SIMULATION OF NONLINEAR VISCO-ELASTICITY

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Abstract

CAE has been applied for the development of various industrial products in many kinds of fields. They are not only products made of metals but also products made of polymeric materials, such as tires, balls for various kinds of sports, press ink rolls, and so on. These materials, such as rubber and synthetic resin have viso-elastic properties. And most of them have nonlinear characteristics. In case of dealing with them, it is very important for accurate simulation to apply nonlinear visco-elasitc material models. Especially it is necessary in the field of Impact. Because impact is in the condition of large strain, or high-speed strain rate, performances as products depend on nonlinearity.

Therefore it is not reasonable for the simulation of impact to input material properties measured by visco-elastic spectrometer well known as a typical visco-elastic measurement. They are in the condition of small strain on the order of few percent and low strain rate on the order of a few [/s]. It is necessary to measure them in large strain on the order of more than ten percent and high strain rate on the order of thousands [/s] same as practical condition.

However there had not been any testers to measure viso-elastic material properties in the condition of large displacements and high deformation rates before. Recently, the split Hopkinson pressure bar, which is originally for evaluation of metals, has been improved for evaluation of polymeric materials. And material properties of some polymeric materials in that condition have been evaluated.

In this study, a nonlinear visco-elastic material model is developed by using the measurement results from the improved split Hopkinson pressure bar. Further, the tests of the split Hopkinson pressure bar are simulated, and good correlation with the experiment is obtained. Finally the restitution tests of golf ball is simulated as an example of application. Agreement between the experiment and the simulation is confirmed.

Introduction

It is well known that deformable behavior under impulsive loading is in the condition of high strain rate. It is different from behavior under static loading. There are many products made of not only metals but also polymeric materials such as rubber and synthetic resin. They are typical visco-elastic materials and particularly occur large deformation in many cases.

Behavior under impulsive loading is a phenomenon during so short time that ends in a few or less than microseconds. In addition to a high-speed phenomenon, some cases using with soft materials occur large deformation on the order of more than ten [%]. Most of them are visco-elastic materials. And accurate simulation under impulsive loading is necessary for effective development of industrial products made of visco-elastic materials.

In general, visco-elastic spectrometer is used for evaluation of visco-elastic materials, such as rubber, synthetic resin, and other polymeric materials. However, material properties measured by it are sometimes not suitable for analysis under impulsive loading. The reason of the inadequacy is that material properties measured by it are much different from material properties in the practical condition. They are in the condition of small strain on the order of 0.1Å`1 percent and low strain rate on the order of 0.1Å`1[/s]. This condition is much different from practical condition under impulsive loading, in the range of strain around 10 to 30 percent, and in the range of strain rate hundreds to thousands [/s]. Because suitable measurement apparatuses had not existed, the material properties measured by visco-elastic spectrometer or other static measurement testers had been used in the analysis.

However, the split Hopkinson pressure bar, which is originally for evaluation of metals, has been improved for evaluation of visco-elastic materials under impulsive loading. (reference 1, 2) And material properties of some polymeric materials in the ranges as mentioned above have been evaluated. They are very effective for not only the evaluation of impact properties of products made of them but also the simulation of the impact analysis for them.

Some of polymeric materials have nonlinear characteristics depending on the range of strain, or the range of strain rate. Therefore it is very important for effective development of such products to develop visco-elastic material models which can express nonlinear characteristics in large and high-speed deformable condition.

In this study, the visco-elastic material model that can express nonlinear characteristics is developed by using measurement results from the split Hopkinson pressure bar modified for visco-elastic materials. And tests of the split Hopkinson pressure bar are simulated to prove adequacy. Finally restitution tests of a golf ball are simulated as an example of application. And good agreement between the experiment and the calculation is obtained.

Approach

Measurement Instrument

Figure 1 shows the schematic diagram of the impact compressive tester, the modified split Hopkinson pressure bar. It was used for measurement of visco-elastic properties in this study. It is improved upon the split Hopkinson pressure bar that was originally for evaluation of metals. It is dealt as a uniaxial phenomenon by using long and slender instrumental bars. Originally they are made of metals to evaluate metallic materials.

At first a collision between an incident bar and a striker one launched by air gun produces a stress wave. Secondly the stress wave propagating through the incident bar causes other collision between the incident bar and a specimen. Finally the last collision between the specimen and a transmitted bar occurs. Electrical resistance strain gauges are attached to the incident bar and the transmitted one to determine the strain wave through them. Strain and stress of the specimen are calculated by using the data of strain gauges and one dimensional wave propagation theory.

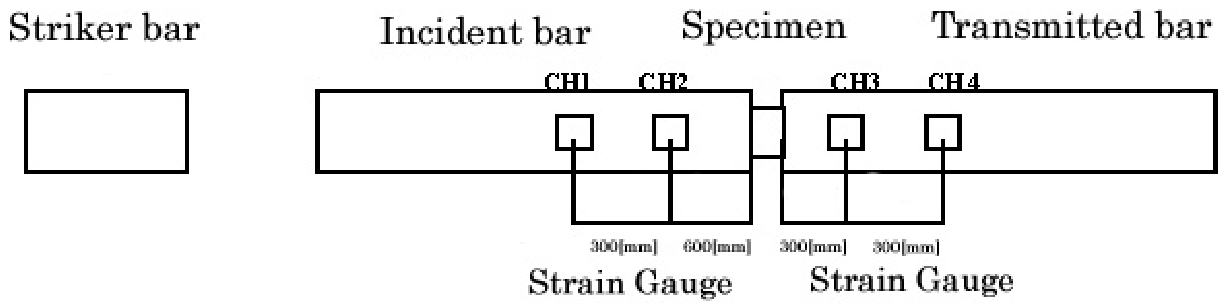

Fig.1 Schematic Diagram of the Impact Compressive Tester

There are some modifications below to measure softer materials.

• The incident bar and the transmitted one made of an acrylic material PMMA(poly-methyl methacrylate resin) are adopted instead of metals in order to minimize the reflection wave due to the difference of the acoustic impedance between these ones and specimen.

(Acoustic impedance is defined as ρ C. ρ:density, C: speed of sound in a solid)

• Unavoidable overlaps of waves are revised.

• 4 gauges are adopted to take dispersion effects in account.

• Longer instrumental bars whose length is 2[m] are adopted to avoid overlaps of waves due to end effects in the incident wave.

Figure 2 shows an example of the strain data directly measured by 4 gauges attached to the incident bar and the transmitted one in a test. Figure 3 shows the propagation of uniaxial wave in the impact compressive tester. Table 1 shows the details of the measurement apparatus in this study.

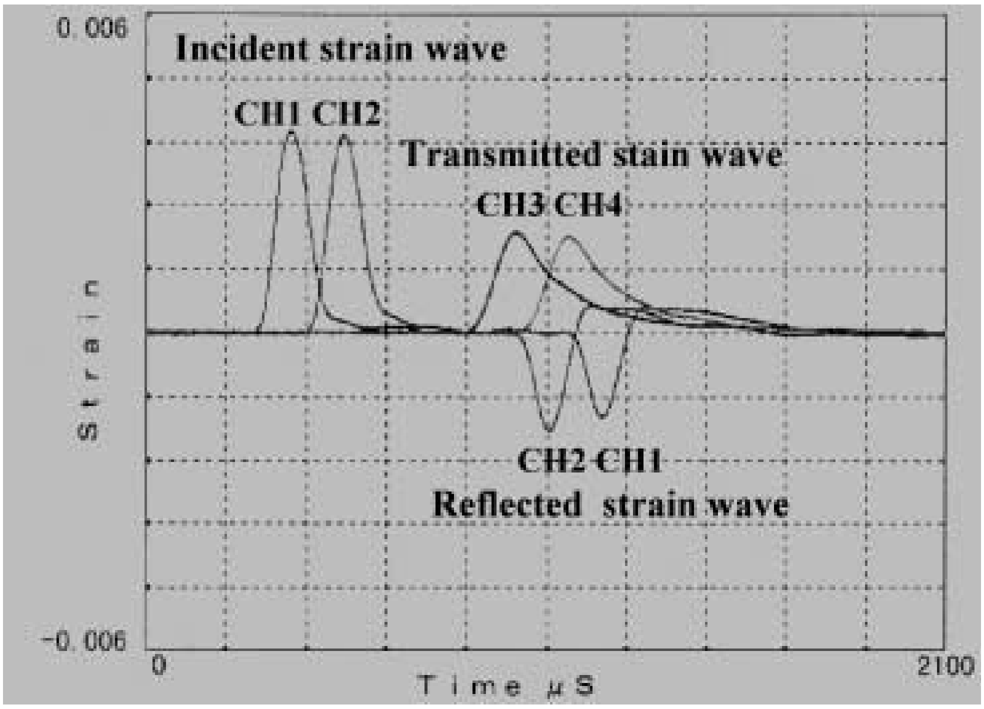

Fig.2 Strain Wave Data

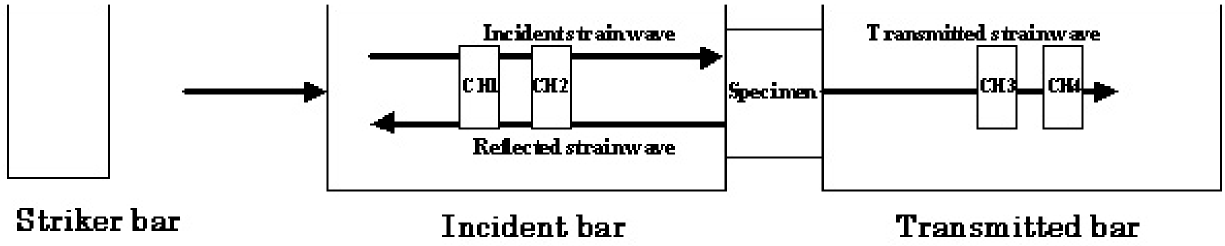

Fig.3 Propagated Strain Wave

Table 1. The Details of the Measurement apparatus

	Material	Dimension
Striker Bar	PMMA	ϕ20[mm]×100[mm]
Incident Bar	PMMA	ϕ20[mm]×2000[mm]
Transmitted Bar	PMMA	ϕ20[mm]×2000[mm]
Specimen	An Elastomer	ϕ18[mm]×4[mm]

The result of the measurement and the calculation of visco-elastic characteristic

Strain and stress of specimen are obtained by means of some process of the strain data from 4 gauges. Figure 4 shows an example of strain data of a specimen. Figure 5 shows an example of stress data of it. The effects of reflection waves are removed in them. Figure 6 shows an example of the curve of stress versus strain response of it.

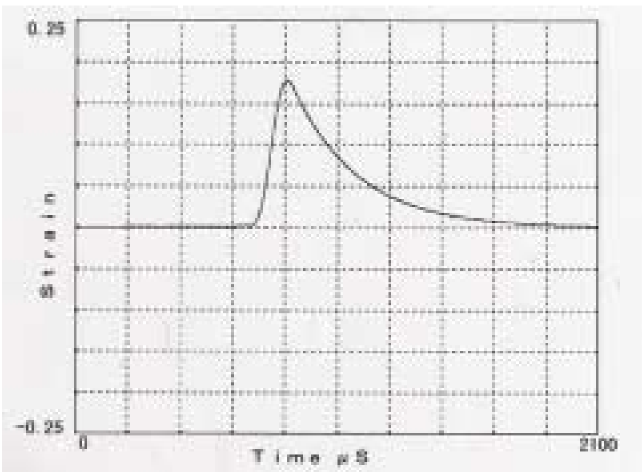

Fig.4 Strain data of Specimen

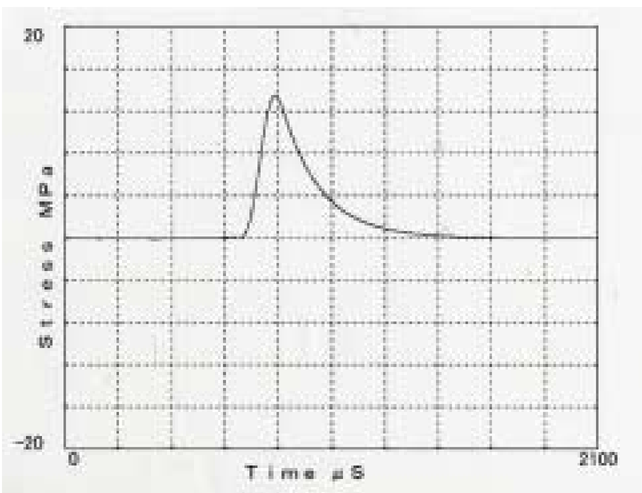

Fig.5 Stress data of Specimen

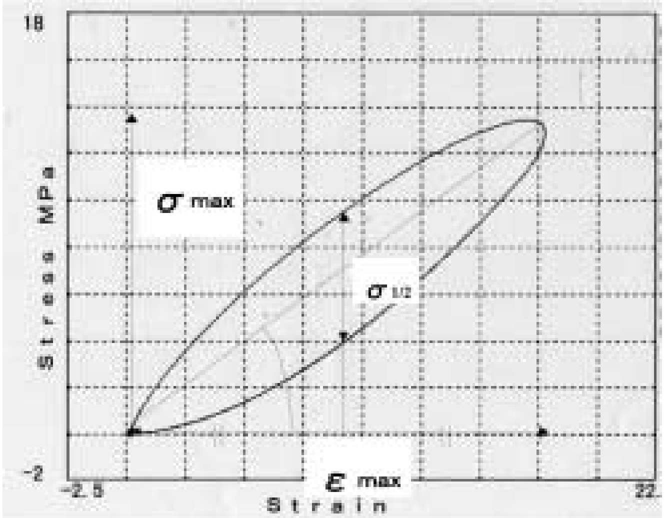

Fig.6 Stress-Strain Curve

Loss factor is related to viscosity. It is one of the characteristics of visco-elastic materials. It is defined as below to evaluate the material properties.

$$\text{Phase Angle } (\delta) = \sin^{-1}(\sigma_{1/2} / \sigma_{max}) \qquad (1)$$

$$\text{Loss Factor} = \tan\delta \cdots\cdots (2)$$

In this study, an elastomer was measured by using the split Hopkinson pressure bar in 4 cases. They are different from striker bar's velocity, 7[m/s], 14[m/s], 20m/s], and 25[m/s]. Figure 7,8,9,10 show the results of stress versus strain response in 4 cases. Table 2 shows values of loss factor in these 4 cases and visco-elastic spectrometer's measurement. Figure 11 shows the transition of values in 4 cases. The values with the split Hopkinson pressure bar are much different from value with visco-elastic spectrometer as shown in table 2. The difference is due to the condition. The maximum value of loss factor with the split Hopkinson pressure bar measurement is about 8 times higher than the value with visco-elastic spectrometer. The higher striker bar's velocity is, the higher the value of loss factor is. And transition of the values is nonlinear in the case of this test sample as shown in Figure 11. Some products made of visco-elastic materials are affected by this nonlinear characteristic of loss factor.

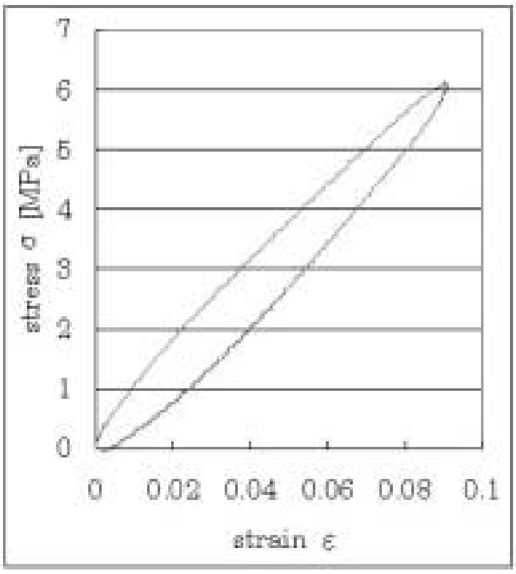

Fig.7 Stress-Strain Curve
(Velocity of Striker bar 7[m/s])

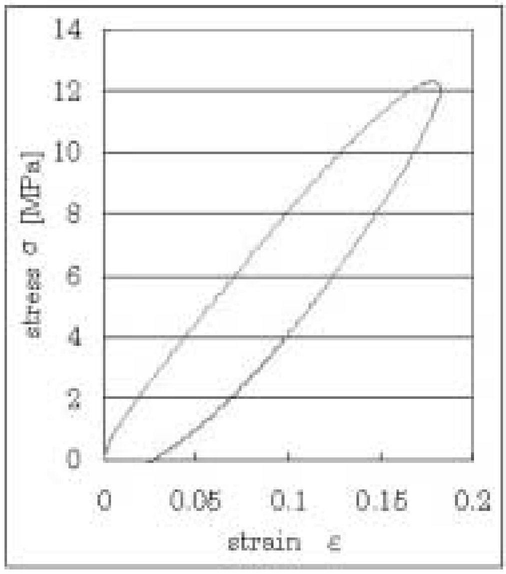

Fig.8 Stress-Strain Curve
(Velocity of Striker bar 14[m/s])

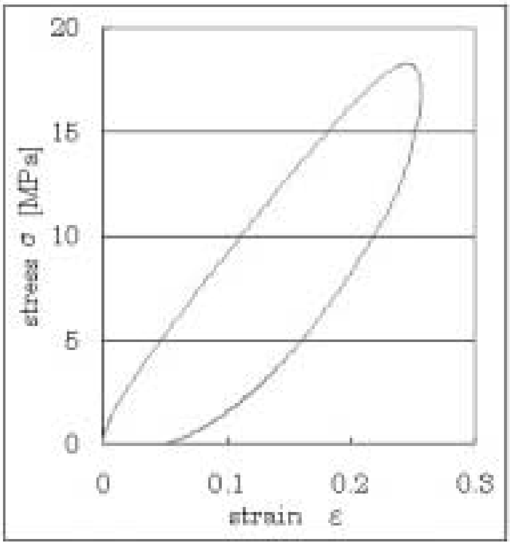

Fig.9 Stress-Strain Curve
(Velocity of Striker bar 20[m/s])

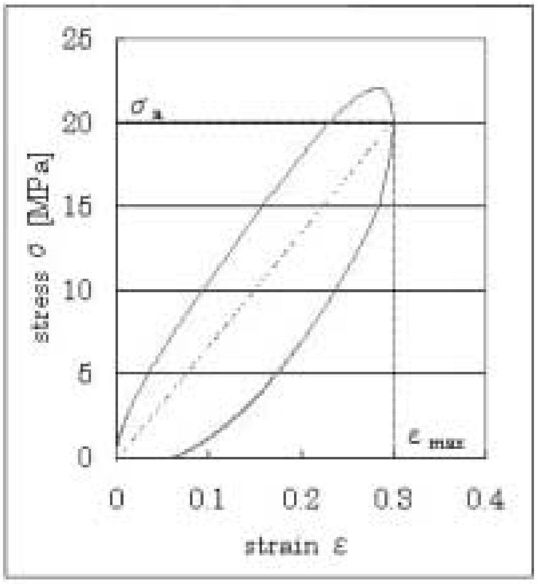

Fig.10 Stress-Strain Curve
(Velocity of Striker bar 25[m/s])

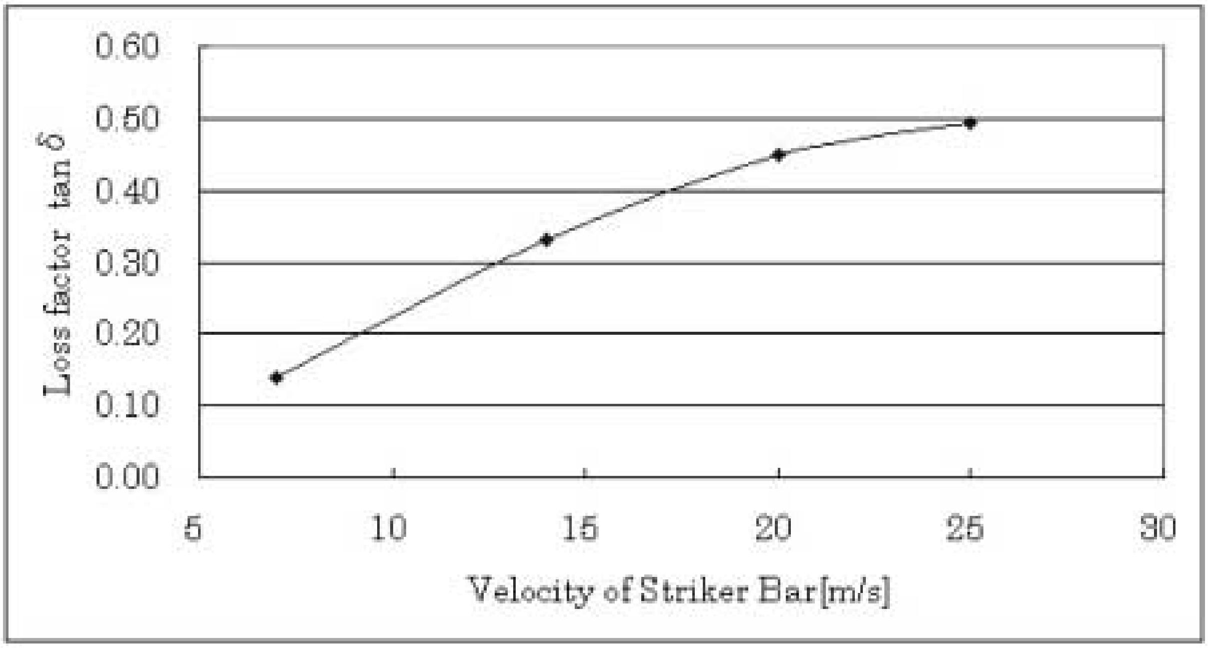

Table2. Loss Factor tan δ of a test sample

Fig.11 Loss Factor tan versus Velocity of Striker Bar

Visco-elastic material model

In this study, the simple Voigt visco-elastic model that consists of a viscous damping and a spring was adopted as the material model as shown in Figure 12.

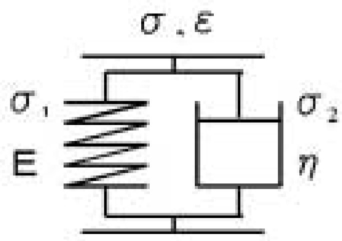

Fig.12 Voigt model

The value, coefficient of viscous damping “η” is changeable. Total stress “σ” of this model is expressed by stress of viscous damping “ 1” and stress of spring “ 2”.

$$\sigma = \sigma_1 + \sigma_2 = E\varepsilon + \eta\dot{\varepsilon} \qquad (3)$$

The values, coefficient of viscous damping “ η”, and stiffness of spring “E”, are calculated by means of the data of strain and stress measured by using the split Hopkinson pressure bar. Modulus of spring “E” is calculated as below.

$$E = \sigma_a / \varepsilon_{max} \qquad (4)$$

The inclination drawn by the dotted line as shown in Figure 10 express “E”.

“ η” is also calculated by these 2 equations above. It is calculated every moment in each case of measurement.

“ η” depends on not only strain but also strain rate. Figure 13 shows the curves of strain rate versus strain in 4 cases. Figure 14 shows the curves of coefficient of viscous damping versus strain in 4 cases.

The value of Coefficient of viscous damping “ η” is determined by referring to the relationship between strain, strain rate and itself as shown in Figure13 and 14.

This method as mentioned above is equivalent to define coefficient of viscous damping “ η” as a function of strain and strain rate.

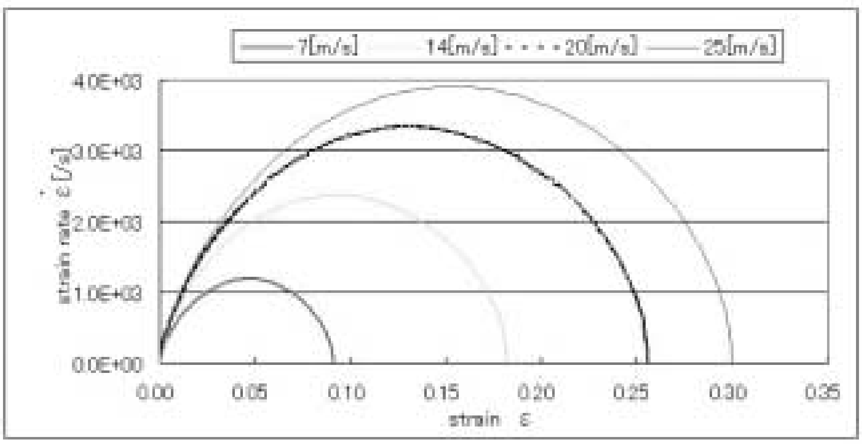

Fig.13 Strain-Strain Rate Curve of 4 cases

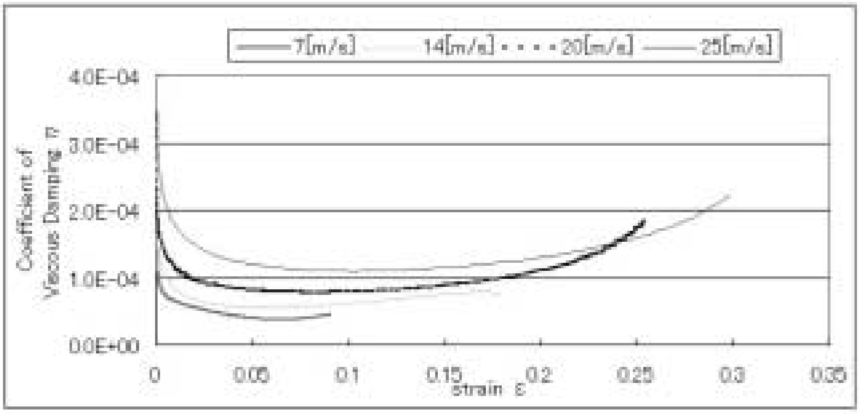

Fig.14 Strain-Coefficient of Viscous Damping Curve of 4 cases

Discussion of Results

The simulation of the split Hopkinson pressure bar measurement

The split Hopkinson pressure bar measurement was simulated to prove the adequacy of the visco-elastic material model as mentioned above.

Analysis model

Figure 15-1,2,3 show the analysis model of this simulation. The incident bar, the transmitted one, and the specimen between them were modeled as the quarter models with the symmetrical boundary condition. The striker bar was not modeled.

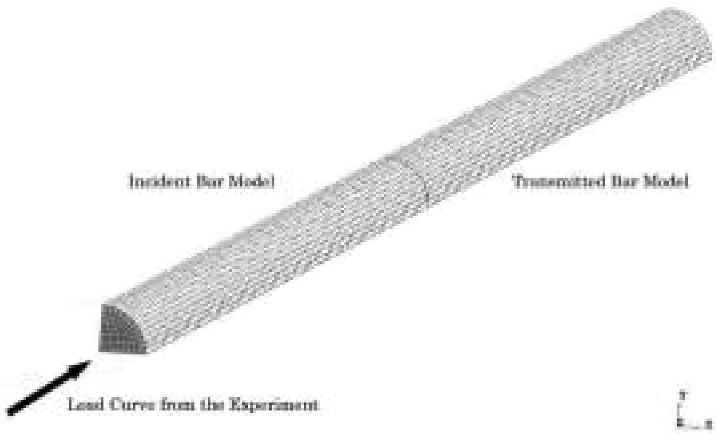

Fig.15-1 Analysis Model of the Split Hopkinson Bar

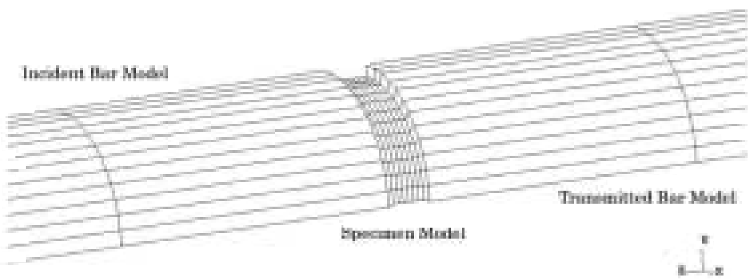

Fig.15-2 Specimen Model between Incident Bar Model and Transmitted Bar Model

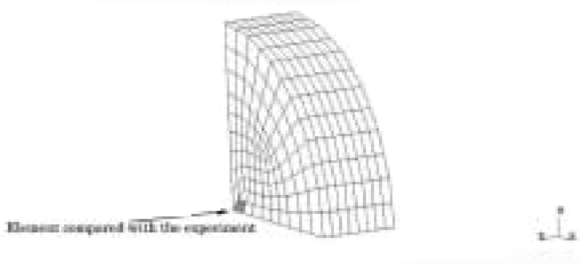

Fig.15-3 Element in the Specimen for comparison with the Experiment

Analysis condition

Instead of the striker bar, the load curve data of stress wave from the experiment was loaded on the end of the incident bar touching with it. The stress wave propagated through the incident bar and transmitted to the specimen.

The results of stress and strain of the element in the specimen model as shown in Figure 15-3 were compared with the results of stress and strain data in the experiment.

Simulation results

The simulation results of 2 cases are shown as some examples of the comparison between simulation and experiment. Figure 16 shows the results of the experiment and the simulation in the case of the striker bar's velocity 14[m/s]. Figure 17 shows the comparison in the case of the striker bar's velocity 25[m/s]. The vibration due to reflection waves at the descending phase is not removed.

We could confirm agreement between the experiment and the simulation as shown in Figure 16 and 17. The loops expressed by the curves of stress versus stress are related to loss factor of visco-elastic materials. It is proved that the visco-elastic material model could express nonlinearity of loss factor depending on the range of strain and strain rate.

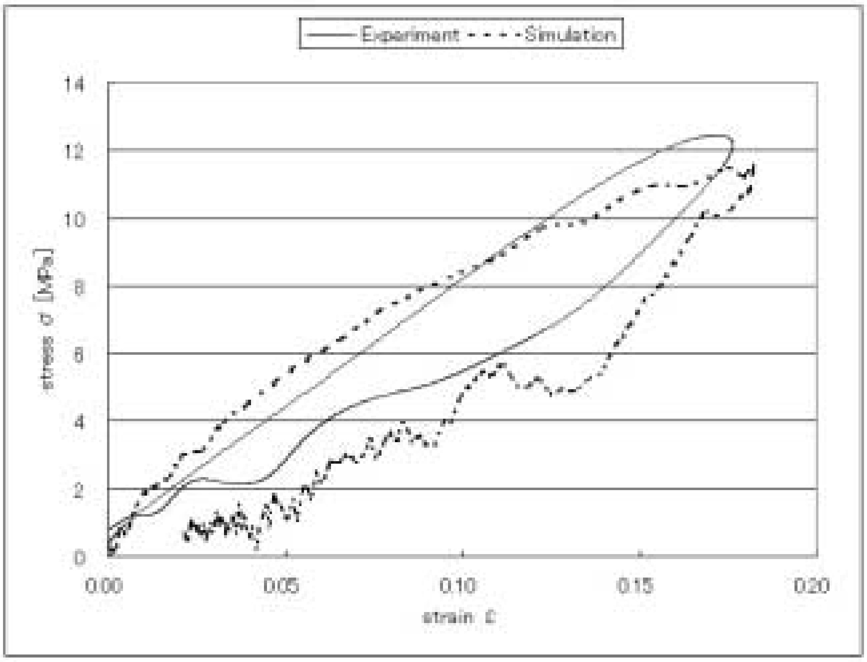

Fig.16 Stress-Strain Curve (Velocity of Striker is 14[m/s])

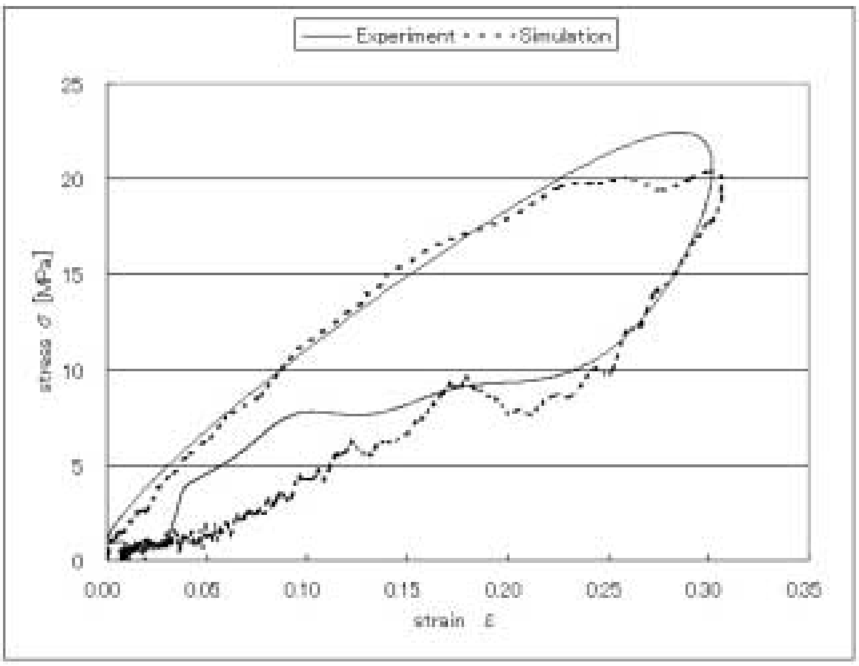

Fig.17 Stress-Strain Curve (Velocity of Striker is 25[m/s])

Example of application

The characteristics of visco-elastic materials is not negligible for the development of products made of them, such as tires, balls for various kinds of sports, press ink rolls, and so on, because it affects the performances of them. It is very important in the case of involving nonlinear characteristics. Therefore visco-elastic material model developed above is effective.

In this study, the restitution tests of a golf ball made of the visco-elastic material simulated in the above section were simulated as an example of the application. Restitution is one of the most important performances of golf balls. 2 objects, an aluminum cylinder and a golf ball are used in the test. A mean to measure their coefficient of restitution is that an aluminum cylinder launched by air gun impacts a golf ball and their velocities are measured before and after the impact by using laser beam.

Analysis model

Figure 18 shows the analysis model of the restitution tests of golf ball. The weight of the aluminum cylinder is almost same as one of usual golf club heads. The golf ball is 42.78[mm] in diameter.

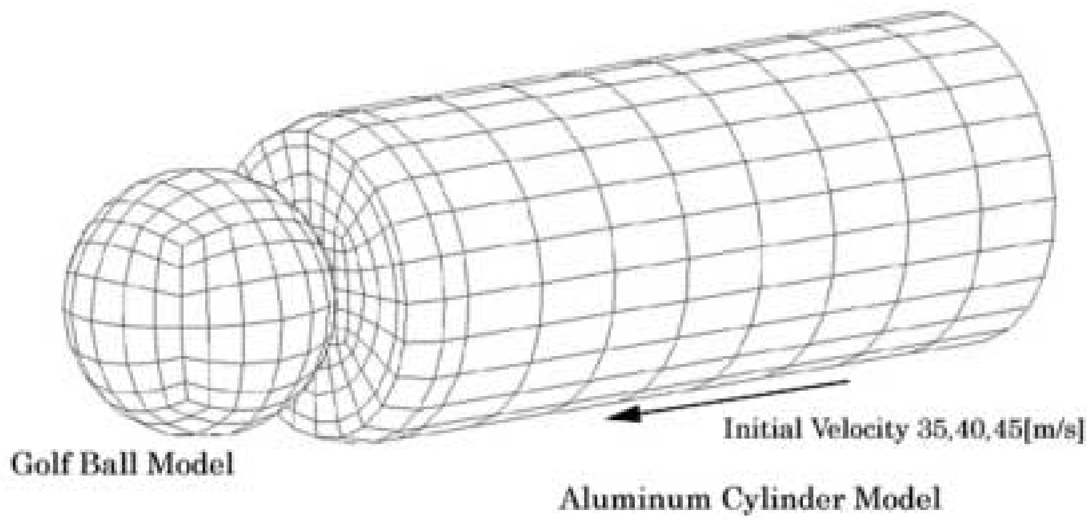

Fig.18 Analysis Model of Restitution Test of Golf Ball

Analysis condition

Three cases of the restitution tests were measured. The difference is the velocity of the aluminum cylinder before impact, 35[m/s], 40[m/s] 45[m/s]. In the simulation, the aluminum cylinder has initial velocity toward the golf ball model same as the experiment.

The values of coefficient of restitution were compared between the experiment and the simulation.

Simulation results

Figure 19 shows the results of the experiment and the simulation. There are two cases of simulation. Simulation 1 is used the usual visco-elastic material model (Mat.6) and material properties measured by a typical visco-elastic measurement apparatus, visco-elastic spectrometer. Simulation 2 is used the visco-elastic material model developed to express nonlinearity in this study.

The values of simulation 1 are much higher than the experiment, because it can not take large loss factor in the condition of large and high-speed deformation in account. And it cannot express transition of coefficient of restitution depending on the cylinder's velocity adequately.

The values of simulation 2 are much closer to the experiment than simulation 1. And it can express transition of coefficient of restitution depending on the cylinder's velocity more adequately than simulation 1. These results proved the effectiveness of the developed visco-elastic material model in this study.

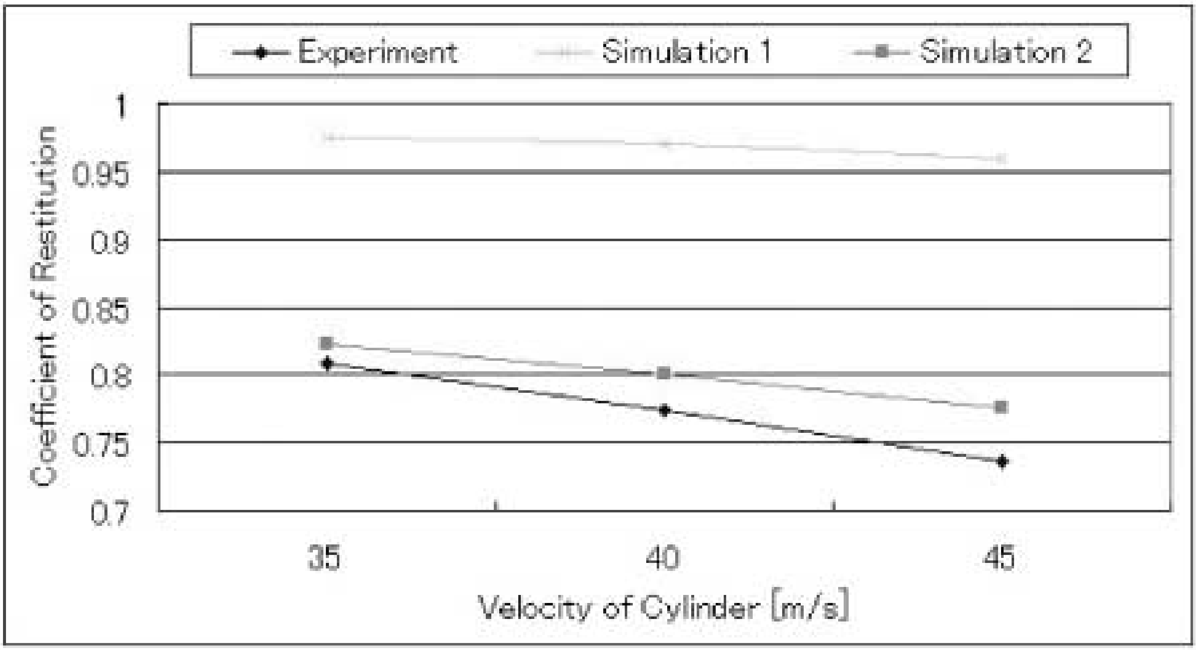

Fig.19 Coefficient of Restitution versus Velocity of Cylinder

Conclusion

In this study, a visco-elastic material model that can express nonlinear characteristics, such as loss factor, was developed by using the measurement results from the split Hopkinson pressure bar improved for visco-elastic materials. Further, the tests of the split Hopkinson pressure bar were simulated, and good correlation with experiment was obtained. Finally as an example of the application of industrial products, the restitution tests of golf ball were simulated. And the transition of coefficient of restitution due to nonlinear viso-elastic characteristics was expressed in the simulation same as the experiment. The effectiveness of the developed visco-elastic material model was proved in large displacements and high deformation rates condition.

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