S.P.H. :
A SOLUTION TO AVOID USING EROSION CRITERION?

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Abbreviations:

S.P.H. **S**moothed **P**article **H**ydrodynamics

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ABSTRACT

A new particle element has been added to LS-DYNA. It is based on Smoothed Particle Hydrodynamics theory. SPH is a meshless lagrangian numerical technique used to modelize the fluid equations of motion. SPH has proved to be useful in certain class of problems where large mesh distortions occur such as high velocity impact, crash simulations or compressible fluid dynamics.

First, the basis principles of the SPH method will be introduced. Then, the model of perforation of a bullet through a thin plate will be presented. Two models are realised: one is made of lagrangian brick elements only, and the second one uses SPH elements for the plate. Finally, a discussion is proposed on the different methods used to deal with the penetration problem.

INTRODUCTION

Meshfree methods have known sizeable developments these last years for solving conservation laws.

S.P.H. (Smoothed Particle Hydrodynamics) is a meshfree lagrangian method developed initially to simulate astrophysical problems. But, the easy way with which it is possible to introduce sophisticated phenomena, has made of SPH a very interesting tool to resolve other physic problems: resolution of continuum mechanics, crash simulations, ductile and brittle fractures in solids.

The high handiness of SPH allows the resolution of many problems that are hardly reproducible with classical methods. It is very simple to obtain a first approach of the kinematics of the problem to study. For instance, due to the absence of mesh, one can calculate problems with large irregular geometry.

From a computational point of view, we represent a fluid with a set of moving particles evolving at the flow velocity. Each SPH particle represents an interpolation point on which all the properties of the fluid are known. The solution of the entire problem is then computed on all the particles with a regular interpolation function, the so-called smoothing length. The equations of conservation are then equivalent to terms expressing flux or inter-particular forces.

In this paper, the basis principles of the method are first given. Then, the penetration of a bullet through a thin plate is considered. In order to simulate this problem, two kinds of methods are tested. Firstly, the target is meshed with finite elements and an erosion criterion is applied. Comparison is done with the second model where the target is meshed with SPH elements.

BASIS PRINCIPLES OF THE SPH METHOD

Particle methods are based on quadrature formulas on moving particles $(x_i(t), w_i(t))_{i \in P}$.

P is the set of the particles, *xi(t)* is the location of particle *i* and *wi(t)* is the weight of the particle. We classically move the particles along the characteristic curves of the field *v* and also modify the weigths with the divergence of the flow to conserve the volume :

$$(i)\ \frac{d}{dt}x_i = v(x_i, t) \qquad (ii)\ \frac{d}{dt}w_i = div(v(x_i, t))w_i \tag{1.1}$$

We can then write the following quadrature formula :

$$\int_\Omega f(x)dx \approx \sum_{j \in P} w_j(t) f(x_j(t)) \tag{1.2}$$

Particle approximation of function

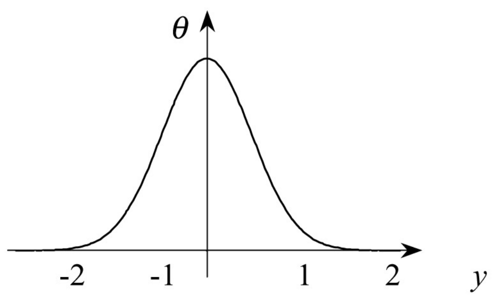
The previous quadrature formula together with the notion of smoothing kernel leads to the definition of the particle approximation of a function.

To define the smoothing kernel, we need first to introduce an auxiliary function q. The most useful function used by the SPH community is the cubic B-spline which has some good properties of regularity.

It is defined by:

$$\theta(y) = C \times \begin{cases} 1 - \frac{3}{2}y^2 + \frac{3}{4}y^3 & \text{for} \quad y \le 1 \\ \frac{1}{4}(2-y)^3 & \text{for} \quad 1 < y \le 2 \\ 0 & \text{for} \quad y > 2 \end{cases} \tag{2.1}$$

where *C* is the constant of normalization that depends on the space dimension.



We have then enough elements to define the smoothing kernel *W*:

$$W(x_i - x_j, \bar{h}) = \frac{1}{\bar{h}}\theta\left(\frac{x_i - x_j}{\bar{h}}\right) \tag{2.2}$$

$W(x_i - x_j, \bar{h}) \longrightarrow \delta$ when $\bar{h} \longrightarrow 0$, where δ is the Dirac function. $\bar{h}$ is a function of x_i and x_j and is the so-called smoothing length of the kernel.

We can now define the particle approximation $\Pi^h u$ of the function *u*, by approximating the integral (1.2):

$$\Pi^h u(x_i) = \sum_{j \in \Omega} w_j(t) u(x_j) W(x_i - x_j, \bar{h}) \tag{2.3}$$

The approximation of gradients is obtained by applying the operator of derivation on the smoothing length. We then obtain :

$$\nabla\Pi^{h}u(x_i) = \sum_{j\in\Omega} w_j(t)u(x_j)\nabla W(x_i - x_j, \overline{h}) \tag{2.4}$$

PERFORATION OF A BULLET THROUGH A THIN PLATE

The case of a bullet impacting a thin target is considered here. The projectile is a 5.56 mm caliber bullet and the target is a 1,5mm thick titanium plate. The projectile has several angles of impact with a velocity of 1010 m/s. However, in this paper only the impact with a 90° angle of incidence is displayed. The difficulty of this test is the good modelisation of the plate's failure. Two solution seems to be convenient: on the first hand, the lagrangian method with an erosion criterion, on the other hand the use of the SPH elements.

For this model, two comparison parameters have been chosen :

- the residual velocity of the projectile
- the shape of the deformed plate

Moreover, an experimental database for the residual velocity is avalaible.

As shown during experimental trials, very few deformation of the projectile have been noticed, the projectile is then assumed to be rigid in the whole calculation.

Numerical model using brick elements with erosion

Two different models are compared. The first one is only realised with brick elements. The projectile is modeled with 33250 brick elements. The target is modeled with 72900 elements. We apply some non-reflecting boundary elements around the plate to simulate an infinite plate. The material data are summed up in the following list:

Projectile	Density 11340 kg/m^3	Young's modulus 112.82 GPa	Poisson ratio 0.30
Target	Density 4510 kg/m^3	Shear modulus 43.4 GPa	Yield stress 0.85 GPa

The projectile is modeled as a rigid body. The plate is modeled using the material elastic plastic hydrodynamic.

In order to represent the perforation of the projectile through the plate, an erosion criterion on the plate is applied. Two tests are performed. One is realised by setting the failure strain for erosion at 1.5 , and the other one by setting this value to 0.5.

Numerical model using SPH elements

In this model, the center of the plate is meshed with SPH elements. Around these elements a set of brick elements is defined. A tied node to surface contact is used between the SPH elements and the brick elements, and an automatic node to surface contact is defined between the particles and the projectile. As in the lagrangian model, some non-reflecting boundaries are defined.

The default values are used for the SPH control and section cards.

The model is realized with 44100 particles in the center of the plate with 9 particles in the thickness of the plate. Some previous tests have shown that 9 particles are enough to represent the deformations of the plate during the perforation.

The main difference with the lagrangian model is that in this case no erosion criterion is necessary. Indeed, the projectile is expected to push away the particles and then to create the crack in the target.

Results

The different models have been run until the projectile has reached a constant velocity. This termination time is 30 microseconds for the two lagrangian models and 25 microseconds for the SPH model.

In both models, the residual velocities obtained are in good agreement with the experimental data. Nevertheless, in the lagrangian models, the final shape of the target is highly related to the value of the erosion criterion as shown on figures 1 and 2. As no erosion criterion is necessary with the SPH technique to simulate failure phenomena, the problem of modelisation that occurs with the lagrangian formalism is not raised here (figure 3).

Conclusions

A new particle element has been added to LS-DYNA. It is based on Smoothed Particle Hydrodynamics theory. In this paper, the use of SPH for modelling perforation problems where failure occurs is described.

If the only evaluation criterion was based on the residual velocity of the projectile, both lagrangian and SPH methods used here would be convenient.

However, if the shape of the plate was another evaluation point, without experimental trials data, the SPH seems to be more suitable.

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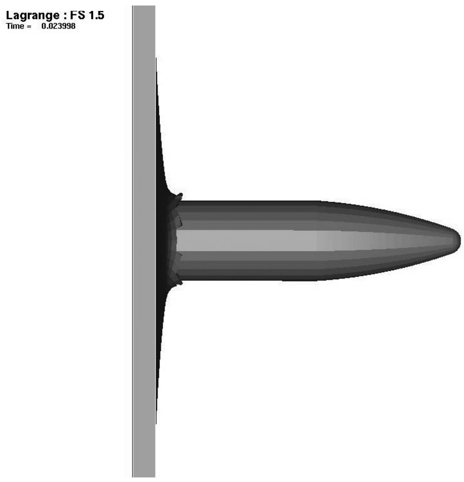

Figure 1: Lagrange model, Failure strain = 0.5

Figure 2: Lagrange model, Failure strain = 1.5

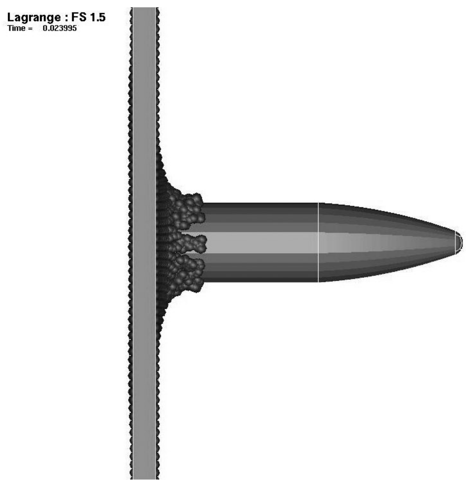

Figure 3: SPH model