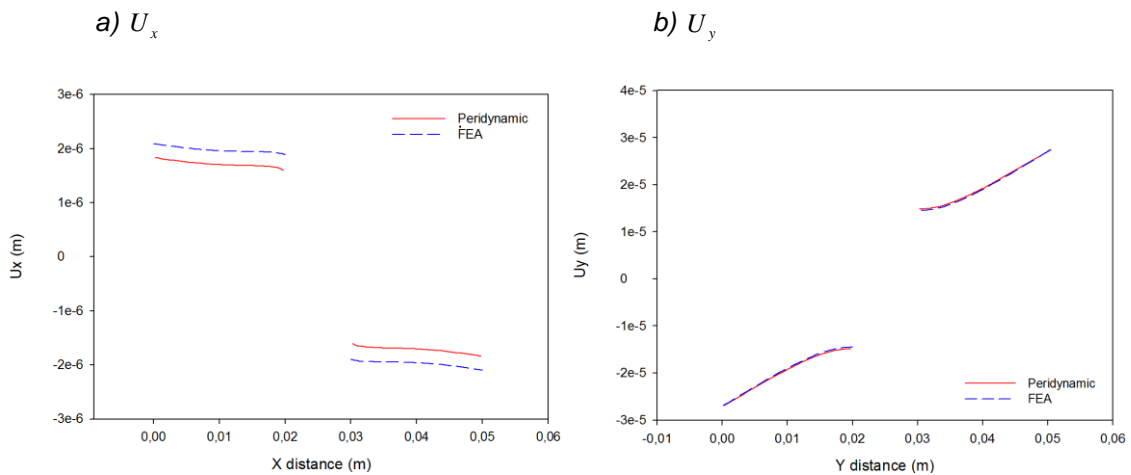


Fig.9: a) U_x and b) U_y displacement graphs for lamina with 90° fiber orientation(for paths see Fig.(6))

5.2 Crack propagation in a lamina with a circular cutout

Composite structures with only one layer is not mechanically stable structures. Therefore, the usage of a single lamina is not common in industrial applications. Fiber reinforced lamina with a circular cutout under in-plane loading is simulated using PTIFEA. Three different fiber orientation angles are used to understand how cracks initiate and propagate. Crack initiation and propagation is clearly captured for different fiber orientation angles. As illustrated in Fig.10-12., crack propagates in fiber direction for fiber angles $\theta = 0, 45, 90$ respectively. This kind of failure mode is also known as splitting fiber mode [2].

6 CONCLUSION

This paper aims to implement a relatively new theory called Peridynamic Theory into LS-DYNA. In this paper it is shown that failure initiation and propagation in a lamina can be modeled by PTIFEA. Implementation process is validated by comparing elastic displacements in conventional FEA and PTIFEA solutions.

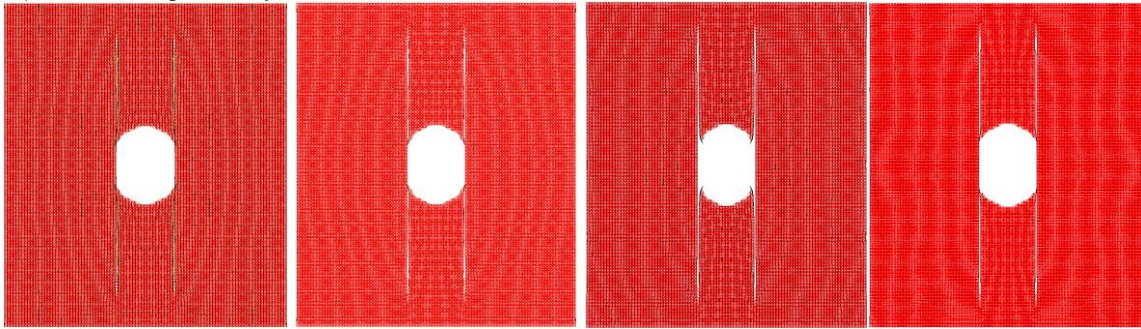
7 SUMMARY

Finite Element Implementation of Peridynamic Theory into LS-DYNA software is achieved in this study. Peridynamic Theory is a nonlocal theory of continuum mechanics which is suitable for modeling of crack initiation and propagation. This theory is implemented in LS-DYNA using mass and truss elements. Peridynamic model of a lamina with a circular cutout under velocity boundary condition is modeled and validated by comparing with the results from conventional FEA approach. PTIFEA results are in agreement with the ones in literature.

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a) Deformed geometry



b) Displacement Contour in y direction

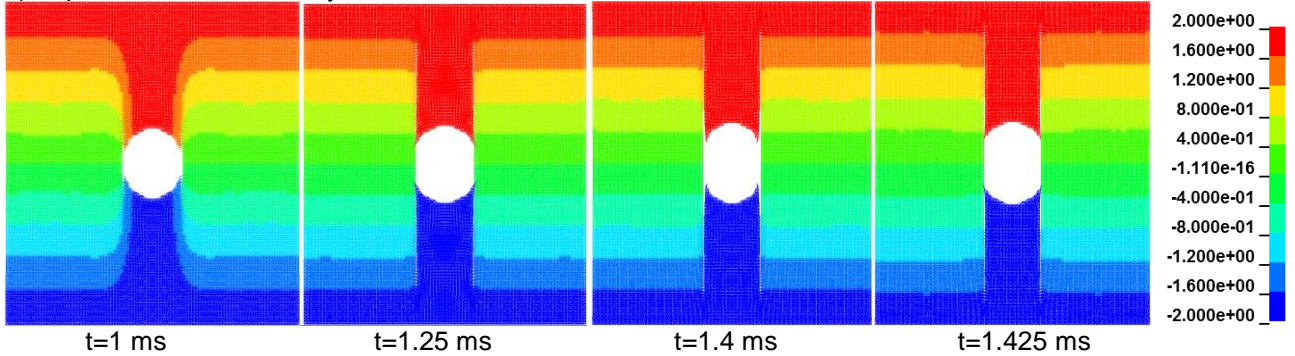
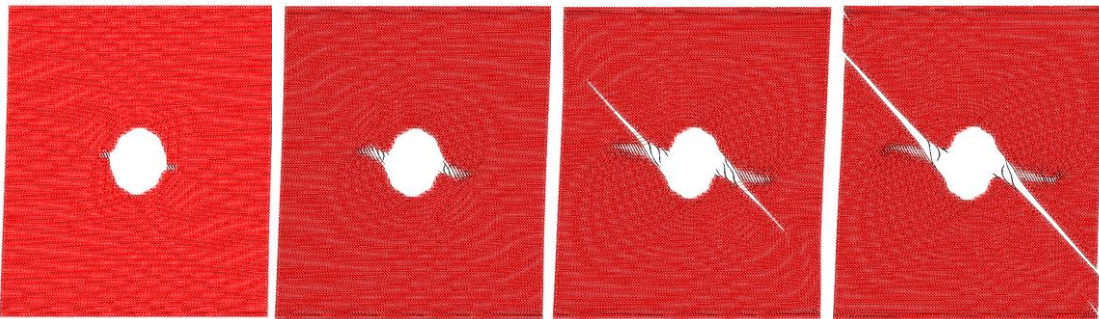


Fig.10: Crack propagation for the composite lamina with 0° fiber orientation a) deformed geometry b) displacement contour in y direction (fringe level in mm)

a) Deformed geometry



b) Displacement contour in y direction

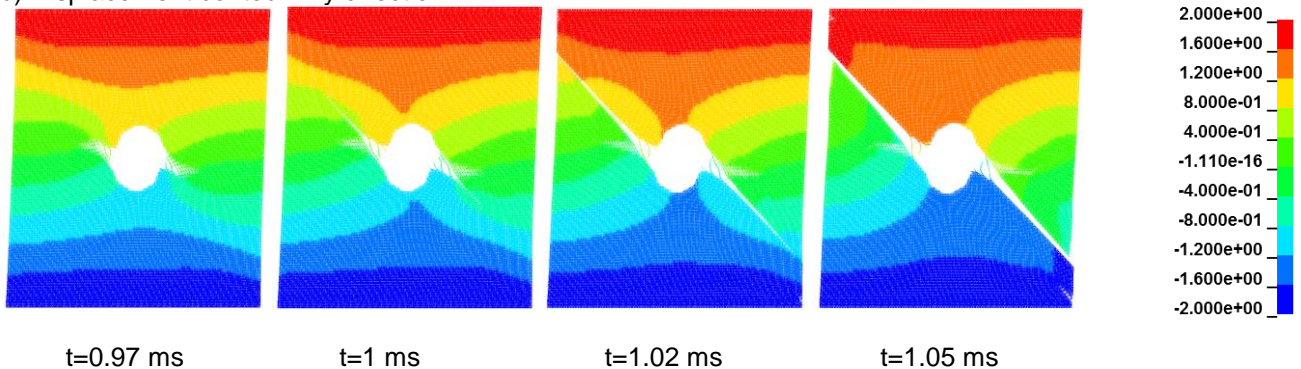


Fig.11: Crack propagation for the composite lamina with 45° fiber orientation a) deformed geometry b) displacement contour in y direction (fringe level in mm)

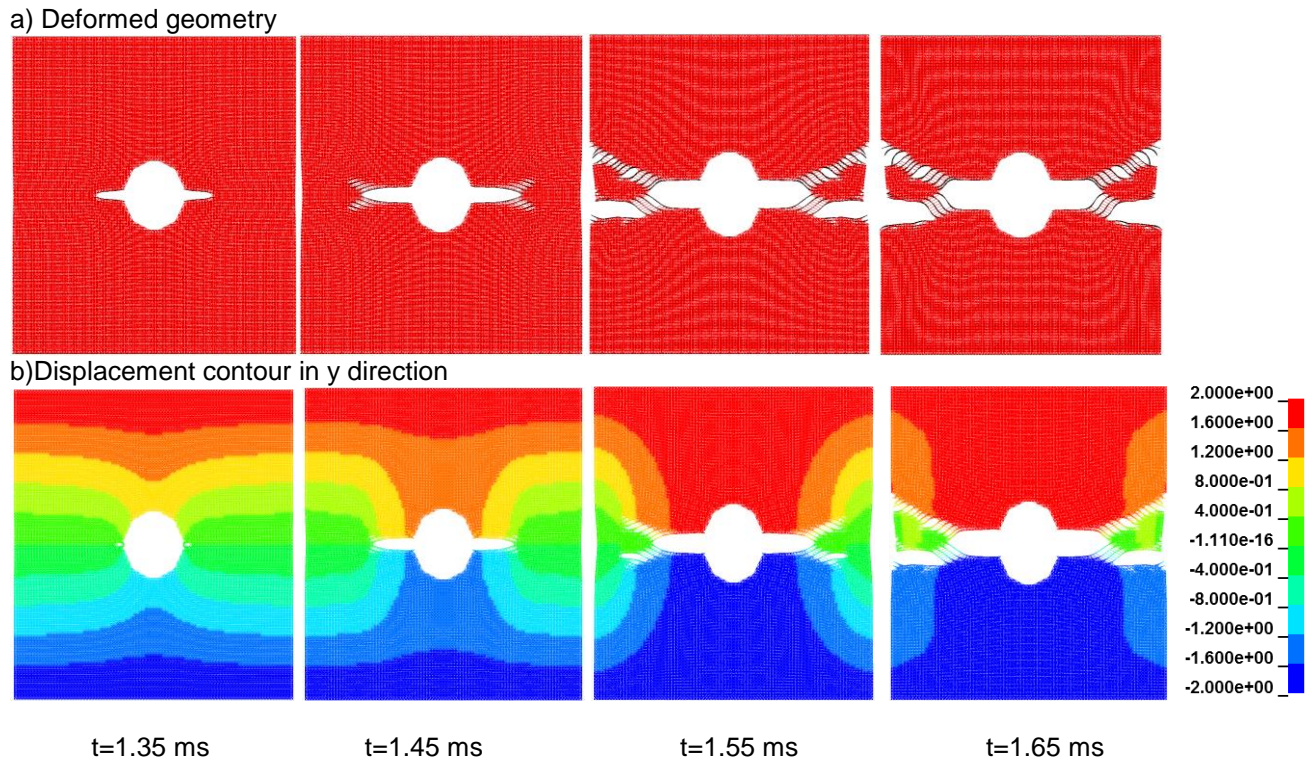


Fig.12: Crack propagation for the composite lamina with 90° fiber orientation a) deformed geometry b) displacement contour in y direction (fringe level in mm)

9 LITERATURE

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