NEW NONLINEAR HIGHER ORDER SHEAR DEFORMATION SHELL ELEMENT FOR METAL FORMING AND CRASHWORTHINESS ANALYSIS: PART II. PERFORMANCE VALIDATION THROUGH STANDARD TESTS

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ABSTRACT

This work presents the results from a set of verification shell problems used to assess the performance of the higher order shear deformation shell elements formulated in part I of the present study. The developed element has been added to the element library of the nonlinear dynamic explicit finite element code DYNA3D. Several standard verification test problems are performed using the code DYNA3D with the developed shell element. Results are presented for different test problems and are compared with experiments and results from other existing shell elements. The good overall performance builds confidence in the formulation and implementation of the proposed higher order shear deformable element. The superior advantage of the developed element is evident in one of the examples presented for representation of plastic flow through the thickness in isotropic materials. The element can be used in crash and metal forming simulations in local areas of high transverse shear stresses. Local areas of crack in crash applications and splitting in metal forming applications can be modeled more accurately with the developed shell element.

Key words: nonlinear higher order shear deformation shell elements, explicit finite element analysis, shell verification test problems, shell element performance

INTRODUCTION

It is well known that to ensure its validity, reliability, and accuracy any newly formulated finite element (FE) has to be put through an extensive testing process. In this manner the theoretical formulation of the new element will be checked and, more importantly, errors in its FE code implementation could be found and corrected. Through different test problems the scope of applicability of the newly developed element could be set, its computational efficiency and accuracy could be assessed. Good results from the standard test problems can

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build confidence in the formulation and implementation of the element. All this has been very well realized by scientists and FE software developers, and therefore, many element validation tests have been developed and published. Verification problems for shell elements can be found in almost all publication on FE shells. Some of them have proven very efficient in assessing shell elements performance and are described in the works of Morino et al. [1], Hughes and Liu [2], Belytschko et al. [3, 4], MacNeal and Harder [5], and MacNeal [6] to mention just a few.

To assess the performance of the higher order shear deformable shell element formulated in part I of the present work a standard set of static and dynamic verification test problems was chosen. Tests were performed on a single element and on element assemblages of regular and irregular shapes with flat initial geometry, and single and double curvatures. Both geometrically linear and nonlinear analyses were performed, and elastic or elastic-plastic materials were used. All tests showed very good accuracy in the shell behavior prediction as compared with experimental results and results from existing shell elements. The higher order shear deformable element supremacy over first order shear deformable shells is illustrated through the transverse shear distribution prediction. Although computationally heavier as compared to the first order shear deformable shell formulations, the implementation of the present element fits quite well in a general explicit FE code, providing both efficient and accurate analysis. The CPU time and the critical integration time step are presented for several test problems and compared to the ones obtained from the default shell element with selectively reduced integration in DYNA3D.

VERIFICATION TESTS: DESCRIPTION AND RESULTS

A set of 6 verification problems was selected from the multitude of published shell problems. The aim was to check the performance with different geometric configurations, loading, boundary conditions, and materials, which could occur often in engineering practice. Problem 1 involves a single flat element under membrane loading. Problem 2 is the well-known patch test with prescribed loading. Problem 3 represents a thick beam for which results for displacements and natural frequencies are presented and compared for linear and nonlinear behavior. Problem 4 is a square plate solved with elastic and elastic-plastic material, and with regular and irregular meshes, while problems 5 and 6 represent shells with single and double curvatures. In addition, a problem is considered for which the transverse shear stresses are the dominant stresses. The load and boundary conditions are chosen to cause plastic deformation and flow. Predictions of the developed shell and the standard default shell in DYNA3D are presented and compared. In what follows description and results of all the test verification problems are presented.

***Problem 1** –Single Element Membrane Test*

A single element is loaded with uniformly distributed normal and shear load. The geometry, loading, boundary conditions, and theoretical results are presented on fig. 1. The results from the present analysis are shown and compared to the exact solution in table 1. As evident from the presented results, the developed higher order shell element passes this test verification problem.

***Problem 2** – The Patch Test*

This is the well-known patch test. A prescribed loading is used in the test as opposed to prescribed displacements because the current implementation does not allow prescribed values for the higher order terms in the velocity field. The test is described in fig. 2 and the results are presented in table 2. The results for the stresses and strains in the table are the same for all elements in the patch assemblage. Table 2 shows the excellent performance of the developed element in this test.

Problem 3 *– Impulsively Loaded Cantilever Beam*

This is Example 5.1 from Belytschko et al. [3]. A thick beam is subjected to suddenly applied lateral pressure. The problem and the FE mesh are shown in fig. 3 and table 3 gives results for the maximum deflection and the period of oscillation for two values of the loading: $p = 0.01$ *psi* corresponding to a linear response, and $p = 2.85$ *psi* corresponding to a nonlinear response. Both linear and nonlinear results with the higher order element are in a very good agreement with the reference analytic and numerical results.

Problem 4 *– Simply Supported Square Plate*

This problem was described by Belytschko et al. [3]. It involves an impulsively loaded square plate analyses with both elastic and elastic-plastic materials. Due to symmetry only one quarter of the plate was analyzed. To check how irregularities in the FE mesh will affect the solution the problem was also solved with irregular mesh geometry for the elastic material. The problem is described in fig. 4. and results from the regular FE mesh solution are presented in fig. 5. Fig. 6 shows results from the irregular mesh solution compared to the regular mesh results. As seen the present results agree very well with the published numerical results and the irregularity of the FE mesh affects the solution negligibly.

Problem 5 *– Cylindrical Panel*

This is an impulsively loaded cylindrical panel shown in fig. 7. The explosive loading is modeled by prescribing initial inward radial velocity of the nodes in the loaded region. This problem tests the ability of the elements to model single curvature shells. It was modeled using a FE mesh with 16 elements along the cylinder side and 12 elements along the circumference. Due to the symmetry only a half of the cylinder was modeled prescribing symmetric boundary conditions along the symmetry line. Results are compared with experimental results published by Morino et al. [1] and other FE solution by Belytschko et al. [4] using a 12×32 element FE mesh. The results are presented on figs. 8 and 9 and show the excellent performance of the higher order shear deformable shell element.

Problem 6 *– Spherical Cap*

The spherical cap shown in fig. 10 is subjected to suddenly applied pressure. This problem tests the element performance for doubly curved shells. Both elastic and elastic-plastic materials a re considered and results are presented on figs. 11 and 12 respectively. At both elastic and elastic-plastic regimes the behavior pattern is captured well by the developed element.

Problem 7 *– Through-Thickness Stress Distribution of a Short Beam*

To illustrate the ability of the third order shell element to correctly predict the through thickness stress distribution, a simple problem is solved. A short cantilever beam of elastic-perfectly-plastic material is loaded with two shearing forces. The magnitude and position of the forces is selected to produce plasticity only in a central section of the shell thickness in the investigated element (element 5 on fig. 13) due to the high transverse shear stresses. Furthermore, throughout the whole length of the beam there is no section, which is completely plasticized. Fig. 13 presents the model, FE mesh and material data. The transverse shear stress distribution in the higher order element and in a shell element based on a first order shear deformation theory (FOSDT) is presented on fig. 14. As seen from this figure due to the incorrect distribution of the shear stresses the FOSDT element almost completely interchanges the elastic and plastic zones throughout the shell thickness. On the other hand, the results from the higher order element look much more realistic and reliable.

Solution CPU Time

Due to its slightly heavier code implementation and mainly to its considerably smaller critical time step, the computational efficiency of the third order shell element is not as good as that of the first order shell elements. The solution CPU time on a SGI Origin 2000 supercomputer and the critical time step of the explicit time integration scheme are presented in table 4. As seen from the table the solution times for the simple models herein investigated are very reasonable, which means that the proposed element can be successfully used in much more complicated models involving many shell elements and longer simulation duration.

CONCLUSIONS

The test problems herein described were used to assess the performance of the higher order shear deformable shell element formulated in part I of the present work and implemented into the explicit FE code DYNA3D. Several standard test verification problems are considered to evaluate the response of the developed shell with respect to linear, geometric nonlinear, elastic, and plastic analysis. As seen from the reported results, the accuracy of the newly formulated element is very good and it can be used in various problems involving shell structures. It is evident that the third order element has better accuracy potentials compared to the first order shear deformation elements, especially if it is used in problems involving nonlinear thickness strain distribution like in plasticity of isotropic shells, or layered composite or sandwich shell problems.

ACKNOWLEDGEMENTS

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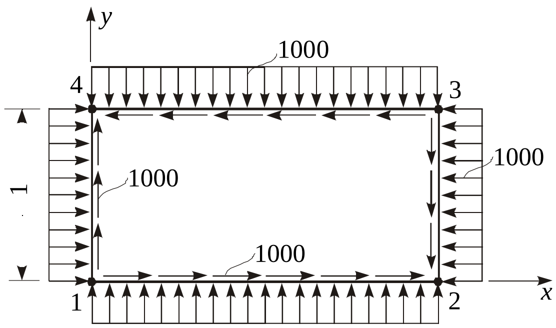


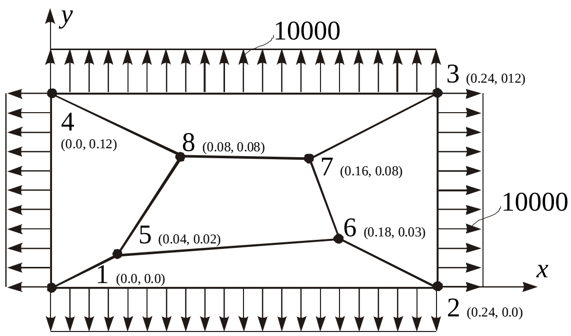
$E = 30\times10^{6}$
$\nu = 0.3$
$t = 1$
Boundary conditions:
$u_{x1} = u_{y1} = u_{y2} = 0$, $u_z = 0$ for all nodes

Results:
Sresses: $\sigma_x = \sigma_y = \sigma_{xy} = -1000$
Strains: $\varepsilon_x = \varepsilon_y = -2.333\times10^{-5}$
$\gamma_{xy} = -8.667\times10^{-5}$
Displacements: $u_x = x\cdot\varepsilon_x + y\cdot\gamma_{xy}$
$u_y = y\cdot\varepsilon_y$

Figure 1. Membrane test on a single flat element – geometry, loading, boundary conditions, and results.

Table 1. Results for Problem 1

Variable	Results		
	Present	Exact	% Error
Normal stress, σ_x	–1000	–1000	0
Normal stress, σ_y	–1000	–1000	0
Shear stress, σ_{xy}	–1000	–1000	0
Normal strain, ε_x	-2.333×10^{-5}	-2.333×10^{-5}	0
Normal strain, ε_y	-2.334×10^{-5}	-2.333×10^{-5}	0.04
Shear strain, γ_{xy}	-8.667×10^{-5}	-8.667×10^{-5}	0
Displacement, u_{x2}	-4.661×10^{-5}	-4.667×10^{-5}	0.13
Displacement, u_{x3}	-1.333×10^{-4}	-1.333×10^{-4}	0
Displacement, u_{y3}	-2.331×10^{-5}	-2.333×10^{-5}	0.09
Displacement, u_{x4}	-8.667×10^{-5}	-8.667×10^{-5}	0
Displacement, u_{y4}	-2.337×10^{-5}	-2.333×10^{-5}	0.17



$E = 1\times10^6$
$\nu = 0.25$
$t = 0.001$
Boundary conditions:
$u_{x1} = u_{y1} = u_{y2} = 0$, $u_z = 0$ for all nodes

Results:
Sresses: $\sigma_x = \sigma_y = 10000$
Strains: $\varepsilon_x = \varepsilon_y = 7.5\times10^{-3}$
Displacements: $u_x = x\cdot\varepsilon_x$, $u_y = y\cdot\varepsilon_y$

Figure 2. The patch test – geometry, loading, boundary conditions, and results.

Table 2. Results for Problem 2

Variable	Results		
	Present	Exact	% Error
Normal stress, σ_x	9916	10000	0.84
Normal stress, σ_y	9923	10000	0.77
Normal strain, ε_x	7.435×10^{-3}	7.500×10^{-3}	0.87
Normal strain, ε_y	7.449×10^{-3}	7.500×10^{-3}	0.68
Displacement, u_{x2}	1.788×10^{-3}	1.800×10^{-3}	0.67
Displacement, u_{x3}	1.789×10^{-3}	1.800×10^{-3}	0.61
Displacement, u_{y3}	8.976×10^{-4}	9.000×10^{-4}	0.27
Displacement, u_{y4}	8.960×10^{-4}	9.000×10^{-4}	0.44
Displacement, u_{x5}	2.979×10^{-4}	3.000×10^{-4}	0.7
Displacement, u_{y5}	1.493×10^{-4}	1.500×10^{-4}	0.47
Displacement, u_{x6}	1.341×10^{-3}	1.350×10^{-3}	0.67
Displacement, u_{y6}	2.242×10^{-4}	2.250×10^{-4}	0.36
Displacement, u_{x7}	1.192×10^{-3}	1.200×10^{-3}	0.67
Displacement, u_{y7}	5.979×10^{-4}	6.000×10^{-4}	0.35
Displacement, u_{x8}	5.953×10^{-4}	6.000×10^{-4}	0.78
Displacement, u_{y8}	5.976×10^{-4}	6.000×10^{-4}	0.4

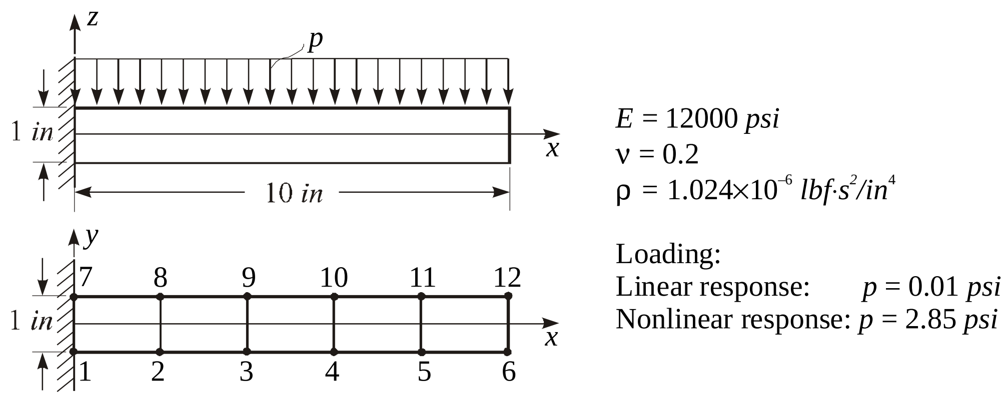
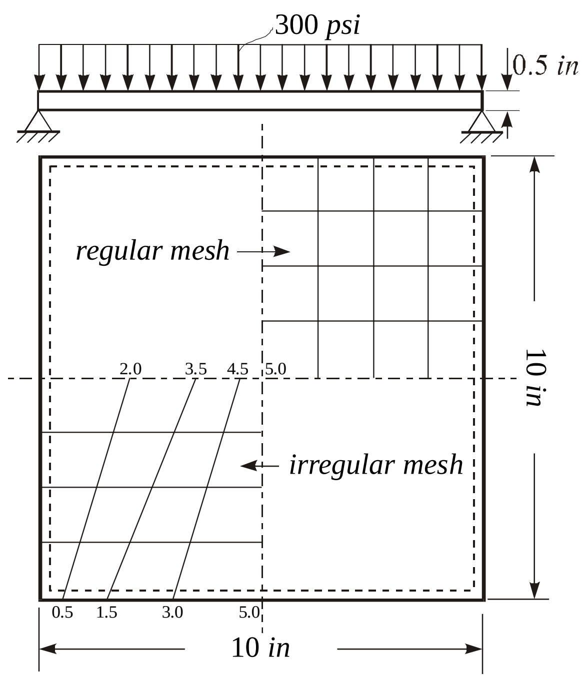


Figure 3. Impulsively loaded beam – geometry, loading, and FE mesh.

Table 3. Results for Problem 3

Analysis	p = 0.01 *psi*		p = 2.85 *psi*	
	Max deflection, *in*	Period, *ms*	Max deflection, *in*	Period, *ms*
Present	0.02495	5.765	6.231	5.770
Belytschko et al. [3]	0.02454	5.680	6.139	5.640
Analytic	0.025	5.719	–	–



$E = 1\times10^7$ *psi*
$\nu = 0.3$
$\rho = 2.588\times10^{-4}$ $lbf{\cdot}s^2/in^4$
$\sigma_{Yield} = 3\times10^4$ *psi*
$E_{plastic} = 0$ *psi*

Figure 4. Simply supported square plate – geometry, loading, regular and irregular FE meshes.

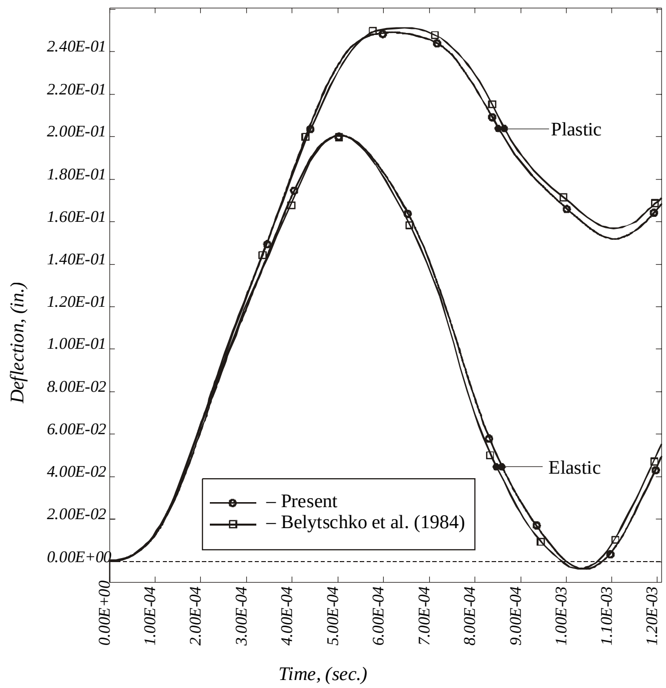


Figure 5. Simply supported square plate – center point deflection for elastic and elastic-perfectly-plastic materials using 5 through thickness integration points

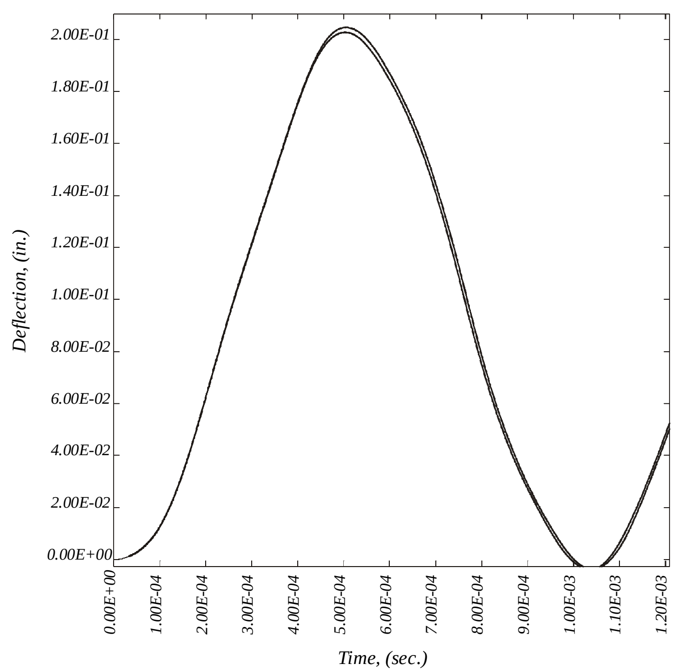


Figure 6. Simply supported square plate – center point deflection for elastic material with regular and irregular FE meshes.

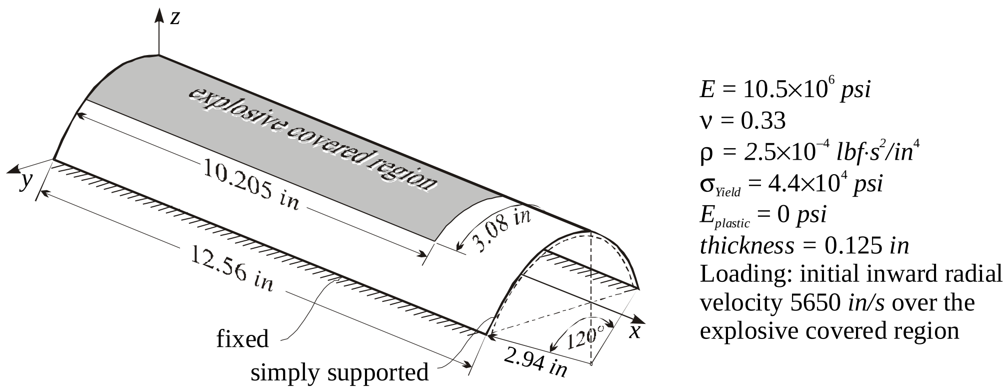


Figure 7. Impulsively loaded cylindrical panel – model description.

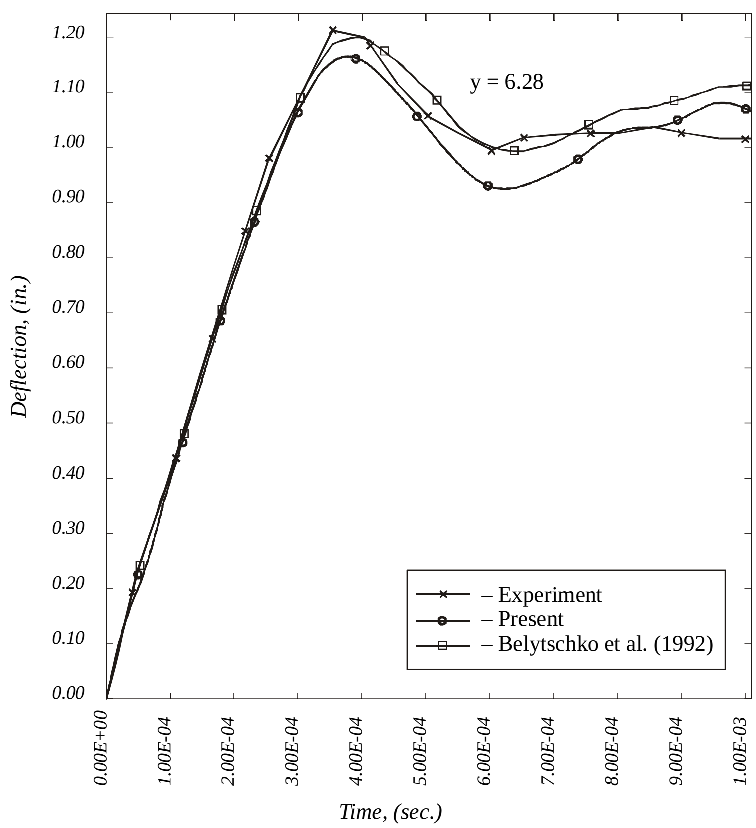


Figure 8. Impulsively loaded cylindrical panel –
deflection curve of a node at (x,y) = (6.28, 0.0)

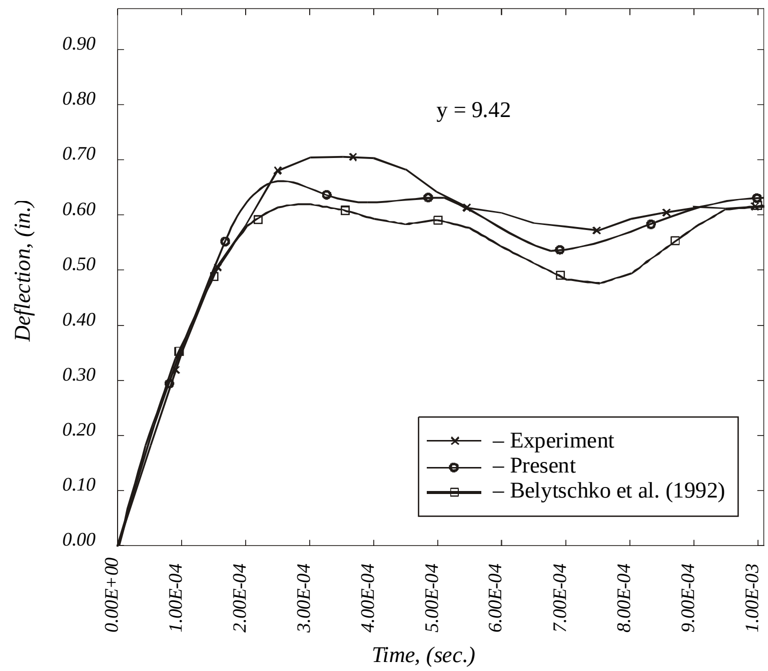


Figure 9. Impulsively loaded cylindrical panel – deflection curve of a node at (x,y) = (9.42, 0.0).

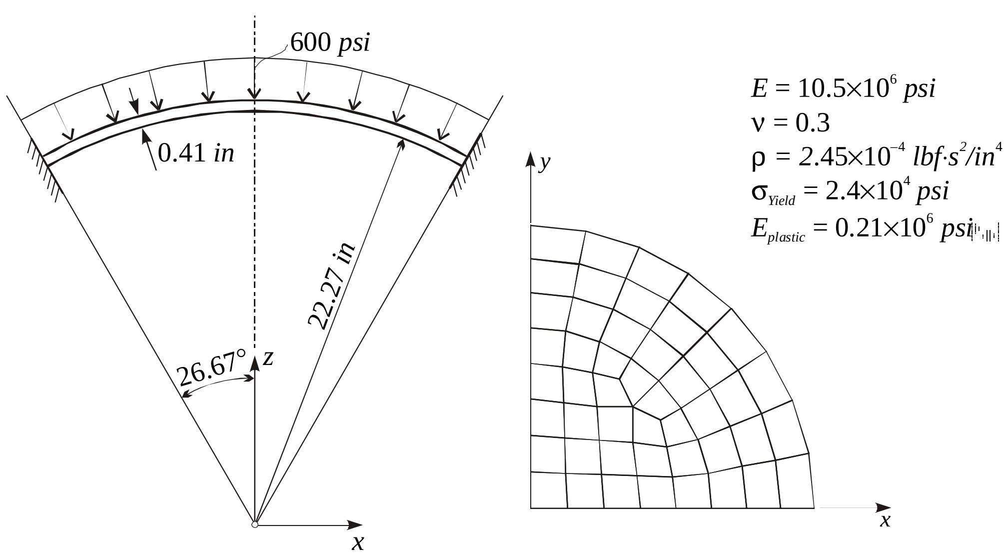


Figure 10. Spherical cap under lateral pressure – model description and FE mesh.

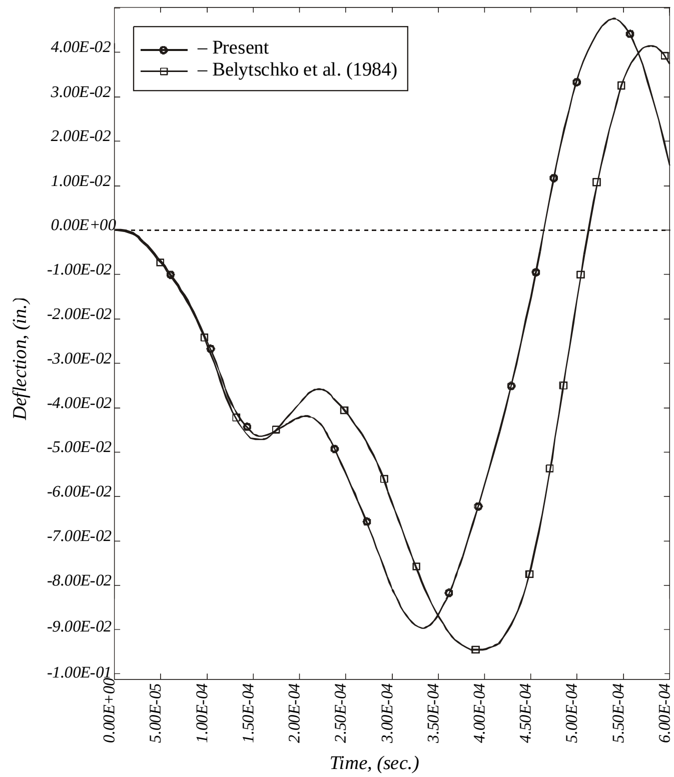


Figure. 11. Spherical cap under lateral pressure –
center point deflection for elastic material.

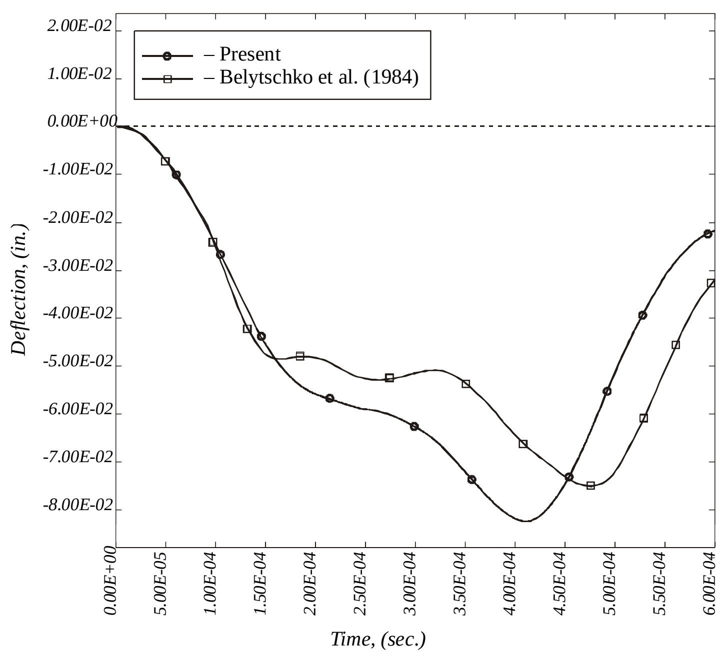


Figure 12. Spherical cap under lateral pressure –
center point deflection for elastic-plastic material.

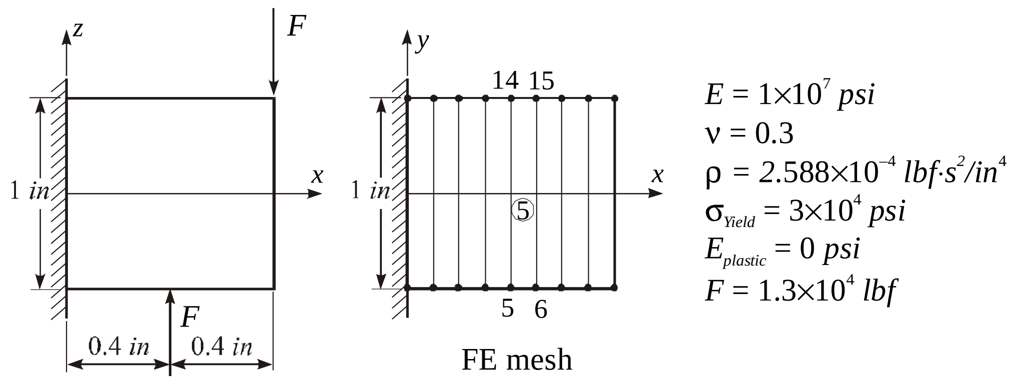


Figure 13. Short beam under shear load – model description and FE mesh.

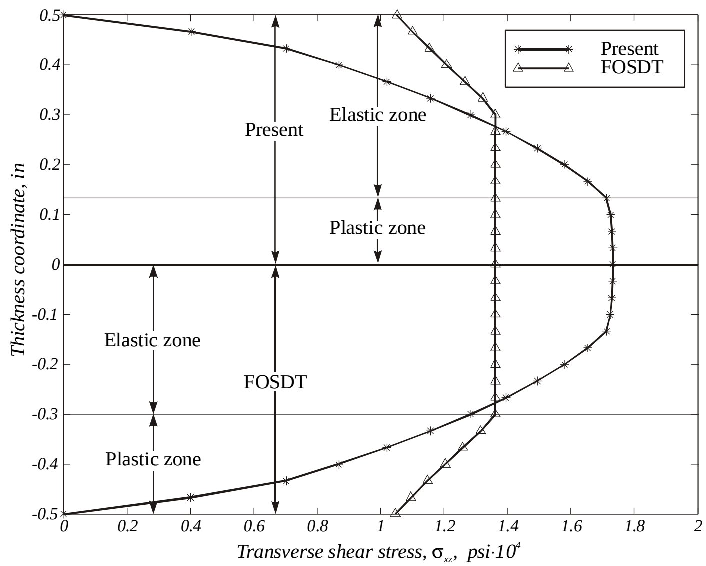


Figure 14. Through-thickness distribution of transverse shear stress of the present approach compared to a first order shear deformable shell element distribution.

Table 4. *Solution CPU Time and Critical Time Step*

Test	Present with selectively reduced integration		FOSDT shell with selectively reduced integration	
	CPU Time, *s*	Critical Δt, μs	CPU Time, *s*	Critical Δt, μs
Problem 3	0.38	3.3	0.13	8.8
Problem 4	1.12	0.84	0.13	6.1
Problem 5	14.0	0.26	1.0	1.8
Problem 6	1.87	0.84	0.18	4.1